2. Conceptual Modeling in UML and First-Order Logic Formalization

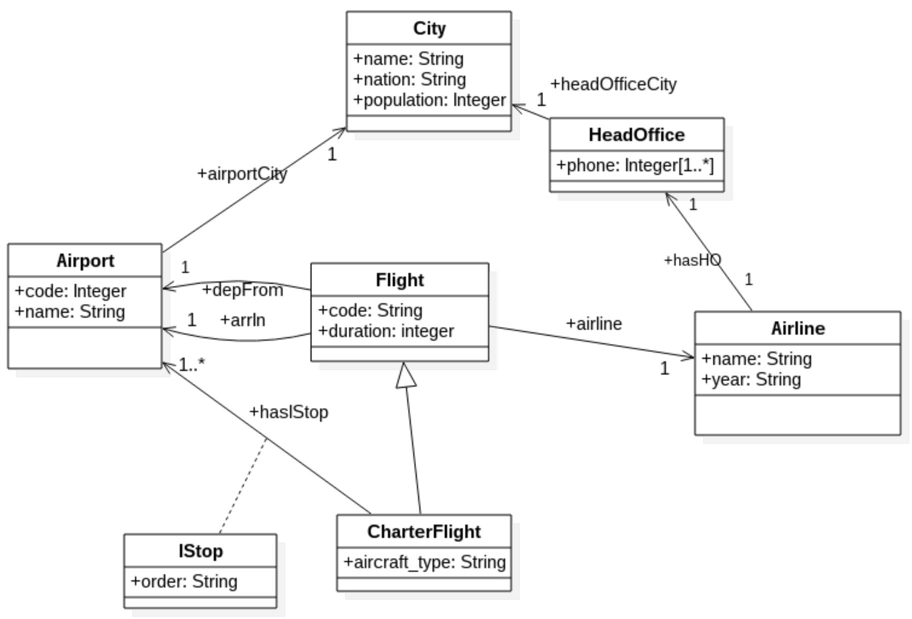
Consider the following description of the information of interest about a **flight** domain.

- The information about each *flight* includes: code, duration in minutes, airline, airport of departure, and airport of arrival.
- The information about each *airport* includes: code, name, city (with name and population), and nation.
- The information about each *airline* includes: name, year of establishment, and city of the head office with its phone number. It is assumed that every office has at least one phone number.
- *Charter flights* are a special kind of flights that foresee intermediate stops at airports. The order of the stops is of interest (for example, flight 124 starting from "Bolzano Airport" and arriving in "Palermo Punta Raisi" has "Rome Ciampino" as intermediate stop 1 and "Napoli Airport" as intermediate stop 2). What is also of interest is the type of the aircraft.

Given the above description, do the following:

Exercise 2.1 Model the flight domain in UML.

Solution:



Exercise 2.2 Formalize the UML diagram in first-order logic.

Solution:

Class Hierarchies

- $\forall F.Charter(F) \rightarrow Flight(F)$

Attributes

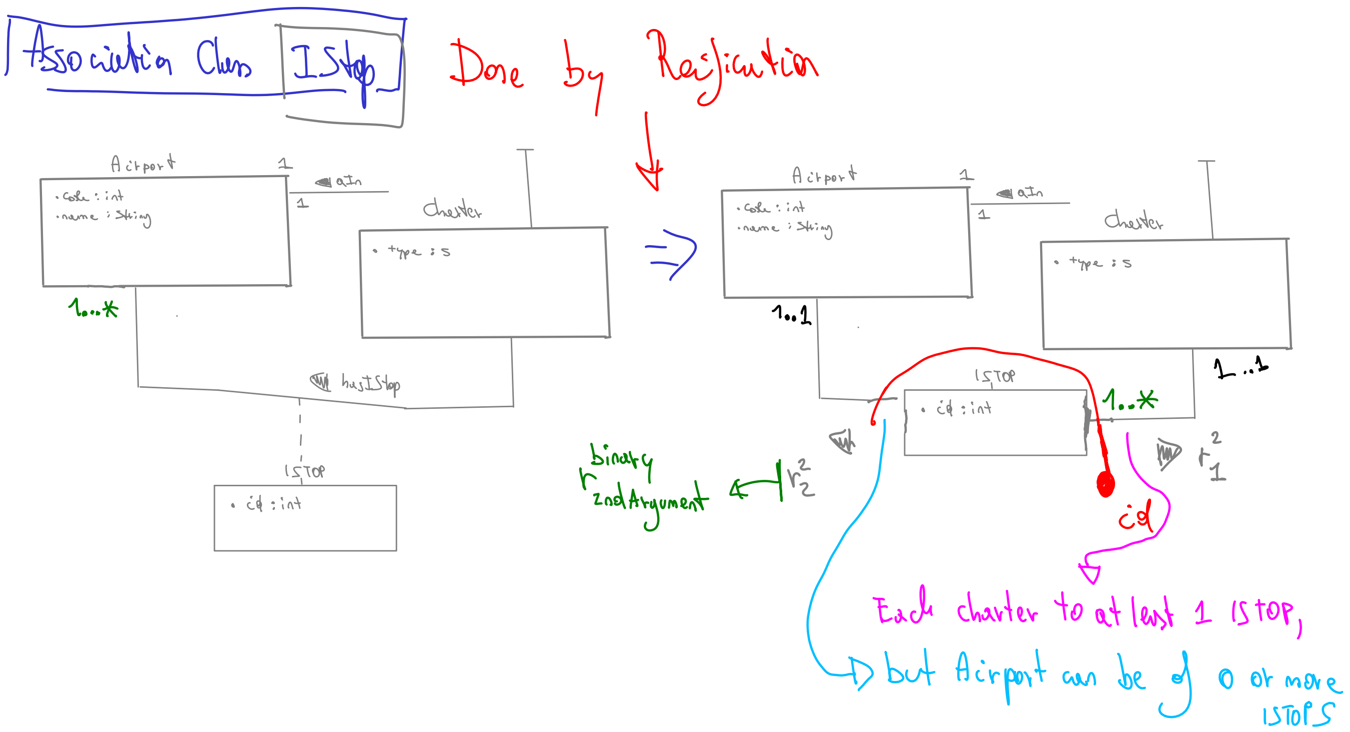
Class City:

- Typing
  - $\forall C \forall N.\texttt{name}_{\texttt{city}}(C, N) \rightarrow \texttt{City}(C) \wedge \texttt{String}(N)$
  - $\forall C \forall P.\texttt{population}_{\texttt{city}}(C, P) \rightarrow \texttt{City}(C) \wedge \texttt{Integer}(P)$
  - $\forall C \forall N.\texttt{nation}_{\texttt{city}}(C, N) \rightarrow \texttt{City}(C) \wedge \texttt{String}(N)$
- Cardinality
  - $\forall C.\texttt{City}(C) \rightarrow (1 \leq \#\{N \mid \texttt{name}_{\texttt{City}}(C, N)\} \leq 1)$
  - $\forall C.\texttt{City}(C) \rightarrow (1 \leq \#\{P \mid \texttt{population}_{\texttt{City}}(C, P)\} \leq 1)$
  - $\forall C.\texttt{City}(C) \rightarrow (1 \leq \#\{N \mid \texttt{nation}_{\texttt{City}}(C, N)\} \leq 1)$

Association airportCity

- Typing
  - $\forall A \forall C.\texttt{airportCity}(A, C) \rightarrow \texttt{Airport}(A) \wedge \texttt{City}(C)$
- Cardinality
  - $\forall A \forall C_1 \forall C_2.\texttt{airportCity}(A, C_1) \wedge \texttt{airportCity}(A, C_2) \rightarrow C_1 = C_2$
  - $\forall A.\texttt{Airport}(A) \rightarrow \exists C.\texttt{airportCity}(A, C)$

Reification



- Reification: Typing
  - $\forall x.\texttt{ISTOP}(x).\forall y.r_2^2(x,y) \rightarrow \texttt{Airport}(y)$
  - $\forall x.\texttt{ISTOP}(x).\forall y.r_1^2(x,y) \rightarrow \texttt{Charter}(y)$
- Reification: 1..1 Cardinalities
  - $\forall x.\texttt{ISTOP}(x) \rightarrow \exists y.r_2^2(x,y)$ $\qquad \forall x.\texttt{ISTOP}(x) \rightarrow (1 \leq \#\{y \mid r_2^2(x,y)\})$
  - $\forall x,y,y'.r_2^2(x,y) \land r_2^2(x,y') \rightarrow y = y'$ $\qquad \forall x.(\#\{y \mid r_2^2(x,y)\} \leq 1)$
  - Same for r_1^2
- Reification: Identification
  - $\forall y,y'.(\#\{x \mid r_2^2(x,y) \land r_1^2(x,y')\} \leq 1)$
- Association-class Multiplicity
  - $\forall y.\texttt{Charter}(y) \rightarrow (1 \leq \#\{x \mid r_1^2(x,y) \land \texttt{ISTOP}(x))\}$

Exercise 2.3 If possible, formalize in first-order logic the following additional constraints (which are not directly expressible in UML):

1. Each flight has a unique code within the airline.
2. Two distinct flights of the same airline must differ either in the airport of arrival or in the airport of departure.
3. A phone number may belong to at most one head office.
4. For each charter flight v with precisely three intermediate stops a_1, a_2, a_3, if o_1, o_2, o_3 are the orders associated to a_1, a_2, a_3, respectively, then $\{o_1, o_2, o_3\} = \{1, 2, 3\}$.

5. For each charter flight v, if $a_1, \ldots, a_n$ are all intermediate stops of v, for some $n \geq 0$, and $o_1, \ldots, o_n$ are the orders associated to $a_1, \ldots, a_n$, respectively, then $\{o_1, \ldots, o_n\} = \{1, \ldots, n\}$.

Solution:

1. $\forall A \forall F_1 \forall F_2 \forall C_1 \forall C_2 (\texttt{flightAirline}(F_1, A) \land \texttt{flightAirline}(F_2, A) \land \texttt{flightCode}(F_1, C_1) \land \texttt{airlineCode}(F_2, C_2) \land C_1 = C_2 \to F_1 = F_2)$
2. $\forall f_1, f_2, a, l_1, l_2 (\texttt{Flight}(x_1) \land \texttt{Flight}(x_2) \land x_1 \neq x_2 \land \texttt{airline}(x_1, a) \land \texttt{airline}(x_2, a) \to (\texttt{dFrom}(f_1, l_1) \land \texttt{dFrom}(f_2, l_2)) \lor (\texttt{arrIn}(f_1, l_1) \land \texttt{arrIn}(f_2, l_2)) \to l_1 \neq l_2)$
3. $\forall p \forall o \forall o' (\texttt{phone}_{\texttt{HeadOffice}}(o, p) \land \texttt{phone}_{\texttt{HeadOffice}}(o', p) \to o = o')$
4. $$\begin{array}{l}
\forall v (\texttt{CharterFlight}(v) \land \\
\quad \exists a_1, a_2, a_3 (r_1^2(a_1, v) \land r_1^2(a_2, v) \land r_1^2(a_3, v) \land a_1 \neq a_2 \land a_1 \neq a_3 \land a_2 \neq a_3 \land \\
\quad\quad \forall a_4 (r_1^2(a_4, v) \to a_4 = a_1 \lor a_4 = a_2 \lor a_4 = a_3) \to \\
\quad\quad \forall o_1, o_2, o_3 (\texttt{order}_{\texttt{ISTOP}}(a_1, o_1) \land \texttt{order}_{\texttt{ISTOP}}(a_2, o_2) \land \texttt{order}_{\texttt{ISTOP}}(a_3, o_3) \to \\
\quad\quad\quad o_1 = 1 \land o_2 = 2 \land o_3 = 3) \\
\quad) \\
)
\end{array}$$
5. Compactness violation