1. Basics of First-Order Logic

Exercise 1.2 [4 points] Convert the following English sentences into sentences of first-order logic. Use meaningful predicate names or state the abbreviation scheme that you are using.

9. There are exactly two coins in the box.
10. The largest coin in the box is a quarter.
11. All students get good grades if they study.

Exercise 1.3 [9 points] Assume N is intended to mean "is a number"; I is intended to mean "is interesting"; $<$ is intended to mean "is less than"; and 0 is a constant symbol intended to denote zero. Translate into first-order logic sentences the English sentences listed below. If the English sentence is ambiguous, you will need more than one translation.

1. Zero is less than any number.
2. If any number is interesting, then zero is interesting.
3. No number is less than zero.
4. Any uninteresting number with the property that all smaller numbers are interesting certainly is interesting.
5. There is no number such that all numbers are less than it.
6. There is no number such that no number is less than it.

Exercise 1.5 [8 points] Consider the following English sentences. Can you formalize them in first-order logic? If yes, how? If not, provide a brief explanation that justifies your answer.

1. "There are three critics who admire only one another."
2. "There are some critics who admire only one another."
3. "It is not the case that there are some natural numbers smaller than 5 among which none is least."
4. "It is not the case that there are some numbers among which none is least."

Exercise 1.7 [9 points] For each group of sentences, give an interpretation in which all sentences are true.

1. $\forall x\,(P(x) \vee Q(x)) \rightarrow \exists x\,R(x)$
   $\forall x\,(R(x) \rightarrow Q(x))$
   $\exists x\,(P(x) \wedge \neg Q(x))$

2. $\forall x\,\neg P(x,x)$
   $\forall x,y,z\,(P(x,y) \wedge P(y,z) \rightarrow P(x,z))$
   $\forall x\,\exists y\,P(x,y)$

3. $\forall x\,\exists y\,P(x,y)$
   $\forall x\,(Q(x) \rightarrow \exists y\,P(y,x))$
   $\exists x\,Q(x)$
   $\forall x\,\neg P(x,x)$

Submission deadline: March 16, 2016, 23:55 by OLE (https://ole.unibz.it/).