Exercise 4

Consider the following problems:

1) Vertex-cover (VC)

Given an undirected graph $G=(V,E)$ and an integer $k \geq 2$, is there a subset C of V with $|C| \leq k$ such that C covers all edges of G (i.e., for each edge $\{v_i, v_j\} \in E$ with $v_i \neq v_j$, $\{v_i, v_j\} \cap C \neq \emptyset$).

2) Independent-set (IS)

Given an undirected graph $G=(V,E)$ and an integer $k \geq 2$, is there a subset C of V with $|C| \geq k$ such that for all $v_i, v_j \in C$ with $v_i \neq v_j$, $\{v_i, v_j\} \notin E$

3) Clique

Given an undirected graph $G=(V,E)$ and an integer $k \geq 2$, is there a subset C of V with $|C| \geq k$ such that for all $v_i, v_j \in C$ with $v_i \neq v_j$, $\{v_i, v_j\} \in E$.

Show that VC, IS, and Clique can be reduced to each other in polynomial time.

N.B. In the definitions of VC, IS, and Clique we have ignored self-loops (since we required $v_i \neq v_j$).

IS $<_{poly}$ Clique

Given an instance (G, k) of IS, we construct an instance (G', k') of Clique as follows:

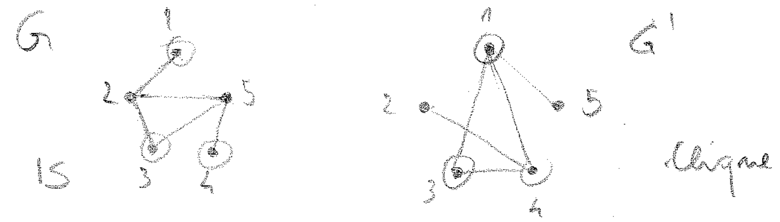
$k' = k$

Let $G = (V, E)$.

Then $G' = (V, E')$, where $E' = V \times V \setminus E$

(i.e., the edges of E' are obtained by connecting all pairs of nodes that are not connected in E.)

E.g.



The reduction works because the maximum independent set of G is precisely the maximum clique in the complement graph of G.

VC $<_{poly}$ IS

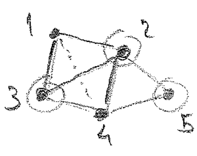
Given an instance (G, k) of VC, we construct an instance (G', k') of IS as follows:

$k' = |V| - k$

$G' = G$

The reduction works because the vertexes in a vertex-cover C cover all edges of G. Hence the set $V \setminus C$ must have no edges between its elements, and is thus an independent set

E.g.



The marked nodes $\{2, 3, 5\}$ are a VC of size 3. Hence, there cannot be an edge $\{1, 4\}$

E 10.3

Clique $<_{poly}$ VC

Given an instance (G, k) of Clique, we construct an instance (G', k') of VC as follows:

$k' = |V| - k$

Set $G = (V, E)$.

Then $G' = (V, E')$, where $E' = V \times V \setminus E$