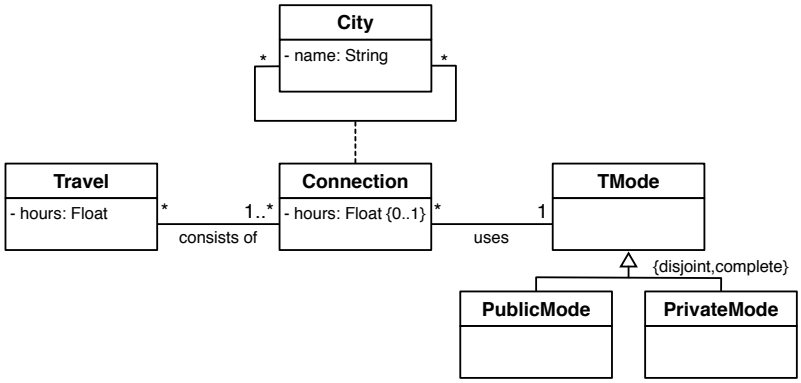
Exercise 1 [6 points] Consider the following UML Class Diagram.



1. Formalize this diagram in *DL-Lite*$_{\mathcal{A},\mathrm{id}}$.
2. Which aspect(s) of this diagram cannot be captured in *DL-Lite*$_{\mathcal{A},\mathrm{id}}$? Provide a justification for your answer.

Exercise 2 [6 points] Consider the following *DL-Lite*$_{\mathcal{A}}$ ontology and queries.

TBox: $\mathcal{T} = \{\ A \sqsubseteq \exists R, \quad \exists R^- \sqsubseteq B, \quad P \sqsubseteq Q,$
$R \sqsubseteq S, \quad B \sqsubseteq \exists P, \quad \mathsf{funct}(R)\ \}$

ABox: $\mathcal{A} = \{A(1)\}$

Queries: $q_1() \leftarrow P(x,y), Q(x,y)$ and $q_2(x) \leftarrow P(x,y), Q(x,y)$

1. Draw the canonical model of $\langle \mathcal{T}, \mathcal{A} \rangle$.
2. Compute the perfect rewriting of q_1 and of q_2 with respect to $\mathcal{T}$.
3. Compute the certain answers to q_1 and q_2 over $\langle \mathcal{T}, \mathcal{A} \rangle$. Explain the difference in the answers.

Exercise 3 [6 points] Consider the problem of answering unions of conjunctive queries in *DL-Lite*$_{\mathcal{A}}$.

1. What is the data complexity of this problem?
2. Suppose now that *DL-Lite*$_{\mathcal{A}}$ is extended by allowing *qualified existential restriction* on the left-hand side of inclusion assertions, i.e., assertions of the form

$$\exists P.A \sqsubseteq B$$

What is the data complexity of the considered problem in this case? Provide a justification for your answer.

Exercise 4 [6 points] Consider the $\mathcal{ALC}$ family of expressive description logics.

1. Define the syntax and the semantics of the description logic $\mathcal{ALCI}$.
2. Define the reasoning problems *concept satisfiability* and *concept subsumption* for a generic description logic $\mathcal{L}$.
3. Show that for $\mathcal{ALC}$, concept satisfiability and concept subsumption can be mutually reduced to each other.
4. What is the complexity of concept satisfiability in $\mathcal{ALC}$. Discuss briefly the sources of complexity.

Exercise 5 [6 points] Check satisfiability of the following $\mathcal{ALC}$-concept using Tableaux.

$$(A_1 \sqcap \exists P.(A_2 \sqcup A_3)) \sqcap \forall P.(\neg A_2)$$

Exercise 6 [4 points] Bonus Question

Introduce two suitable bisimulation relations for $\mathcal{ALCF}$ and $\mathcal{ALCI}$. Use these notions of bisimulation to prove that $\mathcal{ALCF}$ and $\mathcal{ALCI}$ have incomparable expressiveness.