4. Basics of Description Logics

Exercise 4.1 Translate the following DL expressions and axioms into first-order logic:

1. $\mathsf{Father} \sqcap \forall \mathsf{child}.(\mathsf{Doctor} \sqcup \exists \mathsf{managedBy}^-.(\mathsf{Company} \sqcap (\leq 3\, \mathsf{employs}.\mathsf{Doctor})))$
2. $\mathsf{Person} \sqcap \forall \mathsf{child}.\mathsf{HappyPerson} \sqsubseteq \exists \mathsf{child}.\forall \mathsf{child}.\mathsf{HappyPerson}$
3. $\mathsf{Person} \sqcap \exists \mathsf{child}.\mathsf{HappyPerson} \sqsubseteq \mathsf{Happy} \sqcap (\mathsf{Father} \sqcup \mathsf{Mother})$

Exercise 4.2 Translate the following sentences and first-order logic formulas into DL syntax, if possible:

1. Only humans have children that are humans.
2. A node cannot have two distinct P-successors, such that one is a B and the second one is not a B.
3. $\forall x_1, x_2, y_1, y_2.\ P(x_1, y_1) \land P(x_1, y_2) \land P(x_2, y_2) \to x_1 = x_2 \lor y_1 = y_2$
4. $\forall x, y, z.\ P(x, y) \land P(y, z) \land P(z, x) \to A(x)$
5. $\forall x, y, z.\ P(x, y) \land Q(y, z) \to R(x, z)$
6. $\forall x, y, z.\ P(x, y) \land Q(y, z) \to \exists w.\ R(x, w) \land S(w, z)$
7. $\neg(\forall x.\ A(x) \to B(x)) \lor (\forall x.\ A(x) \to C(x))$
8. $\exists x.\forall y.\ R(x, y) \lor S(x, y)$

Exercise 4.3 Compute the certain answers to the query q over the KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$.

1. $q(x) = B(x)$, $\quad \mathcal{A} = \{A(a), B(b), C(c)\}$, $\quad \mathcal{T} = \{A \sqsubseteq B,\ C \sqsubseteq \exists R,\ \exists R^- \sqsubseteq B\}$.
2. $q() = \exists x.B(x)$, $\quad \mathcal{A} = \{A(a)\}$,
   (a) $\mathcal{T} = \{A \sqsubseteq \exists R, \quad \exists R^- \sqsubseteq B\}$.
   (b) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcup \exists S, \quad \exists R^- \sqsubseteq B\}$.
   (c) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcap (\exists S \sqcup \exists Q), \quad \exists R^- \sqsubseteq B, \quad \exists Q^- \sqsubseteq B\}$.
   (d) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcup \exists S, \quad \exists R^- \sqsubseteq B, \quad \exists S^- \sqsubseteq \exists R \sqcup \exists Q, \quad \exists Q^- \sqsubseteq \exists R\}$.
3. $q(x) = \exists y.R(x, y)$, $\quad \mathcal{A} = \{A(a), R(b, c)\}$, $\quad \mathcal{T}$ as in Item 2.
4. $q(x) = \exists y.R(x, y)$, $\quad \mathcal{A} = \{A(a), R(a, c)\}$, $\quad \mathcal{T}$ as in Item 2.