8. Perfect Reformulation and Tableaux

Exercise 8.1 Rewriting of conjunctive queries with respect to a *DL-Lite* ontology with PerfectRef.

1. Compute the perfect reformulation for the following queries:
   (a) $q(x, y) \leftarrow A(x),\ R(x, y),\ C(y)$
   (b) $q(z) \leftarrow S(y, z)$
   (c) $q(z) \leftarrow S(x, y),\ S(y, z)$

   with respect to the TBox $\mathcal{T}$ consisting of the following inclusion assertions:

$$
\begin{aligned}
A &\sqsubseteq \exists R \\
\exists R^- &\sqsubseteq B \\
B &\sqsubseteq A \\
R^- &\sqsubseteq S
\end{aligned}
$$

2. Answer the above queries over the ontology $\langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{A} = \{A(c)\}$.

Exercise 8.2 Consider the following $\mathcal{ALC}$ TBox $\mathcal{T}$:

$$
\begin{array}{lcl}
A & \equiv & \forall R. \neg(\neg B \sqcap C) \\
B & \equiv & \exists R.C
\end{array}
$$

1. Analyze whether $\mathcal{T}$ is cyclic.
2. Determine, using tableaux, whether the concepts
   (a) $\exists R.(A \sqcap B)$ and
   (b) $A \sqcap \exists R.A \sqcap \neg(\exists R.\neg C \sqcup \exists R.\exists R.C)$

   are satisfiable w.r.t. $\mathcal{T}$.
3. Suppose that the assertion $C \equiv A \sqcup D$ is added to $\mathcal{T}$. Discuss whether the same tableaux technique used before can be applied, motivating your answer. If not, explain how the tableaux technique has to be extended.