## 1. Basics of First-Order Logic

**Exercise 1.1** In the following first-order sentences, Bird(x) means "x is a bird", Flies(x) means "x flies", Person(x) means "x is a person", and Mother(x,y) means "x is the mother of y". Translate the sentences into English:

- 1.  $\forall x \ (Bird(x) \to Flies(x))$
- 2.  $\forall x \; \exists y \; (Person(x) \to Mother(y, x))$
- 3.  $\exists x \ \forall y \ (Person(x) \land Mother(x, y))$

**Exercise 1.2** Convert the following English sentences into sentences of first-order logic. Use meaningful predicate names or state the abbreviation scheme that you are using.

- 1. All cats are mammals.
- 2. No cat is a reptile.
- 3. All computer scientists like some operating system.
- 4. The only good extraterrestrial is a drunk extraterrestrial.
- 5. The Barber of Seville shaves all men who do not shave themselves.
- 6. There are at least two mountains in England.
- 7. No mountain is higher than itself.
- 8. There is exactly one coin in the box.
- 9. There are exactly two coints in the box.
- 10. The largest coin in the box is a quarter.
- 11. All students get good grades if they study.

Exercise 1.3 Assume N is intended to mean "is a number"; I is intended to mean "is interesting"; < is intended to mean "is less than"; and 0 is a constant symbol intended to denote zero. Translate into first-order logic sentences the English sentences listed below. If the English sentence is ambiguous, you will need more than one translation.

- 1. Zero is less than any number.
- 2. If any number is interesting, then zero is interesting.
- 3. No number is less than zero.
- 4. Any uninteresting number with the property that all smaller numbers are interesting certainly is interesting.
- 5. There is no number such that all numbers are less than it.
- 6. There is no number such that no number is less than it.

**Exercise 1.4** For each of the following English sentences, write a corresponding sentence in FOL.

- 1. P is a person; T is a time, F(x, y) means that you can fool x at time y.
  - (a) You can fool some of the people all of the time.
  - (b) You can fool all of the people some of the time.
  - (c) You can't fool all of the people all of the time.
- 2. J is a job; a designates Adam; D(x, y) means that x can do y right.
  - (a) Adam can't do every job right.
  - (b) Adam can't do any job right.
- 3. Nobody likes everybody. (L(x, y) means x likes y.)

**Exercise 1.5** Consider the following English sentences. Can you formalize them in first-order logic? If yes, how?

- 1. "There are three critics who admire only one another."
- 2. "There are some critics who admire only one another."
- 3. "It is not the case that there are some natural numbers smaller than 5 among which none is least."
- 4. "It is not the case that there are some numbers among which none is least."

**Exercise 1.6** For each group of sentences, write an interpretation under which the last sentence is false and all the rest are true.

1. 
$$\forall x (P(x) \to Q(x))$$
  
 $\forall x (R(x) \to Q(x))$   
 $\exists x (R(x) \land P(x))$ 

2. 
$$\forall x \exists y \ P(x,y)$$
  
 $\exists y \ \forall x \ P(x,y)$ 

3. 
$$\forall x (P(x) \to Q(a))$$
  
 $(\forall x P(x)) \to Q(a)$ 

**Exercise 1.7** For each group of sentences, give an interpretation in which all sentences are true.

1. 
$$\forall x \ (P(x) \lor Q(x)) \to \exists x \ R(x)$$
  
 $\forall x \ (R(x) \to Q(x))$   
 $\exists x \ (P(x) \land \neg Q(x))$ 

2. 
$$\forall x \neg P(x, x)$$
  
 $\forall x, y, z \ (P(x, y) \land P(y, z) \rightarrow P(x, z))$   
 $\forall x \exists y \ P(x, y)$ 

3. 
$$\forall x \exists y \ P(x,y)$$
  
 $\forall x \ (Q(x) \to \exists y \ P(y,x))$   
 $\exists x \ Q(x)$   
 $\forall x \ \neg P(x,x)$