CONTEXT-FREE GRAMMARS

Home exercise
E 14.1

EXERCISE 1

Define context-free grammars for the following languages:

a) $\{0^i 1^j \mid i \geq 0, i \leq j \leq 2i\}$;

b) $\{0^m 1^n 1^n 0^m \mid m + n > 0\}$;

c) $\{0^n 1^m 0^{m+n} \mid m + n > 0\}$.

EXERCISE 2

Define a context-free grammar that generates strings of balanced parentheses. Examples of strings of balanced parentheses are (()), ()(), (()()) and ε, while)(and (() are not.

EXERCISE 3

Is the following context-free grammar ambiguous?

$$S \to a \mid S;S \mid iS \mid iSeS$$

One may think of the grammar as a rudimentary programming language where i stands for "if-then" and e for "else". Motivate your answer.

EXERCISE 4

Define a context-sensitive grammar for the language $\{a^i b^j c^k \mid 0 < i < j < k\}$. Provide derivations of the strings abbbccccc and aabbbcccc.

SOLUTIONS (08/01/2008)

E 14.2

1)a) $S \to \varepsilon \mid 0S1 \mid 0S11$

1)b) $S \to 00 \mid 0S0 \mid A$

$A \to 11 \mid 1A1$

1)c) $S \to A \mid B$

$A \to 00 \mid 0A0 \mid B$

$B \to 10 \mid 1B0$

2) ① $B \to BB$

② $B \to (B)$

③ $B \to \varepsilon$

$$B \overset{2}{\Rightarrow} (\underline{B}) \overset{1}{\Rightarrow} (\underline{B}B) \overset{2}{\Rightarrow} ((\underline{B})B) \overset{1}{\Rightarrow} ((BB)\underline{B}) \overset{2}{\Rightarrow} ((\underline{B}B)(B))$$
$$\overset{2}{\Rightarrow} (((B)\underline{B})(B)) \overset{3}{\Rightarrow} (((B))(\underline{B})) \overset{3}{\Rightarrow} (((\underline{B}))()) \overset{3}{\Rightarrow} ((())())$$

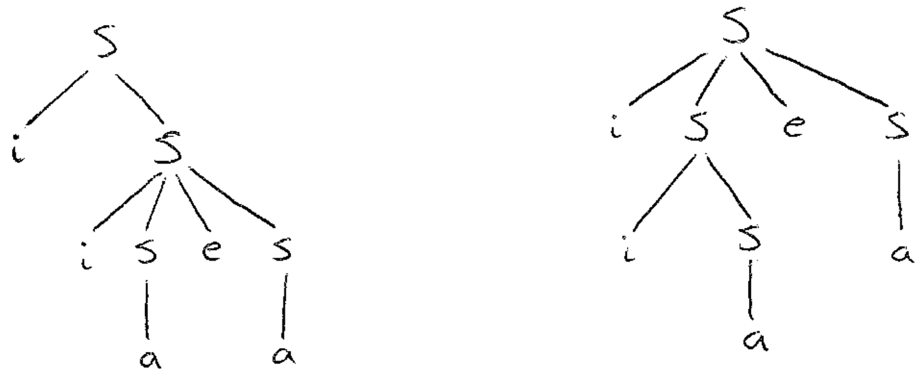
3) The context-free grammar is ambiguous. Indeed, the sentential form $iiSeS$ has two derivations from S:

a) $S \Rightarrow i\underline{S} \Rightarrow iiSeS$;

b) $S \Rightarrow i\underline{S}eS \Rightarrow iiSeS$.

From this one can easily see that the string $iiaea$ has two different parse trees.

3) con't



4) ① $S \longrightarrow aSBC$

② $S \longrightarrow abBCC$

③ $CB \longrightarrow BC$

④ $bB \longrightarrow bbBC$

⑤ $bB \longrightarrow bb$

⑥ $bC \longrightarrow bCc$

⑦ $bC \longrightarrow bcc$

⑧ $cC \longrightarrow cc$

Derivation of abbbccccc:

$$S \xRightarrow{2} a\underline{bB}CC \xRightarrow{4} ab\underline{bB}CCC \xRightarrow{5} abb\underline{bC}CC$$
$$\xRightarrow{6} abb\underline{bC}cCC \xRightarrow{7} abbbcc\underline{cC}C$$
$$\xRightarrow{8} abbbcc\underline{cC} \xRightarrow{8} abbbccccc$$

Derivation of aabbbcccc:

$$S \xRightarrow{1} a\underline{S}BC \xRightarrow{2} aabBC\underline{CB}C \xRightarrow{3} aabB\underline{CB}CC$$
$$\xRightarrow{3} aa\underline{bB}BCCC \xRightarrow{5} aab\underline{bB}CCC \xRightarrow{5} aabb\underline{bC}CC$$
$$\xRightarrow{7} aabbbc\underline{cC}C \xRightarrow{8} aabbbcc\underline{cC} \xRightarrow{8} aabbbcccc$$