Answers to the Sample Exam 20.01.2010

1) a) False, As a Counter-example Consider $L_1 = \Sigma^*$, and L_2 to be an arbitary non-regular language. Then L_1 and $L_2 = L_1 \cup L_2 = \Sigma^*$ are regular language, and L_2 is not.

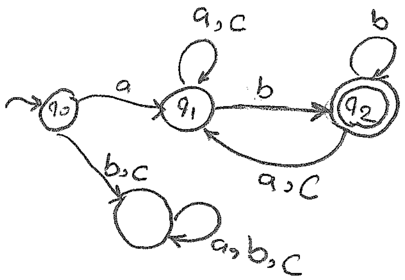
b) False, As a Counter-example Consider $L_1 = L_2 = \{\varepsilon\}$, and $L_3 = \emptyset$. As a result, $L_1^* = L_2^* = L_3^* = \{\varepsilon\}$, so $L_1 \subseteq L_2$ and $L_2^* \subseteq L_3^*$ but $L_1 \not\subseteq L_3$

c) True, It is enough to define a finite state automaton, or a regular expression that accepts the language.

Assume that the language is $\mathcal{L}_f = \{w_1, w_2 \ldots, w_n\}$ where n is the size of the language. Then the following regular expression accepts $\mathcal{L}_f$:

$$R_f = w_1 + w_2 + \ldots + w_n$$

2) a)



b) There are a lot regular expressions that accept the given language Some of them are as follows:

$$((x+z)^* \cdot (yz)^* \cdot (x+z)^*)^*$$

or

$$(x^* z^* (yz)^*)^*$$

or

$$((x+z+\varepsilon)(yz+\varepsilon)(x+z+\varepsilon))^*$$

or

$$(x+z+yz)^*$$

3) a) Having a $A_\varepsilon = (Q, \Sigma, \delta_\varepsilon, q_0, F)$ as an ε-NFA, the NFA whic Accepts the same language is $A_N = (Q, \Sigma, \delta_N, q_0, F)$ with δ_N defined as follows:

$$\forall q \in Q, \quad \forall a \in \Sigma, \quad \delta_N(q,a) = \hat{\delta}_\varepsilon(q,a) = \mathrm{Eclose}\Big(\bigcup_{p_i \in \mathrm{EClose}(q)} \delta_\varepsilon(q_i, a)\Big)$$

First we compute EClose(a) for all $a \in \Sigma$

$\mathrm{Eclose}(A) = \{A, B\}$ $\quad$ $\mathrm{EClose}(B) = \{B\}$ $\quad$ $\mathrm{EClose}(C) = \{A, B, C\}$

Now: $\delta_N(A,0) = \mathrm{EClose}(\{A, C\}) = \{A, B, C\}$

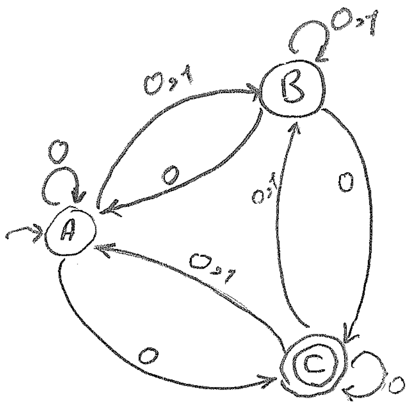
$\delta_N(A,1) = \mathrm{EClose}(\{B\}) = \{B\}$

$\delta_N(B,0) = \mathrm{EClose}(\{C\}) = \{A, B, C\}$

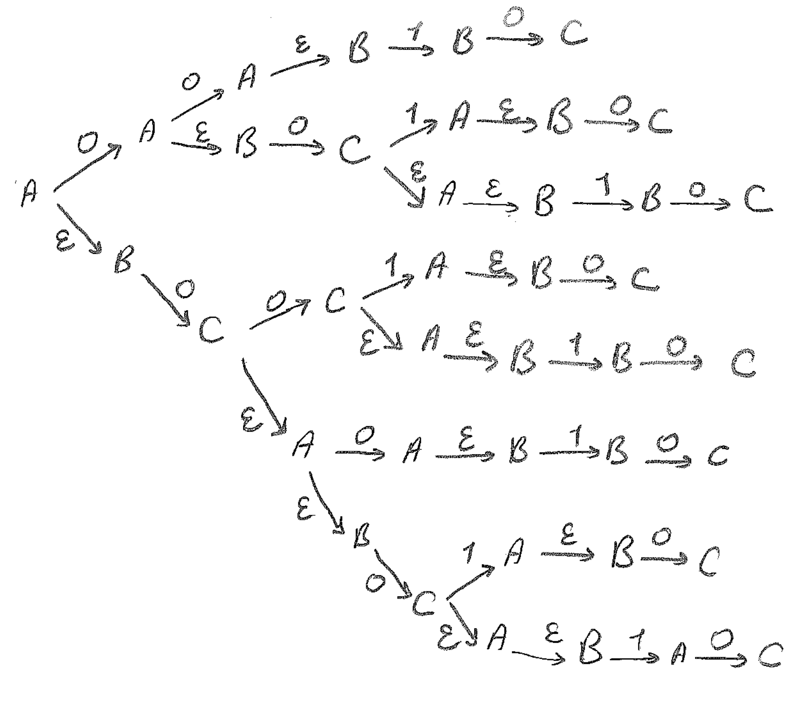
$\delta_N(B,1) = \mathrm{EClose}(\{B\}) = \{B\}$

$\delta_N(C,0) = \mathrm{Eclose}(\{A, C\}) = \{A, B, C\}$

$\delta_N(C,1) = \mathrm{EClose}(\{A, B\}) = \{A, B\}$

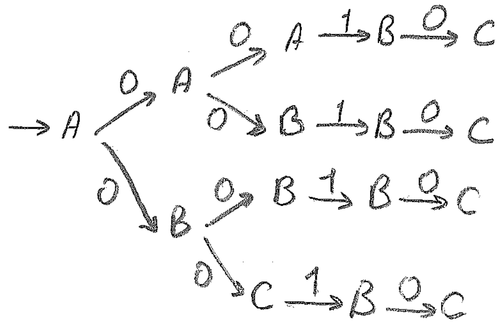


b) N_1:



3) b) Con.t.

transitions over N2:



4) a) we start with the set of all states, in step 0, the set will be separated into two sets of final and non-final states:

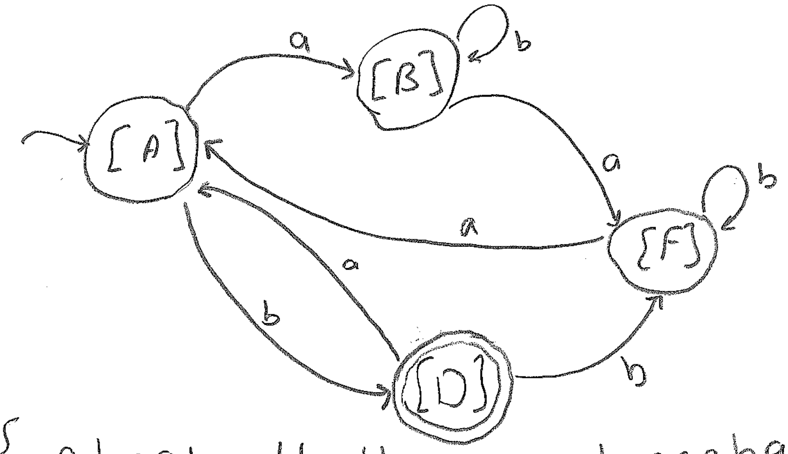
$\{A, B, C, D, E, F\}$

$\{A, B, C, F\}$ $\{D, E\}$

$\{A, C\}$ $\{B, F\}$ $\{D, E\}$

$\{A, C\}$ $\{B\}$ $\{F\}$ $\{D, E\}$

$\{A, C\}$ $\{B\}$ $\{F\}$ $\{D, E\}$ (no changes)



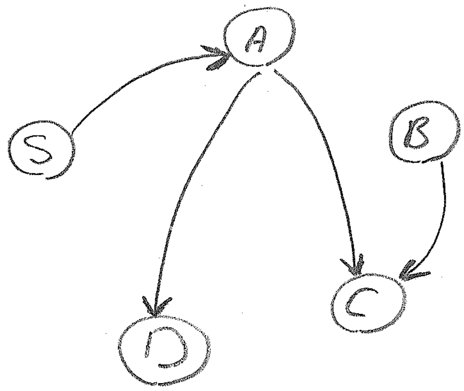
b) $\{ abaab, abbabbab, aaab, aaabab \} \subseteq \mathcal{L}(A)$
in $\mathcal{L}(A)$

$\{ aaaa, aaaba, ababa, bbbba \} \cap \mathcal{L}(A) = \emptyset$
not in $\mathcal{L}(A)$

notice that any strin which ends with a is not in $\mathcal{L}(A)$.

5) a)

Graph of unit Productions:



reachability: $S \overset{*}{\Rightarrow} A$, $S \overset{*}{\Rightarrow} C$, $S \overset{*}{\Rightarrow} D$

$A \overset{*}{\Rightarrow} C$, $A \overset{*}{\Rightarrow} D$

$B \overset{*}{\Rightarrow} C$

we get $G_1 = (V_N, V_T, P_1, S)$ where

$V_N = \{S, A, B, C, D\}$, $V_T = \{a, b, c, d\}$, P defines as follows:

$S \longrightarrow AaBb \mid Da \mid Aa \mid AB \mid aB \mid Ca \mid d \mid dA$

$A \longrightarrow Aa \mid AB \mid aB \mid Ca \mid d \mid dA$

$B \longrightarrow BD \mid Cd \mid AB \mid aB \mid Ca$

$C \longrightarrow AB \mid aB \mid Ca$

$D \longrightarrow d \mid dA$

b)

Generating symbols It 0 : $\{a, b, c, d\}$

It 1 : $\{a, b, c, d, S, A, D\}$

It 2 : $\{a, b, c, d, S, A, D\}$

So we Get $G_2 = (\{S, D\}, \{a, b, c, d\}, P_2, S\}$ where P_2 defines as follows:

$S \longrightarrow Da \mid Aa \mid d \mid dA$

$A \longrightarrow Aa \mid d \mid dA$

$D \longrightarrow d \mid dA$