# FINITE AUTOMATA VS. REGULAR EXPRESSIONS

## EXERCISE 1

Convert the following regular expressions to ε-NFA's.

a) $(\varepsilon+1)(01)^*(\varepsilon+0)$  b) $a^*b^*c^*$

## EXERCISE 2

Convert the following DFA to a regular expression, using the state elimination technique.

| | 0 | 1 |
|---|---|---|
| $\rightarrow * p$ | s | p |
| q | p | s |
| r | r | q |
| s | q | r |

## EXERCISE 3

Repeat exercise 2, eliminating the states in a different order. Then, verify that the following binary strings (accepted by the above DFA) are in the language of the regular expressions obtained here and in exercise 2.

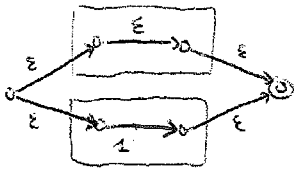
a) 0010110010 1  b) 01000 110 1110

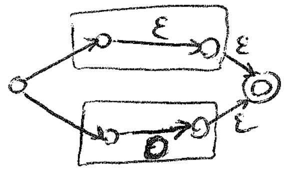
E6.2

# SOLUTIONS (27/11/2008)

1) a) ε-NFA's for ε+1, ε+0, and (01)* are as follows:

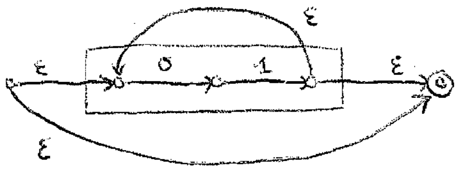
ε+1

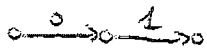




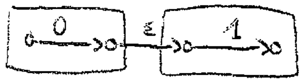
ε+0

(01)*

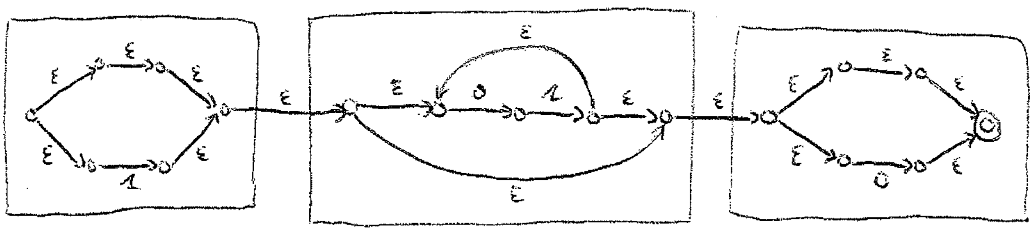




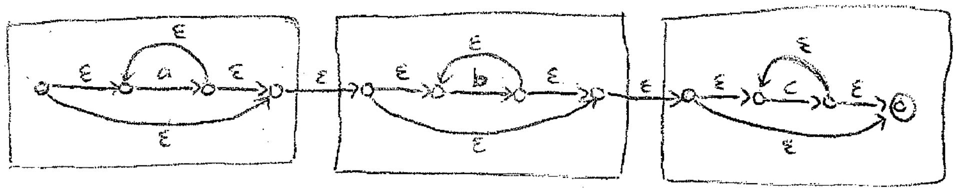
is shorthand for



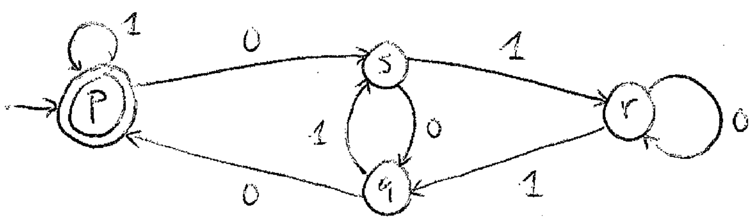
Composition of the above ε-NFA's yields:



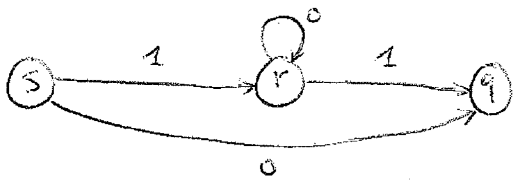
1) b) We get the following ε-NFA:



2) The DFA looks as follows:



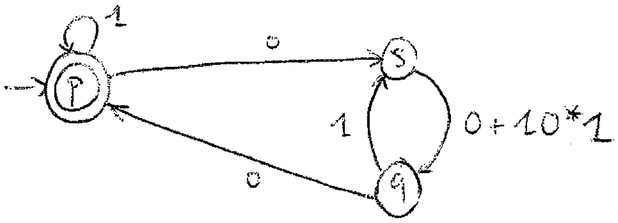
First, we eliminate the state r



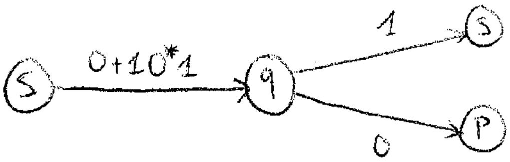
$(0+10^*1$ for $s \to q)$

2) con't

and obtain



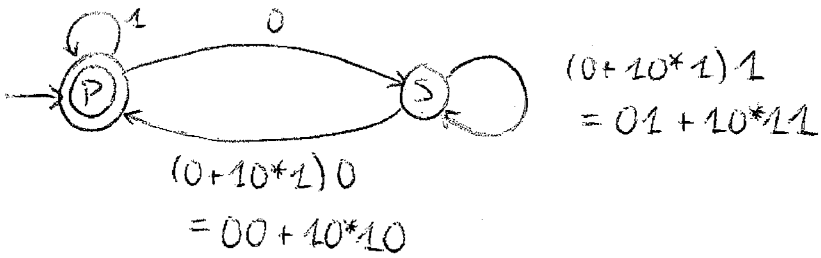

Second, we eliminate the state q



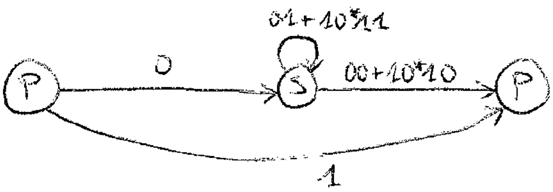

$((0+10^*1)\cdot 1$ for $s \to s)$

$((0+10^*1)\cdot 0$ for $s \to p)$

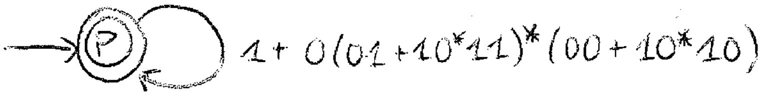
and obtain




Third, we eliminate the state s




and obtain




The regular expression is therefore

$$(1+0(01+10^*11)^*(00+10^*10))^* = E$$

E6.4

3) Note that, if we eliminate first the state r, but then s before q, we get the following regular expression:

$$E' = (1 + (00 + 010^*1)(10 + 110^*1)^*0)^*$$

3) a) $00101100101 \in \mathcal{L}(E)$:

$$(0\underbrace{(01)(01)}_{\text{from } (01 + 10^*11)^*}\underbrace{(10010)}_{\text{from } (00 + 10^*10)})1$$

$$00101100101 \in \mathcal{L}(E')$$

$$(\underbrace{(00)}_{\text{from } (00 + 010^*1)}\underbrace{(10)(11001)}_{\text{from } (10 + 110^*1)^*}0)1$$

3) b) $0\underbrace{(100011)(01)}_{\text{from } (01 + 10^*11)^*}\underbrace{(110)}_{\text{from } (00 + 10^*10)} \in \mathcal{L}(E)$

$$\underbrace{(010001)}_{\text{from } (00 + 010^*1)}\underbrace{(10)(111)}_{\text{from } (10 + 110^*1)^*}0 \in \mathcal{L}(E')$$