12/11/2010
E4.1

Automata with ε-Transitions

Exercise 1

Design ε-NFA's for the following languages:

a) The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

b) The set of all strings consist of either 01 repeated one or more times, or 010 repeated one or more times.

Exercise 2

Consider the following ε-NFA:

	ε	a	b	c
→p	{q,r}	∅	{q}	{r}
q	∅	{p}	{r}	{p,q}
*r	∅	∅	∅	∅

ε-NFA$_1$

a) Compute the ε-closure of each state.

b) which of following strings are accepted by the automata:

bb   bba   a   ε   aba

c) Convert the automata to a DFA.

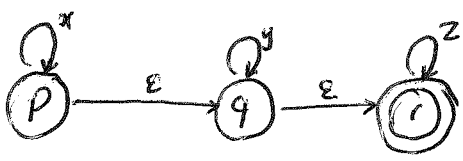
d) Repeat steps a), b), c) for the following ε-NFA:

	ε	a	b	c
→p	∅	{p}	{q}	{r}
q	{p}	{q}	{r}	∅
*r	{q}	{r}	∅	{p}

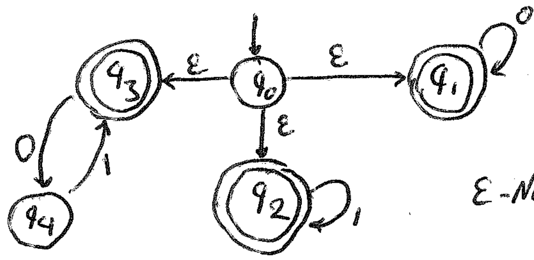
ε-NFA$_2$

Exercise 3:

Convert the following NFA's to a DFA's:



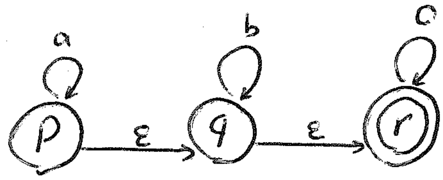
ε-NFA$_1$



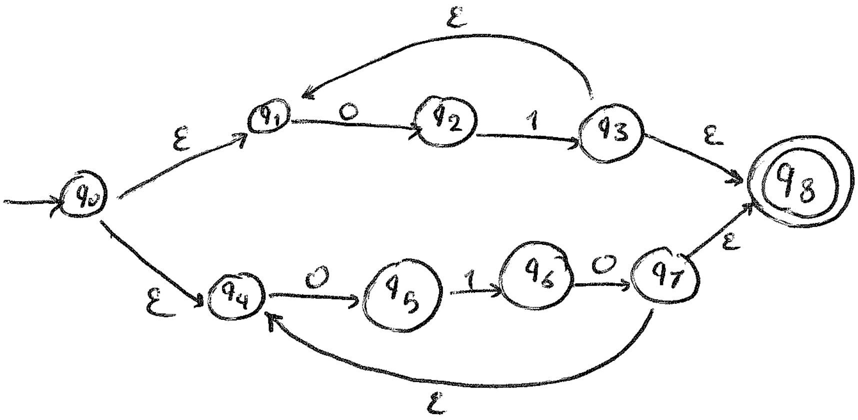
ε-NFA$_2$

E4.2

1) a)



1) b)



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2) a) Remember that: $\begin{cases} q \in ECLOSE(q) \\ p \in ECLOSE(q) \;\&\; r \in \delta(p,\varepsilon) \Rightarrow r \in ECLOSE(q) \end{cases}$

ε-NFA_1: $ECLOSE(p)=\{p,q,r\}$, $ECLOSE(q)=\{q\}$, $ECLOSE(r)=\{r\}$

2) b) bb ✓ bba ✗ a ✓ ε ✓ aba ✓

2) c) From ε-NFA to NFA

Remember that $\delta_N(q,a) = \hat{\delta}_\varepsilon(q,a) = ECLOSE\Big(\bigcup_{p \in ECLOSE(q)} \delta(p,a)\Big)$

The calculation of ε-NFA go a follow:

$\hat{\delta}_\varepsilon(p,a) = ECLOSE(\delta(p,a) \cup \delta(q,a) \cup \delta(r,a)) = ECLOSE(\emptyset \cup \{p\} \cup \emptyset) = \{p,q,r\}$

$\hat{\delta}_\varepsilon(p,b) = ECLOSE(\delta(p,b) \cup \delta(q,b) \cup \delta(r,b)) = ECLOSE(\{q\} \cup \{r\} \cup \emptyset) = \{q,r\}$

$\hat{\delta}_\varepsilon(p,c) = ECLOSE(\delta(p,c) \cup \delta(q,c) \cup \delta(r,c)) = ECLOSE(\{r\} \cup \{p,q\} \cup \emptyset) = \{p,q,r\}$

$\hat{\delta}(q,a) = ECLOSE(\delta(q,a)) = ECLOSE(\{p\}) = \{p,q,r\}$

$\hat{\delta}(q,b) = ECLOSE(\delta(q,b)) = ECLOSE(\{r\}) = \{r\}$

$\hat{\delta}(q,c) = ECLOSE(\delta(q,c)) = ECLOSE(\{p,q\}) = \{p,q,r\}$

$\hat{\delta}(r,a) = ECLOSE(\delta(r,a)) = ECLOSE(\emptyset) = \emptyset$

$\hat{\delta}(r,b) = ECLOSE(\delta(r,b)) = ECLOSE(\emptyset) = \emptyset$

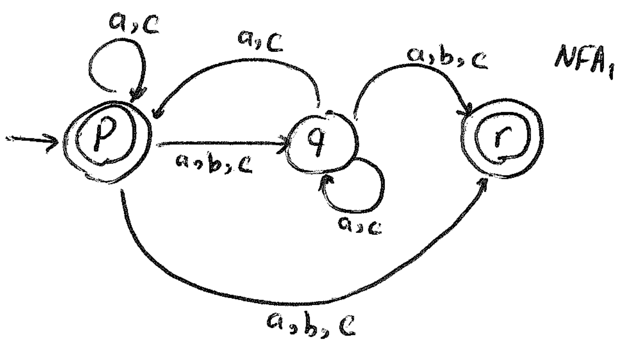
$\hat{\delta}(r,c) = ECLOSE(\delta(r,c)) = ECLOSE(\emptyset) = \emptyset$

2) c) Cont.

we get the following NFA:

δ_N	a	b	c
$\overset{*}{\rightarrow} p$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$
q	$\{p,q,r\}$	$\{r\}$	$\{p,q,r\}$
$* r$	$\emptyset$	$\emptyset$	$\emptyset$

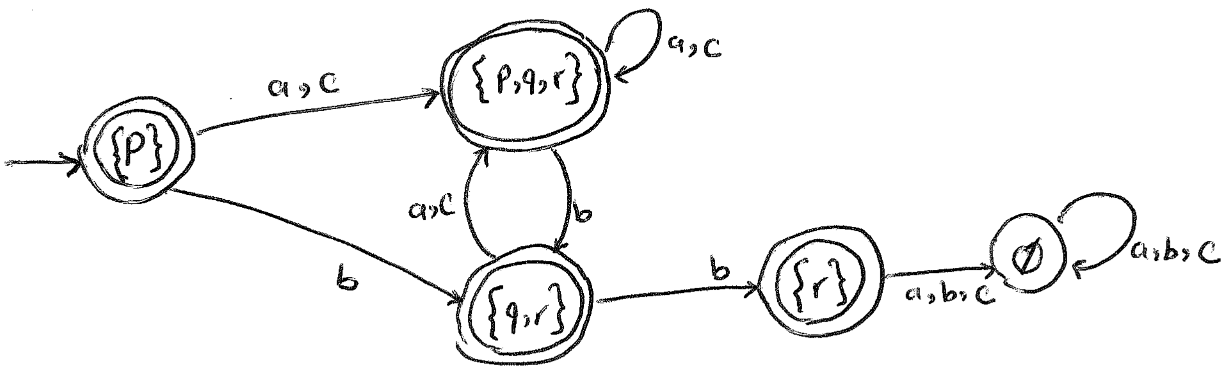
Note that ε-NFA$_1$ accepts the empty string ε.



From NFA to DFA

The subset construction yields:

	a	b	c
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\overset{*}{\rightarrow} \{p\}$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$
$\{q\}$	$\{p,q,r\}$	$\{r\}$	$\{p,q,r\}$
$* \{r\}$	$\emptyset$	$\emptyset$	$\emptyset$
$* \{p,q\}$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$
$* \{p,r\}$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$
$\{q,r\}$	$\{p,q,r\}$	$\{r\}$	$\{p,q,r\}$
$* \{p,q,r\}$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$



Attention: the states: $\{q\}$, $\{p,q\}$, $\{p,r\}$ are not reachable from the initial state, so they are excluded from the NFA diagram.

2)d)

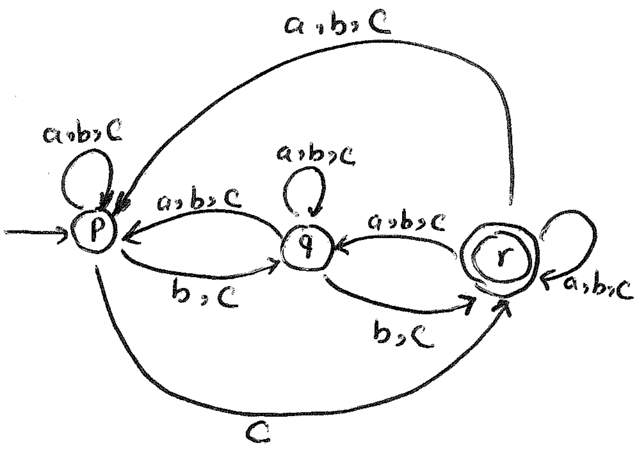
E4.4

a) ε-NFA2: $ECLOSE(p)=\{p\}$, $ECLOSE(q)=\{p,q\}$, $ECLOSE(r)=\{p,q,r\}$

b) bb ✓ bba ✓ a ✗ ε ✗ aba ✓

c) <u>ε-NFA $\longrightarrow$ NFA:</u>

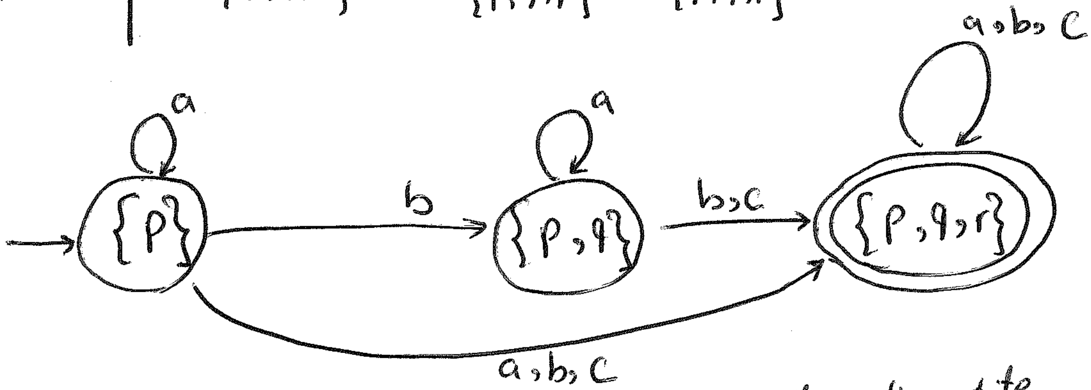
δ_N	a	b	c
$\rightarrow p$	$\{p\}$	$\{p,q\}$	$\{p,q,r\}$
q	$\{p,q\}$	$\{p,q,r\}$	$\{p,q,r\}$
$*r$	$\{p,q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$



<u>NFA $\longrightarrow$ DFA:</u>

The subset Construction yiealds:

	a	b	c
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\rightarrow \{p\}$	$\{p\}$	$\{p,q\}$	$\{p,q,r\}$
$\{q\}$	$\{p,q\}$	$\{p,q,r\}$	$\{p,q,r\}$
$\{r\}$	$\{p,q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$
$\{p,q\}$	$\{p,q\}$	$\{p,q,r\}$	$\{p,q,r\}$
$\{p,r\}$	$\{p,q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$
$\{q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$
$*\{p,q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$



· other states are not reachable from the initial state.

3) ε-NFA$_1$

From ε-NFA$_1$ to NFA$_1$ (as above):

	x	y	z
$\rightarrow p$	$\{p,q,r\}$	$\{q,r\}$	$\{r\}$
q	$\emptyset$	$\{q,r\}$	$\{r\}$
$* r$	$\emptyset$	$\emptyset$	$\{r\}$

From NFA$_1$ to DFA$_1$ (subset Construction):

δ_D	x	y	z
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\rightarrow \{p\}$	$\{p,q,r\}$	$\{q,r\}$	$\{r\}$
$\{q\}$	$\emptyset$	$\{q,r\}$	$\{r\}$
$* \{r\}$	$\emptyset$	$\emptyset$	$\{r\}$
$\{p,q\}$	$\{p,q,r\}$	$\{q,r\}$	$\{r\}$
$* \{p,r\}$	$\{p,q,r\}$	$\{q,r\}$	$\{r\}$
$* \{q,r\}$	$\emptyset$	$\{q,r\}$	$\{r\}$
$* \{p,q,r\}$	$\{p,q,r\}$	$\{q,r\}$	$\{r\}$

ε-NFA$_2$

we only provide the final DFA: