AUTOMATA WITH ε-TRANSITIONS

6/11/2009
E4.1

EXERCISE 1

Design ε-NFA's for the following languages.

a) The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

b) The set of all strings that consist of either 01 repeated one or more times or 010 repeated one or more times.

EXERCISE 2

Consider the following ε-NFA's.

ε-NFA$_1$

	ε	a	b	c
→ p	{q,r}	∅	{q}	{r}
q	∅	{p}	{r}	{p,q}
* r	∅	∅	∅	∅

ε-NFA$_2$

	ε	a	b	c
→ p	∅	{p}	{q}	{r}
q	{p}	{q}	{r}	∅
* r	{q}	{r}	∅	{p}

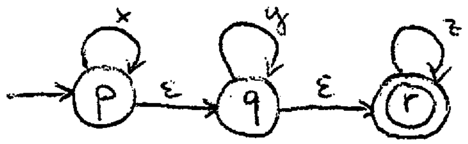
a) Compute the ε-closure of each state.

b) Give all the strings of length three or less accepted by the automata.

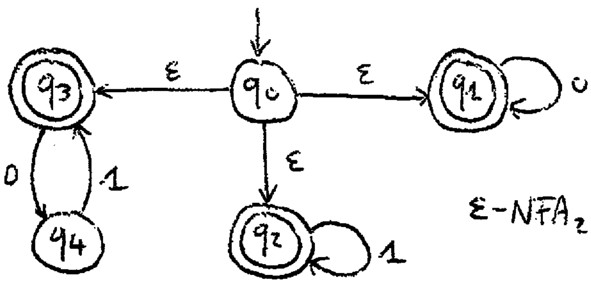
c) Convert the automata to DFA's.

EXERCISE 3

Convert the following ε-NFA's to a DFA's.



ε-NFA$_1$

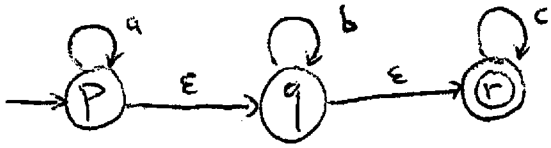


ε-NFA$_2$

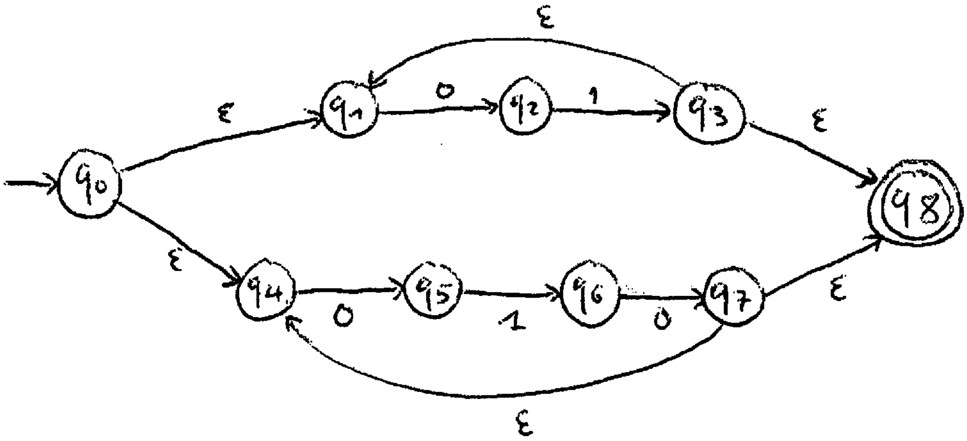
E4.2

SOLUTIONS (13/11/2008)

1) a)



1) b)



2) a) Remember that: $\begin{cases} q \in ECLOSE(q) \\ p \in ECLOSE(q) \ \& \ r \in \delta(p, \varepsilon) \Rightarrow r \in ECLOSE(q) \end{cases}$

ε-NFA_1: $ECLOSE(p) = \{p, q, r\}$, $ECLOSE(q) = \{q\}$, $ECLOSE(r) = \{r\}$

ε-NFA_2: $ECLOSE(p) = \{p\}$, $ECLOSE(q) = \{p, q\}$, $ECLOSE(r) = \{p, q, r\}$

2) b) ε-NFA_1

All strings exept for: bba, bbb, bbc

ε-NFA_2

All strings except for: ε, a, b, aa, ab, ba, aaa, aab, aba, baa

2) c) From ε-NFA to NFA

Remember that $\delta_N(q, a) = \hat{\delta}_E(q, a) = ECLOSE\left(\bigcup_{p \in ECLOSE(q)} \delta(p, a)\right)$

2) c) con't

The calculations for ε-NFA$_1$ go as follows:

$\hat{\delta}_\varepsilon(p,a) = ECLOSE(\delta(p,a) \cup \delta(q,a) \cup \delta(r,a)) = ECLOSE(\emptyset \cup \{p\} \cup \emptyset) = \{p,q,r\}$

$\hat{\delta}_\varepsilon(p,b) = ECLOSE(\delta(p,b) \cup \delta(q,b) \cup \delta(r,b)) = ECLOSE(\{q\} \cup \{r\} \cup \emptyset) = \{q,r\}$

$\hat{\delta}_\varepsilon(p,c) = ECLOSE(\delta(p,c) \cup \delta(q,c) \cup \delta(r,c)) = ECLOSE(\{r\} \cup \{p,q\} \cup \emptyset) = \{p,q,r\}$

$\hat{\delta}_\varepsilon(q,a) = ECLOSE(\delta(q,a)) = ECLOSE(\{p\}) = \{p,q,r\}$

$\hat{\delta}_\varepsilon(q,b) = ECLOSE(\delta(q,b)) = ECLOSE(\{r\}) = \{r\}$

$\hat{\delta}_\varepsilon(q,c) = ECLOSE(\delta(q,c)) = ECLOSE(\{p,q\}) = \{p,q,r\}$

$\hat{\delta}_\varepsilon(r,a) = ECLOSE(\delta(r,a)) = ECLOSE(\emptyset) = \emptyset$

$\hat{\delta}_\varepsilon(r,b) = ECLOSE(\delta(r,b)) = ECLOSE(\emptyset) = \emptyset$

$\hat{\delta}_\varepsilon(r,c) = ECLOSE(\delta(r,c)) = ECLOSE(\emptyset) = \emptyset$

By doing the same calculations for ε-NFA$_2$ we get the following NFA's

δ_N	a	b	c	
$* \rightarrow p$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$	NFA$_1$ (note that ε-NFA$_1$ accepts the empty string ε)
q	$\{p,q,r\}$	$\{r\}$	$\{p,q,r\}$	
$* r$	$\emptyset$	$\emptyset$	$\emptyset$	
$\rightarrow p$	$\{p\}$	$\{p,q\}$	$\{p,q,r\}$	NFA$_2$
q	$\{p,q\}$	$\{p,q,r\}$	$\{p,q,r\}$	
$* r$	$\{p,q,r\}$	$\{p,q,r\}$	$\{p,q,r\}$	

From NFA to DFA

This was already done in exercise E4 (06/11/2008)

E4.4

3) ε-NFA_1

From ε-NFA_1 to NFA_1 (as above)

δ_N	x	y	z
$^*\!\!\rightarrow p$	$\{p,q,r\}$	$\{q,r\}$	$\{r\}$
q	$\emptyset$	$\{q,r\}$	$\{r\}$
$*\, r$	$\emptyset$	$\emptyset$	$\{r\}$

From NFA_1 to DFA_1 (subset construction)

δ_D	x	y	z
$A = \emptyset$	A	A	A
$^*\!\!\rightarrow B = \{p\}$	H	G	D
$C = \{q\}$	A	G	D
$*\, D = \{r\}$	A	A	D
$*\, E = \{p,q\}$	H	G	D
$*\, F = \{p,r\}$	H	G	D
$*\, G = \{q,r\}$	A	G	D
$*\, H = \{p,q,r\}$	H	G	D

ε-NFA_2

We only provide the final DFA