DETERMINISTIC FINITE AUTOMATA

EXERCISE 1

Give a DFA accepting the following language over the alphabet $\{a, b\}$: the set of all strings such that the second last symbol is b.

EXERCISE 2

Give a DFA accepting the following language over the alphabet $\{x, y\}$: the set of strings that either begin or end (or both) with yx.

EXERCISE 3

Give a DFA accepting the following language over the alphabet $\{0, 1\}$: the set of strings such that the number of 0's is divisible by five and the number of 1's is divisible by three.

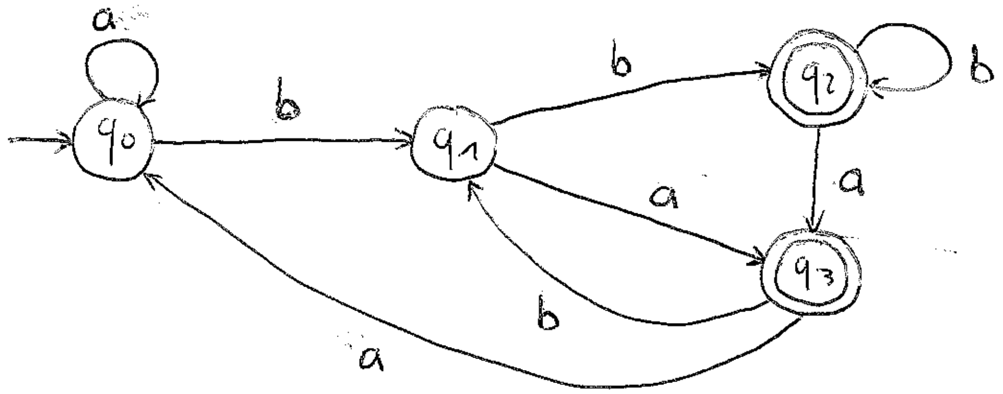
EXERCISE 4

Give a DFA accepting the following language over the alphabet $\{a, b, c, d\}$: the set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

SOLUTIONS

E2.2

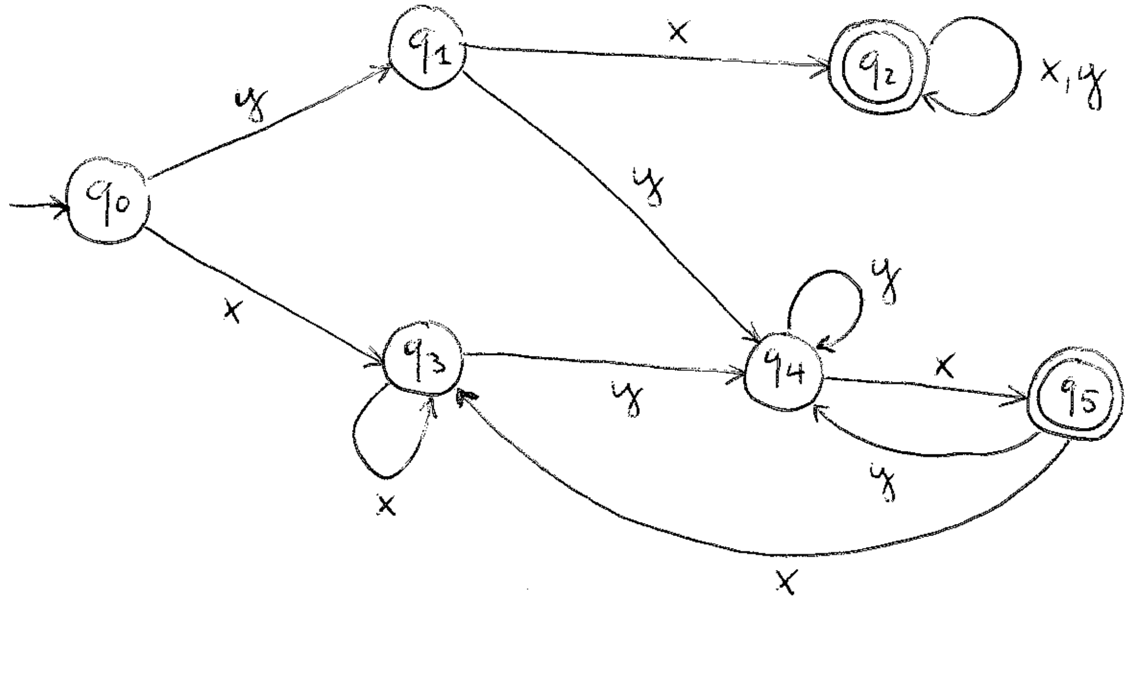
1) The DFA looks as follows:



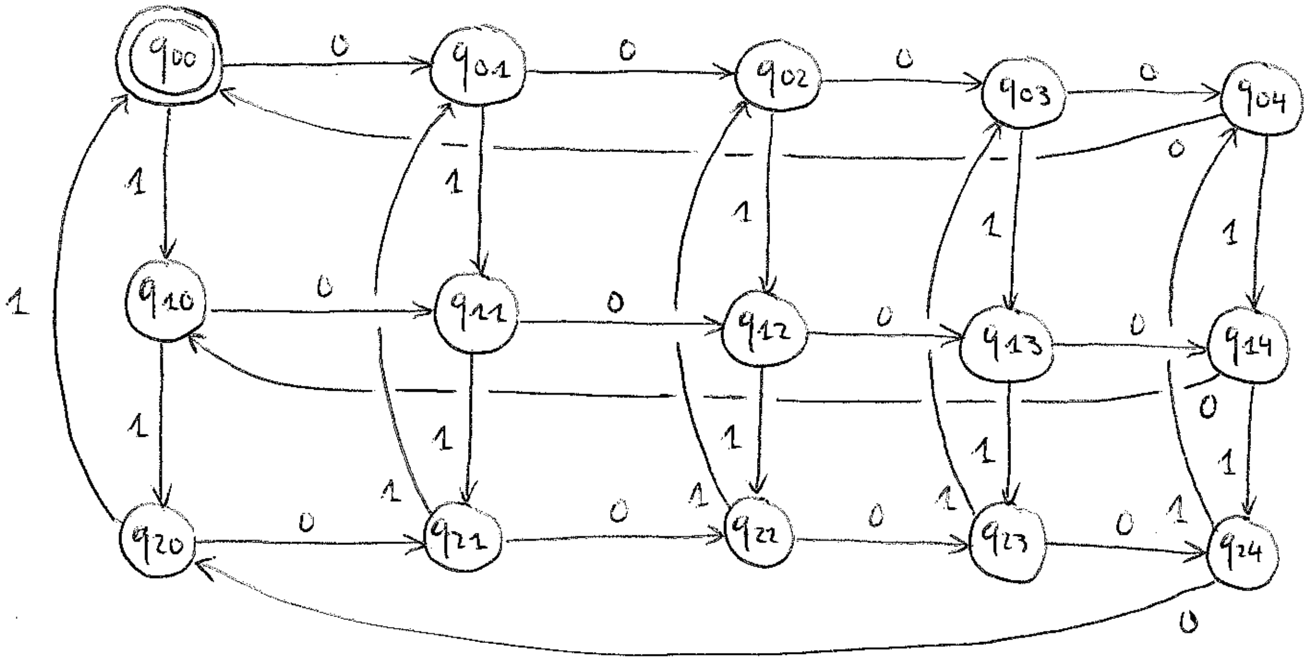
We show that it accepts bbaaba but not abbbaa

- $(q_0, b) \rightarrow q_1$, $(q_1, b) \rightarrow q_2$, $(q_2, a) \rightarrow q_3$, $(q_3, a) \rightarrow q_0$
  $(q_0, b) \rightarrow q_1$, $(q_1, a) \rightarrow q_2$ $\quad$ q_2 is a final state
- $(q_0, a) \rightarrow q_0$, $(q_0, b) \rightarrow q_1$, $(q_1, b) \rightarrow q_2$, $(q_2, b) \rightarrow q_2$
  $(q_2, a) \rightarrow q_3$, $(q_3, a) \rightarrow q_0$ $\quad$ q_0 is not a final state

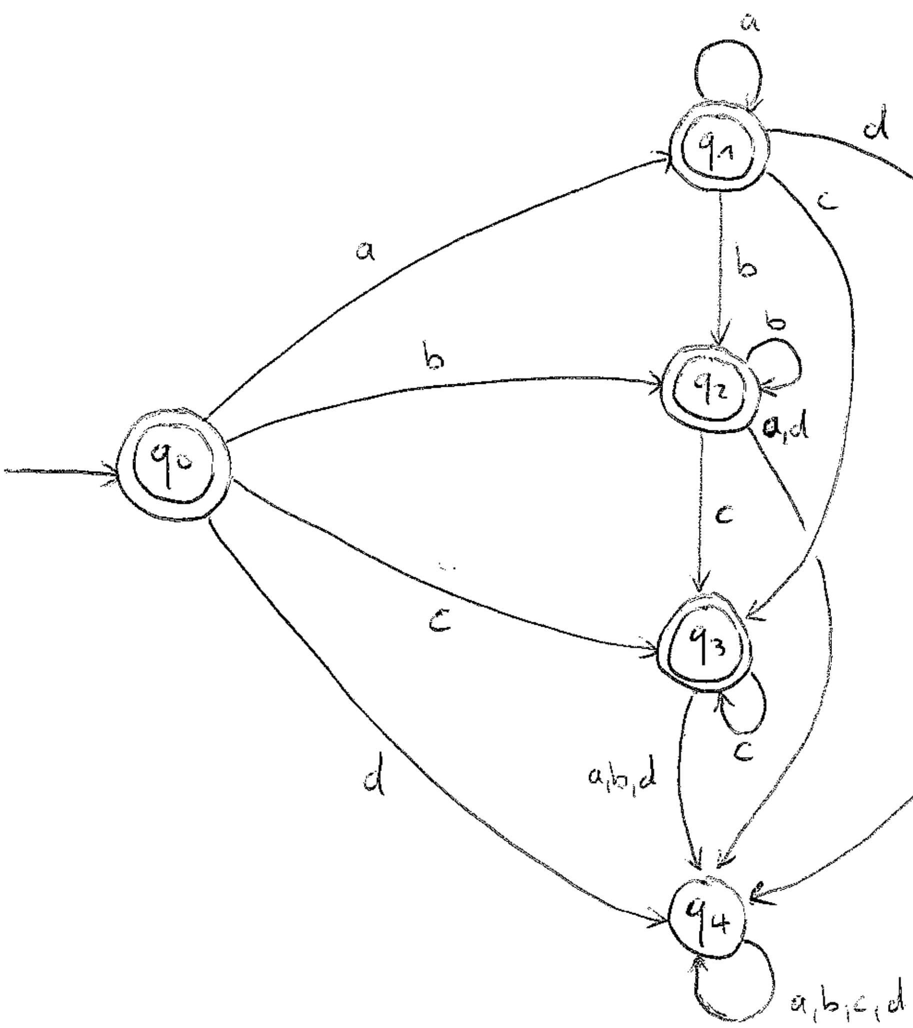
2) The DFA looks as follows:



3) The DFA looks as follows: (EZ.3)



4) The DFA looks as follows:



NON-DETERMINISTIC FINITE AUTOMATA

EXERCISE 1

Pick out one of the DFA's from exercise E2 (16/10/2008) and two strings of length at least five over the corresponding alphabet. Show whether the strings are accepted or not by using the extended transition function.

EXERCISE 2

Give a NFA accepting the following language over the alphabet {a,b}: the set of strings that end with ba, bb, or baa. Then show that the string baab is not accepted by the NFA.

EXERCISE 3

Give NFA's accepting the following languages:

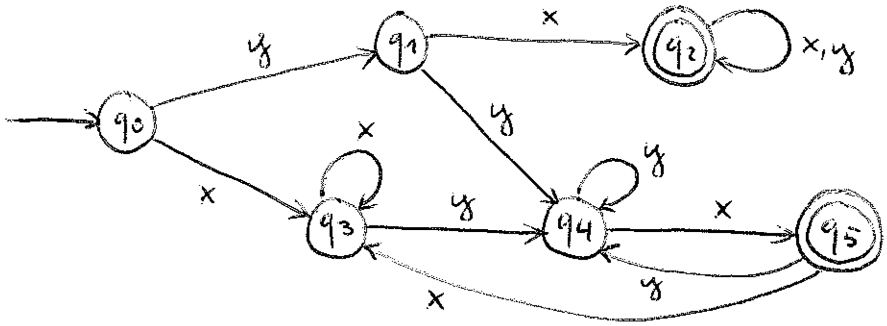
a) the set of strings over {0,1,..., 9} such that the final digit has appeared before;

b) the set of strings over {0,1,..., 9} such that the final digit has <u>not</u> appeared before;

c) the set of strings over {0,1} such that there are two 0's separated by a number of positions that is a multiple of four.

SOLUTIONS

1) We choose the DFA from exercise 2



and show that: $yyxyx$ is accepted; $xyxxy$ is not accepted.
In other words, we show that: $\hat{\delta}(q_0, yyxyx) = q_5 \in F$;
$\hat{\delta}(q_0, xyxxy) = q_4 \notin F$.

$\hat{\delta}(q_0, \varepsilon) = q_0$

$\hat{\delta}(q_0, y) = \delta(\underline{\hat{\delta}(q_0, \varepsilon)}, y) = \delta(q_0, y) = q_1$

$\hat{\delta}(q_0, yy) = \delta(\underline{\hat{\delta}(q_0, y)}, y) = \delta(q_1, y) = q_4$

$\hat{\delta}(q_0, yyx) = \delta(\underline{\hat{\delta}(q_0, yy)}, x) = \delta(q_4, x) = q_5$

$\hat{\delta}(q_0, yyxy) = \delta(\underline{\hat{\delta}(q_0, yyx)}, y) = \delta(q_5, y) = q_4$

$\hat{\delta}(q_0, yyxyx) = \delta(\underline{\hat{\delta}(q_0, yyxy)}, x) = \delta(q_4, x) = q_5$

$\hat{\delta}(q_0, \varepsilon) = q_0$

$\hat{\delta}(q_0, x) = \delta(\underline{\hat{\delta}(q_0, \varepsilon)}, x) = \delta(q_0, x) = q_3$

$\hat{\delta}(q_0, xy) = \delta(\underline{\hat{\delta}(q_0, x)}, y) = \delta(q_3, y) = q_4$

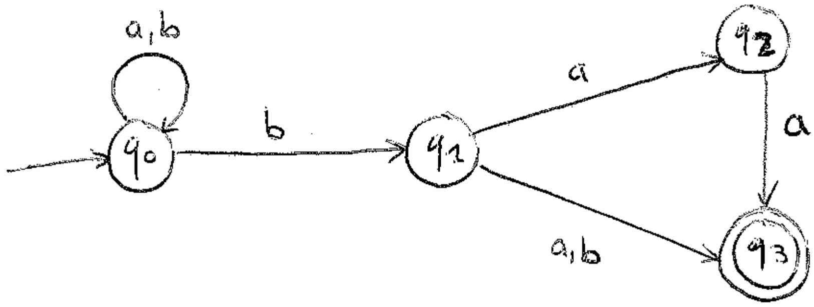
$\hat{\delta}(q_0, xyx) = \delta(\underline{\hat{\delta}(q_0, xy)}, x) = \delta(q_4, x) = q_5$

$\hat{\delta}(q_0, xyxx) = \delta(\underline{\hat{\delta}(q_0, xyx)}, x) = \delta(q_5, x) = q_3$

$\hat{\delta}(q_0, xyxxy) = \delta(\underline{\hat{\delta}(q_0, xyxx)}, y) = \delta(q_3, y) = q_4$

Note that we have underlined the recursive calls of the extended transition function $\hat{\delta}$ in our calculations.

2) The NFA looks as follows:



We show that baab is not accepted, i.e. that $\hat{\delta}(q_0, baab) = \{q_0, q_1\}$ and thus $q_3 \notin \hat{\delta}(q_0, baab)$.

$\hat{\delta}(q_0, \varepsilon) = \{q_0\}$

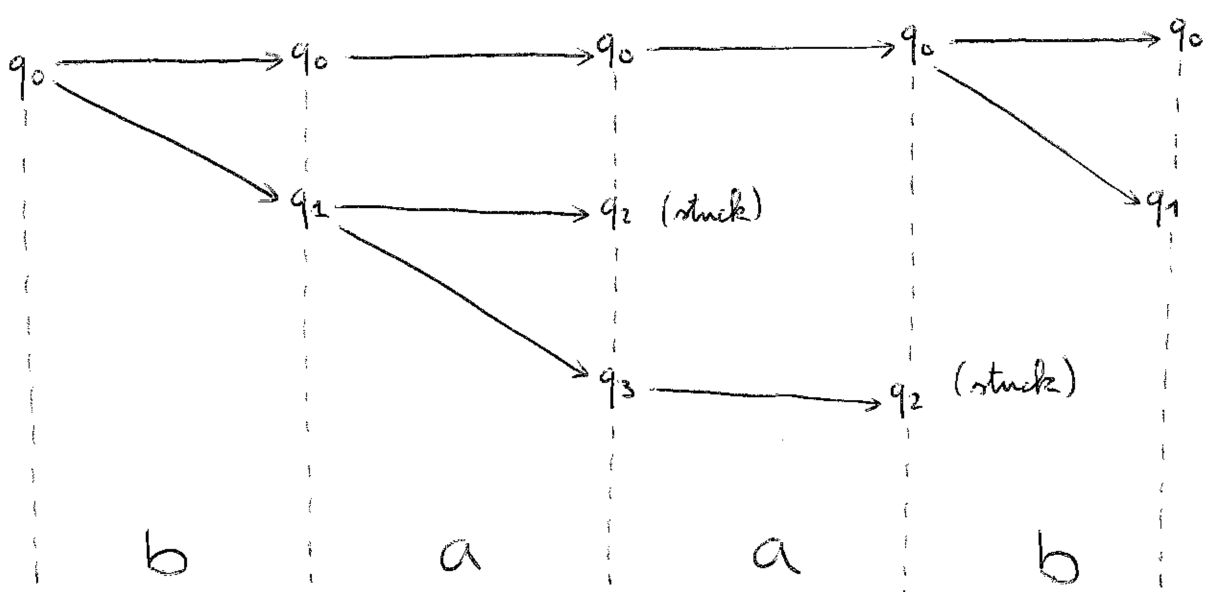
$\hat{\delta}(q_0, b) = \delta(q_0, b) = \{q_0, q_1\}$

$\hat{\delta}(q_0, ba) = \delta(q_0, a) \cup \delta(q_1, a) = \{q_0\} \cup \{q_2, q_3\} = \{q_0, q_2, q_3\}$

$\hat{\delta}(q_0, baa) = \delta(q_0, a) \cup \delta(q_2, a) \cup \delta(q_3, a) = \{q_0\} \cup \{q_3\} \cup \emptyset = \{q_0, q_3\}$

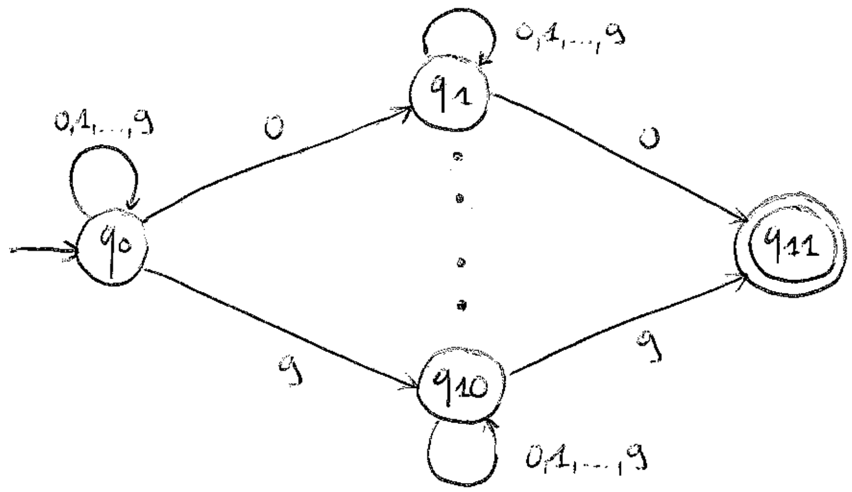
$\hat{\delta}(q_0, baab) = \delta(q_0, b) \cup \delta(q_3, b) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$

The following graph might help to get a more intuitive understanding of what is going on.

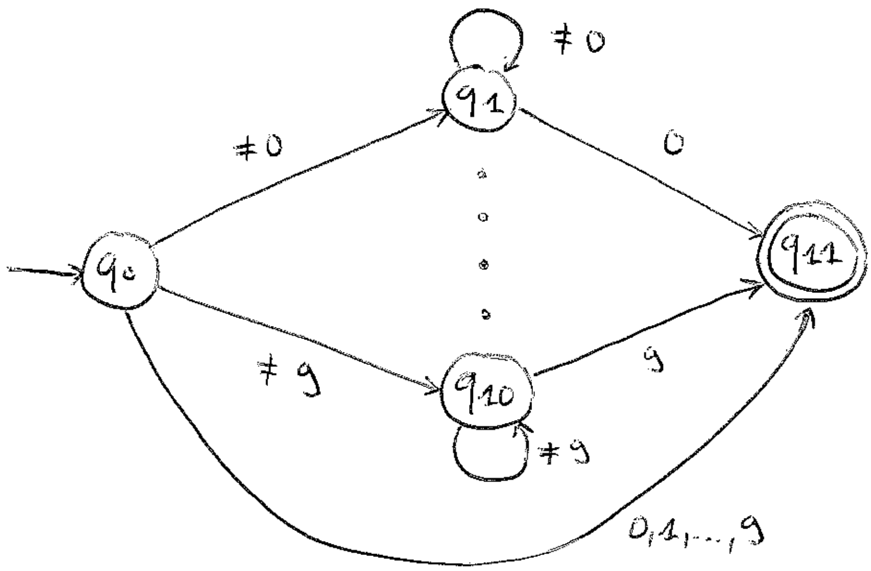


3) The NFA's look as follows:

a)



b)



c)