CONTEXT-FREE GRAMMARS

08/01/2009

E11.1

EXERCISE 1

Define context-free grammars for the following languages:

a) $\{0^i 1^j \mid i \geq 0, i \leq j \leq 2i\}$;

b) $\{0^m 1^n 1^n 0^m \mid m+n > 0\}$;

c) $\{0^n 1^m 0^{m+n} \mid m+n > 0\}$.

EXERCISE 2

Define a context-free grammar that generates strings of balanced parentheses. Examples of strings of balanced parentheses are (()), ()(), (()()) and ε, while)(and (() are not.

EXERCISE 3

Is the following context-free grammar ambiguous?

$S \rightarrow a \mid S;S \mid iS \mid iSeS$

One may think of the grammar as a rudimentary programming language where i stands for "if-then" and e for "else". Motivate your answer.

EXERCISE 4

Define a context-sensitive grammar for the language $\{a^i b^j c^k \mid 0<i<j<k\}$. Provide derivations of the strings abbbccccc and aabbbcccc.

SOLUTIONS (08/01/2008)

E11.2

1)a) $S \to \varepsilon \mid 0S1 \mid 0S11$

1)b) $S \to 00 \mid 0S0 \mid A$

$A \to 11 \mid 1A1$

1)c) $S \to A \mid B$

$A \to 00 \mid 0A0 \mid B$

$B \to 10 \mid 1B0$

2) ① $B \to BB$

② $B \to (B)$

③ $B \to \varepsilon$

$$B \overset{2}{\Rightarrow} (\underline{B}) \overset{1}{\Rightarrow} (\underline{B}B) \overset{2}{\Rightarrow} ((\underline{B})B) \overset{1}{\Rightarrow} ((BB)\underline{B}) \overset{2}{\Rightarrow} ((\underline{B}B)(B))$$
$$\overset{2}{\Rightarrow} (((B)\underline{B})(B)) \overset{3}{\Rightarrow} (((B))(\underline{B})) \overset{3}{\Rightarrow} (((\underline{B}))()) \overset{3}{\Rightarrow} ((())())$$

3) The context-free grammar is ambiguous. Indeed, the sentential form $iiSeS$ has two derivations from S:

a) $S \Rightarrow i\underline{S} \Rightarrow iiSeS$;

b) $S \Rightarrow i\underline{S}eS \Rightarrow iiSeS$.

From this one can easily see that the string $iiaea$ has two different parse trees.

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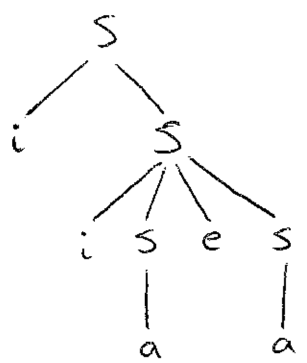

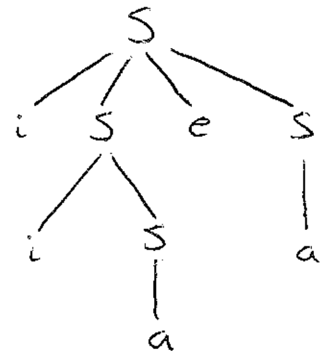

4) (1) $S \longrightarrow aSBC$

(2) $S \longrightarrow abBCC$

(3) $CB \longrightarrow BC$

(4) $bB \longrightarrow bbBC$

(5) $bB \longrightarrow bb$

(6) $bC \longrightarrow bCc$

(7) $bC \longrightarrow bcc$

(8) $cC \longrightarrow cc$

Derivation of abbbccccc:

$S \stackrel{2}{\Longrightarrow} a\underline{bB}CC \stackrel{4}{\Longrightarrow} ab\underline{bB}CCC \stackrel{5}{\Longrightarrow} abb\underline{bC}CC$

$\stackrel{6}{\Longrightarrow} abb\underline{bC}cCC \stackrel{7}{\Longrightarrow} abbbcc\underline{cC}C$

$\stackrel{8}{\Longrightarrow} abbbccc\underline{cC} \stackrel{8}{\Longrightarrow} abbbccccc$

Derivation of aabbbcccc:

$S \stackrel{1}{\Longrightarrow} a\underline{S}BC \stackrel{2}{\Longrightarrow} aabBC\underline{CB}C \stackrel{3}{\Longrightarrow} aabB\underline{CB}CC$

$\stackrel{3}{\Longrightarrow} aa\underline{bB}BCCC \stackrel{5}{\Longrightarrow} aab\underline{bB}CCC \stackrel{5}{\Longrightarrow} aabb\underline{bC}CC$

$\stackrel{7}{\Longrightarrow} aabbbc\underline{cC}C \stackrel{8}{\Longrightarrow} aabbbcc\underline{cC} \stackrel{8}{\Longrightarrow} aabbbcccc$