

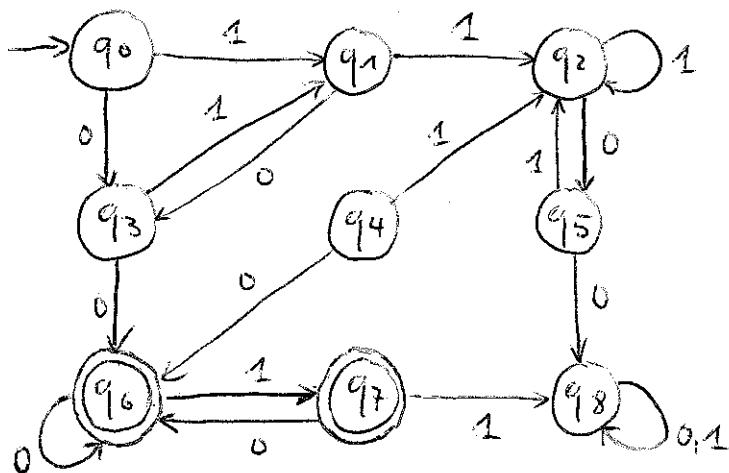
STATE MINIMIZATION

18/12/2008

E10.1

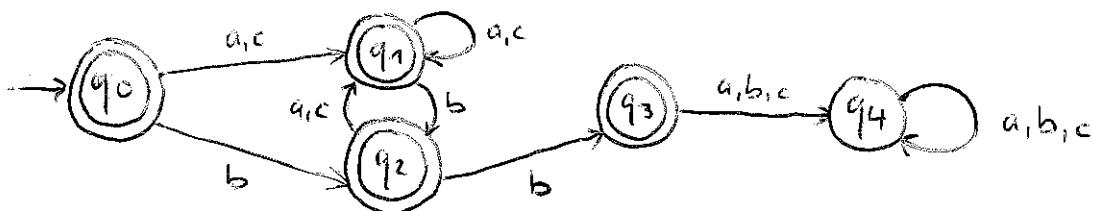
EXERCISE 1

Minimize the following DFA.



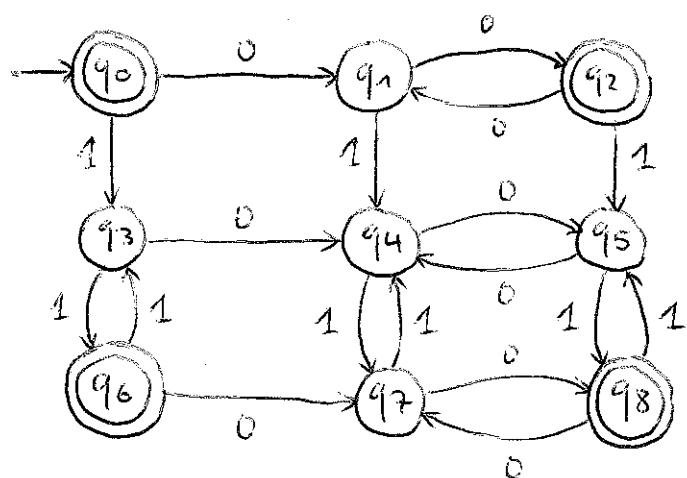
EXERCISE 2

Minimize the following DFA.



EXERCISE 3

Minimize the following DFA.

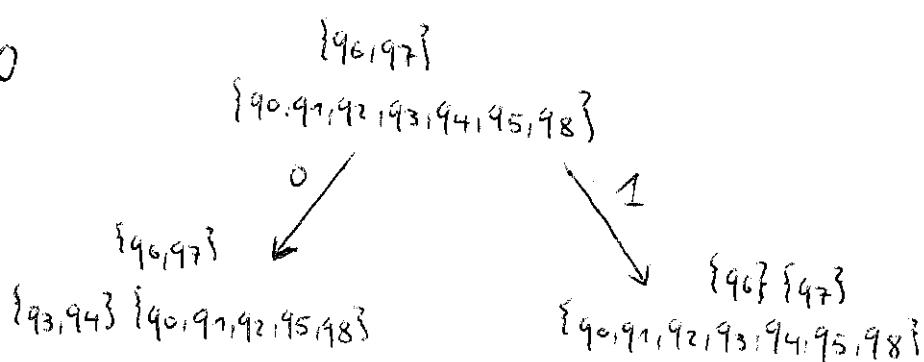


SOLUTIONS (18/12/2008)

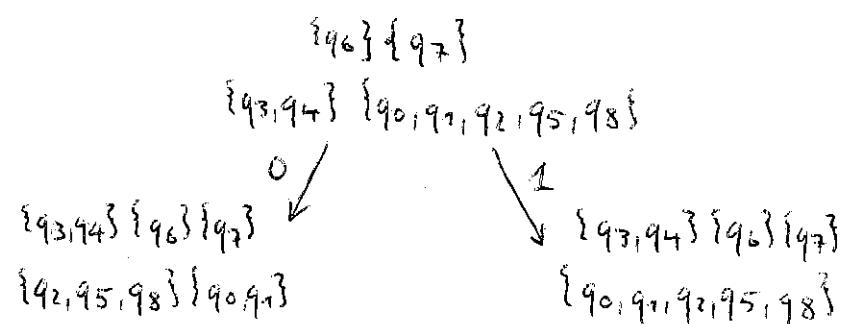
E10.2

- 1) We start with the following initial partition:
- $$C_1 = F = \{q_6, q_7\} \text{ and } C_2 = Q - F = \{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}.$$

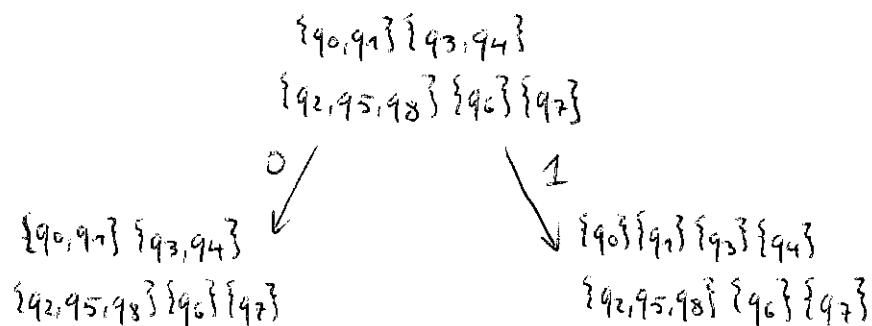
Step 0



Step 1

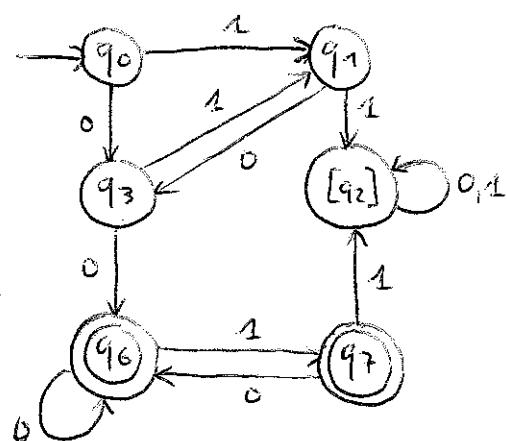


Step 2



Final partition: $\{q_0\} \{q_1\} \{q_2, q_5, q_8\} \{q_3\} \{q_4\} \{q_6\} \{q_7\}$

The minimized automaton looks as follows:



Note: q_4 is not reachable, thus it has been omitted from the minimized automaton

E10.3

2) Initial partition:

$$C_1 = \{q_0, q_1, q_2, q_3\}, C_2 = \{q_4\}$$

Step 0

$$\{q_0, q_1, q_2, q_3\} \{q_4\}$$

↓
a,b,c

Step 1

$$\{q_0, q_1, q_2\} \{q_3\} \{q_4\}$$

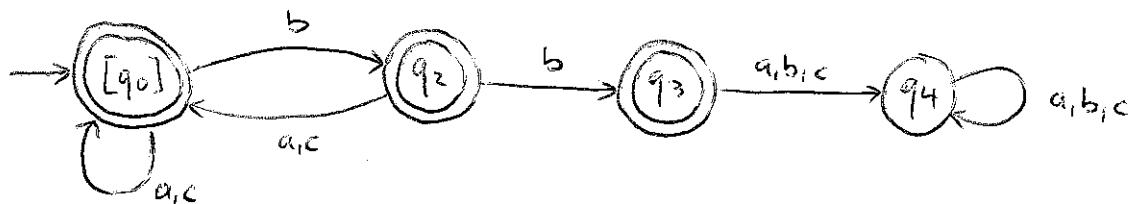
a,c b

$$\begin{array}{c} \{q_0, q_1, q_2\} \{q_3\} \{q_4\} \\ \hline \{q_0, q_1\} \{q_2\} \{q_3\} \{q_4\} \end{array}$$

$$\{q_0, q_1\} \{q_2\} \{q_3\} \{q_4\}$$

Final partition: $\{q_0, q_1\} \{q_2\} \{q_3\} \{q_4\}$

The minimized automaton looks as follows:



3) Initial partition:

$$C_1 = \{q_0, q_2, q_6, q_8\}, C_2 = \{q_1, q_3, q_4, q_5, q_7\}$$

Step 0

$$\{q_0, q_2, q_6, q_8\}$$

$$\{q_1, q_3, q_4, q_5, q_7\}$$

0

1

$$\{q_0, q_2, q_6, q_8\}$$

$$\{q_1, q_7\} \{q_3, q_4, q_5\}$$

$$\{q_0, q_2, q_6, q_8\}$$

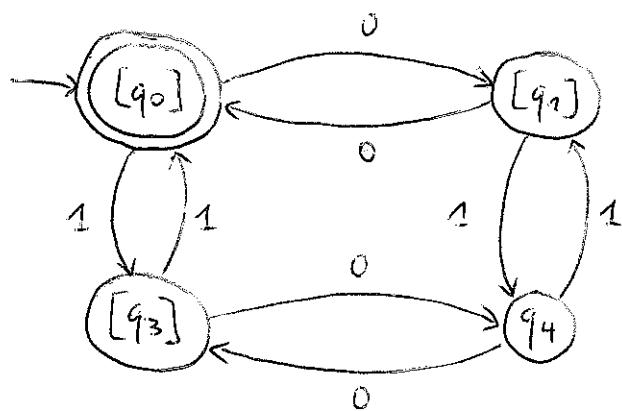
$$\{q_3, q_5\} \{q_1, q_4, q_7\}$$

Final partition: $\{q_0, q_2, q_6, q_8\} \{q_1, q_7\} \{q_3, q_5\} \{q_4\}$

3) cont

E10.4

The minimized automaton looks as follows:



Alternative solution:

One could first minimize the automata A_L^* and A_R^* of the last exercise in E8 (12/12/2008) and then apply the so-called product construction