# FINITE AUTOMATA VS. REGULAR EXPRESSIONS

27/11/2008

E7.1

## EXERCISE 1

Convert the following regular expressions to $\varepsilon$-NFA's.

a) $(\varepsilon+1)(01)^*(\varepsilon+0)$ b) $a^*b^*c^*$

## EXERCISE 2

Convert the following DFA to a regular expression, using the state elimination technique.

| | 0 | 1 |
|---|---|---|
| $\rightarrow * p$ | s | p |
| q | p | s |
| r | r | q |
| s | q | r |

## EXERCISE 3

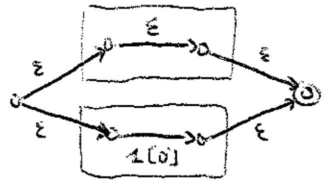
Repeat exercise 2, eliminating the states in a different order. Then, verify that the following binary strings (accepted by the above DFA) are in the language of the regular expressions obtained here and in exercise 2.

a) 00101100101 b) 010001101110

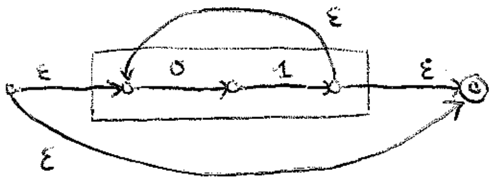
SOLUTIONS (27/11/2008) 

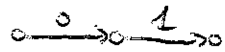
1) a) ε-NFA's for $\varepsilon+1$, $\varepsilon+0$, and $(01)^*$ are as follows:

$\varepsilon+1$
$[\varepsilon+0]$

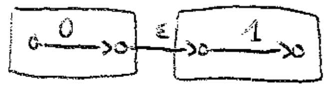



$(01)^*$

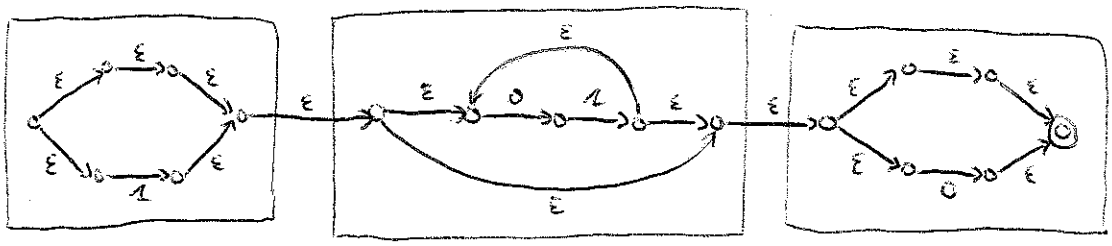


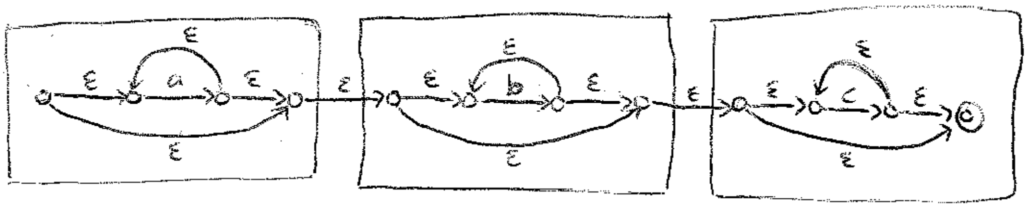

is shorthand for



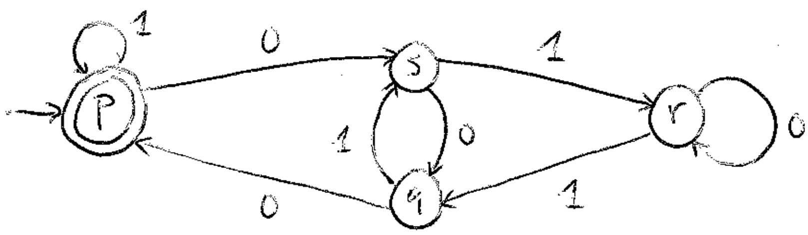

Composition of the above ε-NFA's yields:



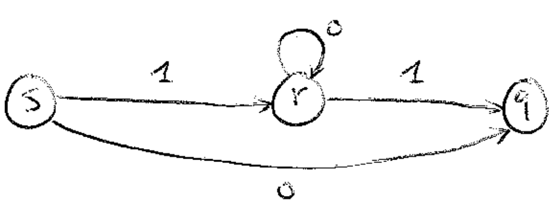
1) b) We get the following ε-NFA:




2) The DFA looks as follows:



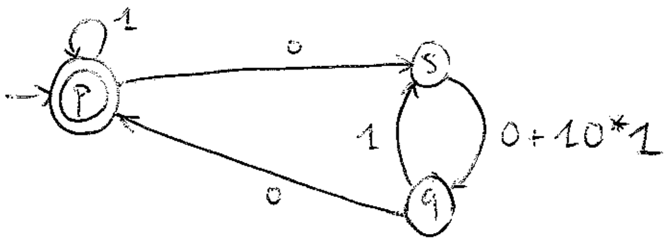

First, we eliminate the state r



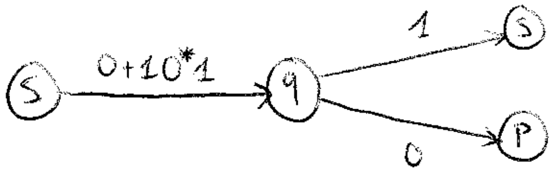

$(0+10^*1$ for $s \to q)$

2) con't E7.3

and obtain



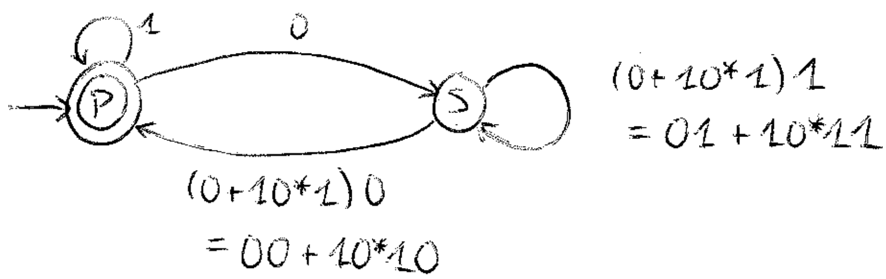

Second, we eliminate the state q



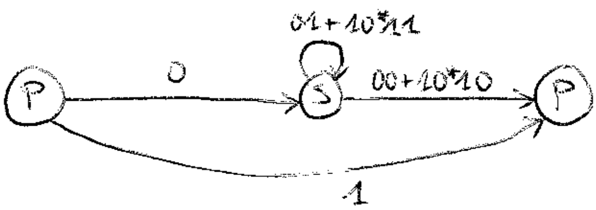

((0+10*1)·1 for s→s)

((0+10*1)·0 for s→p)

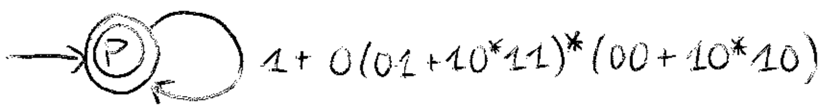
and obtain




Third, we eliminate the state s




and obtain




The regular expression is therefore

$$(1+0(01+10^*11)^*(00+10^*10))^* = E$$

E7.4

3) Note that, if we eliminate first the state r, but then s before q, we get the following regular expression:

$$E' = (1 + (00+010^*1)(10+110^*1)^*0)^*$$

3) a) $00101100101 \in \mathcal{L}(E)$:

$$(0\underbrace{(01)(01)}_{\text{from } (01+10^*11)^*}\underbrace{(10010)}_{\text{from } (00+10^*10)})1$$

$00101100101 \in \mathcal{L}(E')$

$$(\underbrace{(00)}_{\text{from } (00+010^*1)}\underbrace{(10)(11001)}_{\text{from } (10+110^*1)^*}0)1$$

3) b) $0\underbrace{(100011)(01)}_{\text{from } (01+10^*11)^*}\underbrace{(110)}_{\text{from } (00+10^*10)} \in \mathcal{L}(E)$

$\underbrace{(010001)}_{\text{from } (00+010^*1)}\underbrace{(10)(111)}_{\text{from } (10+110^*1)^*}0 \in \mathcal{L}(E')$