FINITE AUTOMATA

E1.1

22/10/2007

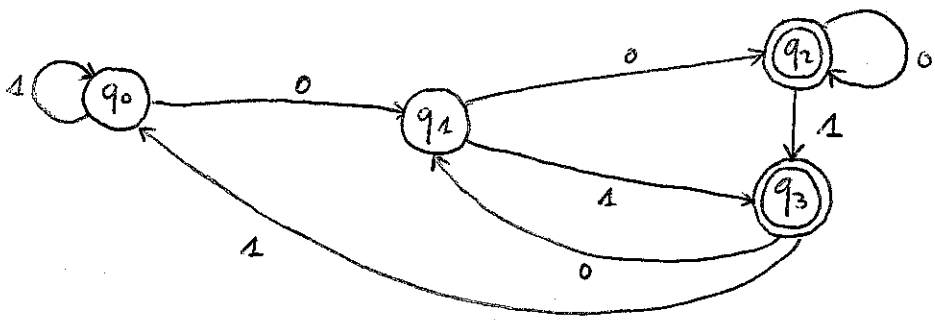
Exercise 1

a) Give a DFA accepting the following language over the alphabet $\{0,1\}$: the set of all strings such that the second last symbol is 0.

b) Show that the string 001101 is accepted by the DFA.

Solution:

a)



b) We show that $\hat{\delta}(q_0, 001101) = q_3$

- $\hat{\delta}(q_0, \varepsilon) = q_0$
- $\hat{\delta}(q_0, 0) = \delta(\hat{\delta}(q_0, \varepsilon), 0) = \delta(q_0, 0) = q_1$
- $\hat{\delta}(q_0, 00) = \delta(\hat{\delta}(q_0, 0), 0) = \delta(q_1, 0) = q_2$
- $\hat{\delta}(q_0, 001) = \delta(\hat{\delta}(q_0, 00), 1) = \delta(q_2, 1) = q_3$
- $\hat{\delta}(q_0, 0011) = \delta(\hat{\delta}(q_0, 001), 1) = \delta(q_3, 1) = q_0$
- $\hat{\delta}(q_0, 00110) = \delta(\hat{\delta}(q_0, 0011), 0) = \delta(q_3, 0) = q_1$
- $\hat{\delta}(q_0, 001101) = \delta(\hat{\delta}(q_0, 00110), 1) = \delta(q_1, 1) = q_3$

Exercise 2 (2.2.5 from textbook)

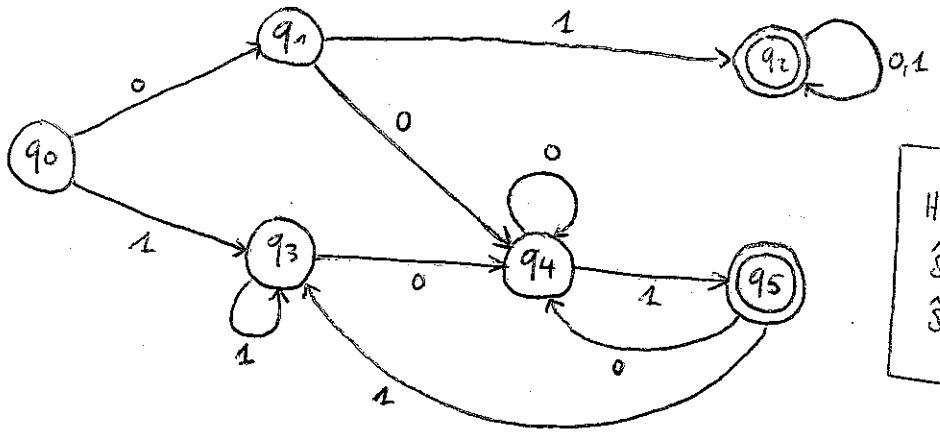
Give DFA's accepting the following languages over the alphabet $\{0,1\}$:

a) The set of strings that either begin or end (or both) with 01;

b) The set of strings such that the number of 0's is divisible by five, and the number of 1's is divisible by three.

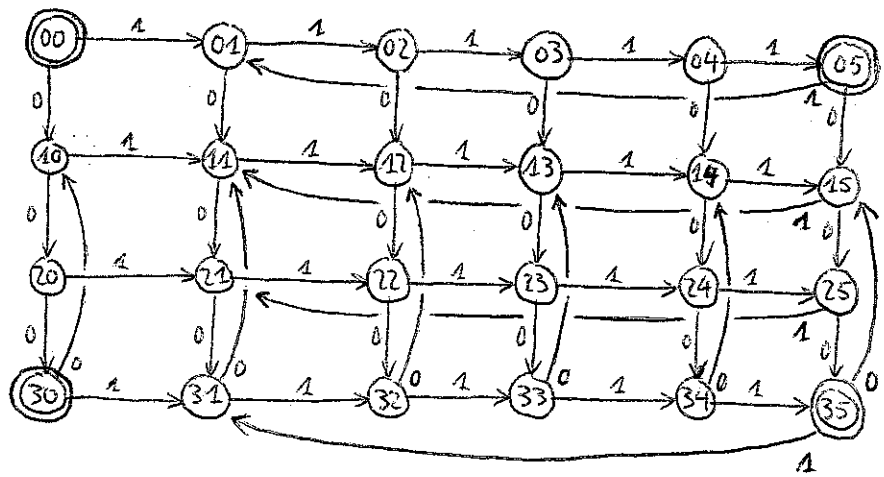
Solution:

a)



HW: Show that
$\hat{\delta}(q_0, 0001) = q_5$
$\hat{\delta}(q_0, 0101) = q_2$

b)

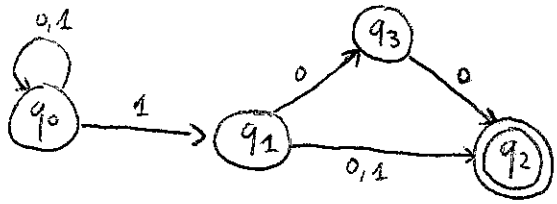


HW: Give an automaton that uses a smaller number of states

Exercise 3a

Give a NFA accepting the following language over the alphabet $\{0,1\}$: the set of strings that end with 10, 11, or 100.

Solution:



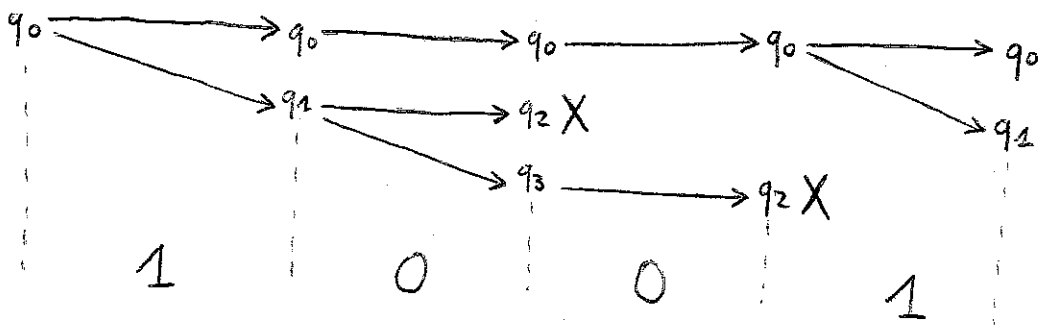
Exercise 3b

Show that the string 1001 is not accepted by the [redacted] above NFA.

Solution:

We show that $q_2 \notin \hat{\delta}(q_0, 1001)$.

- $\hat{\delta}(q_0, \varepsilon) = \{q_0\}$
- $\hat{\delta}(q_0, 1) = \delta(q_0, 1) = \{q_0, q_1\}$
- $\hat{\delta}(q_0, 10) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0\} \cup \{q_2, q_3\} = \{q_0, q_2, q_3\}$
- $\hat{\delta}(q_0, 100) = \delta(q_0, 0) \cup \delta(q_2, 0) \cup \delta(q_3, 0) = \{q_0\} \cup \emptyset \cup \{q_2\} = \{q_0, q_2\}$
- $\hat{\delta}(q_0, 1001) = \delta(q_0, 1) \cup \delta(q_2, 1) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$



Exercise 4 (2.3.4 from textbook)

Give NFA's accepting the following languages:

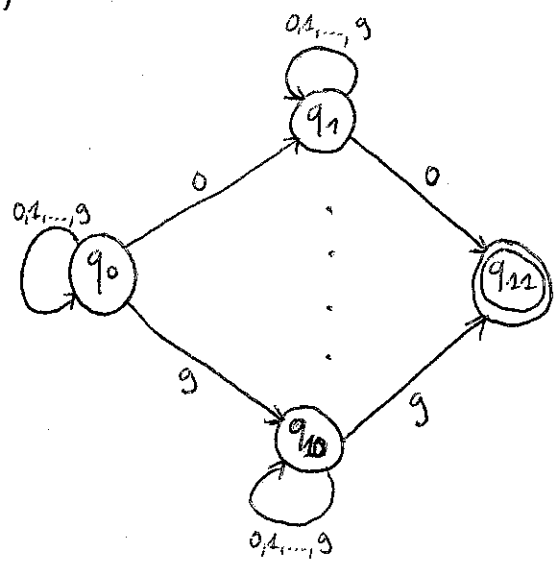
a) The set of strings over $\{0,1,\dots,9\}$ such that the final digit has appeared before;

b) The set of strings over $\{0,1,\dots,9\}$ such that the final digit has <u>not</u> appeared before;

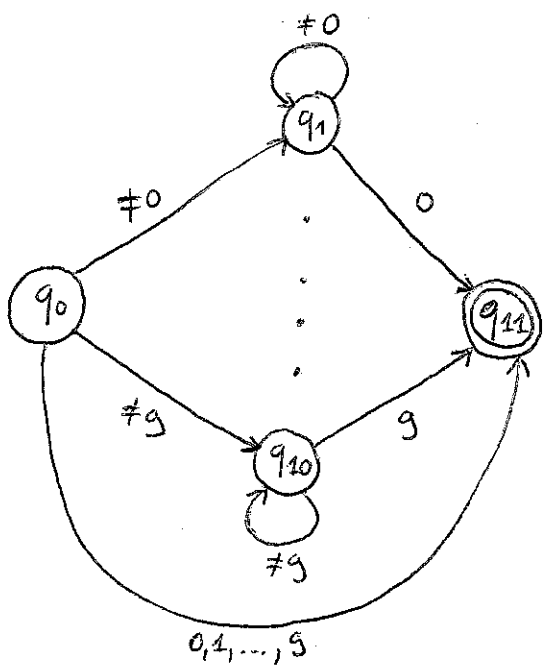
c) The set of strings over $\{0,1\}$ such that there are two 0's separated by a number of positions that is a multiple of 4.

Solution:

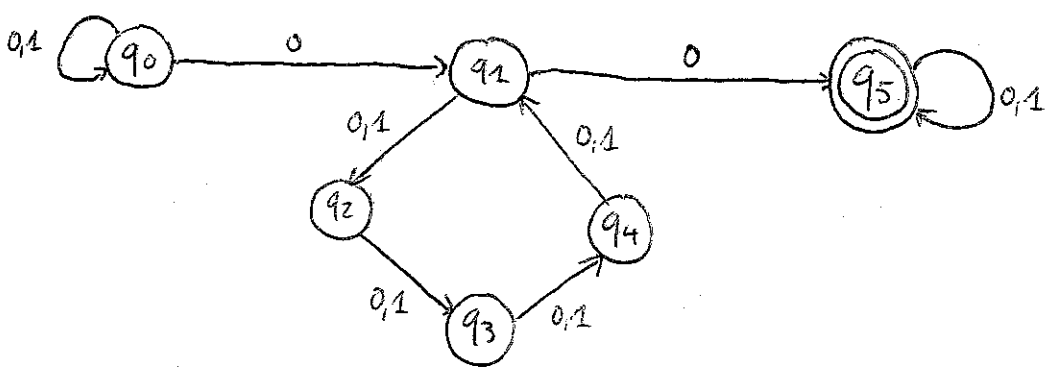
a)



b)



c)



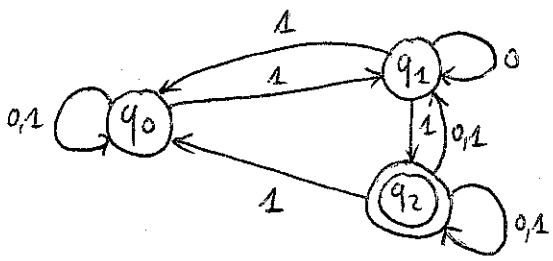
Exercise 5

Convert to a DFA the following NFA:

		0	1
→	q_0	$\{q_0\}$	$\{q_0, q_1\}$
	q_1	$\{q_1\}$	$\{q_0, q_2\}$
*	q_2	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$

Solution:

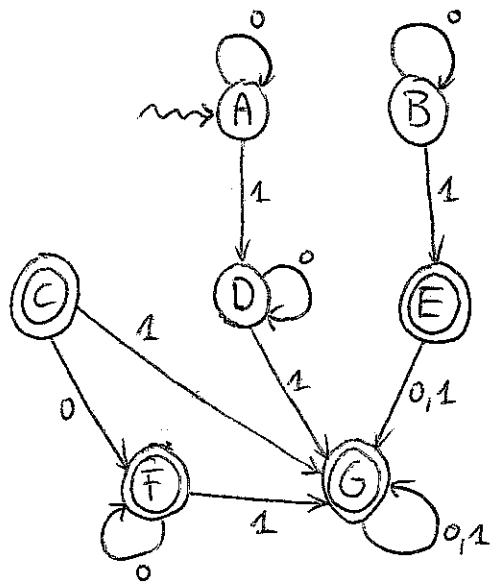
1) The NFA looks as follows:



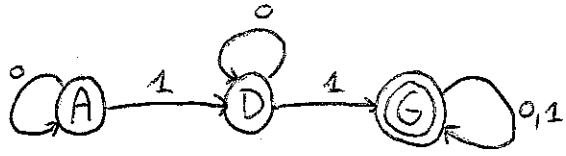
2) The subset construction yields:

		0	1
	$\emptyset$	$\emptyset$	$\emptyset$
→	[A =] $\{q_0\}$	$\{q_0\}$ [A]	$\{q_0, q_1\}$ [D]
	[B =] $\{q_1\}$	$\{q_1\}$ [B]	$\{q_0, q_2\}$ [E]
*	[C =] $\{q_2\}$	$\{q_1, q_2\}$ [F]	$\{q_0, q_1, q_2\}$ [G]
	[D =] $\{q_0, q_1\}$	$\{q_0, q_1\}$ [D]	$\{q_0, q_1, q_2\}$ [G]
*	[E =] $\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$ [G]	$\{q_0, q_1, q_2\}$ [G]
*	[F =] $\{q_1, q_2\}$	$\{q_1, q_2\}$ [F]	$\{q_0, q_1, q_2\}$ [G]
*	$\{q_0, q_1, q_2\}$ [G]	$\{q_0, q_1, q_2\}$ [G]	$\{q_0, q_1, q_2\}$ [G]

3) The DFA look as follows:



4) If we delete inaccessible states we get:



Thus the automaton accepts strings with at least two 1's.