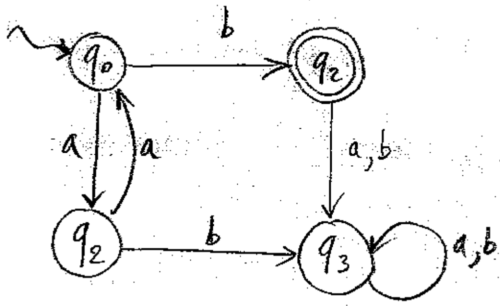
Exercise

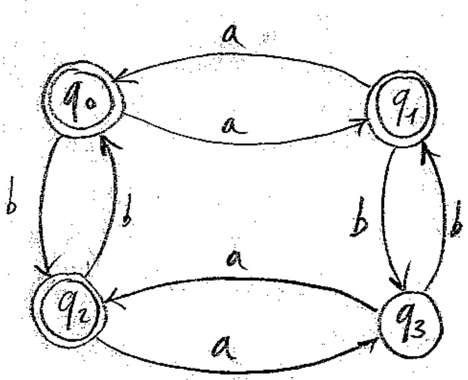
Construct a DFA that accepts the language $\{a^{2n} b\}$, $n \geq 0$



q_3 is a sort of "error state"; it can be omitted for brevity.

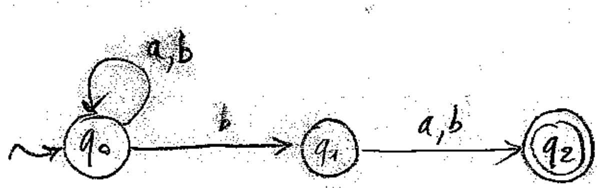
Exercise

Construct a DFA that accepts strings containing an even number of a or (not exclusive) an even number of b, on the alphabet $\{a, b\}$.



Exercise

Construct a NFA accepting strings on alphabet $\{a,b\}$ in which the symbol before the last one is b.



Exercise (2.3.4 from textbook)

Give non-deterministic finite automata that accept the following languages:

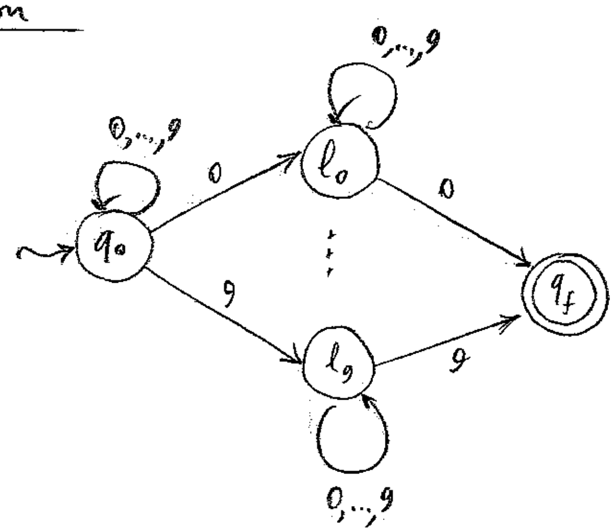
(a) strings over $\{0,\dots,9\}$ such that the last digit has appeared before

(b) strings over $\{0,\dots,9\}$ such that the last digit has not appeared before

(c) strings over $\{0,1\}$ such that there are two zeros separated by a number of digits that is a multiple of four (including 0)

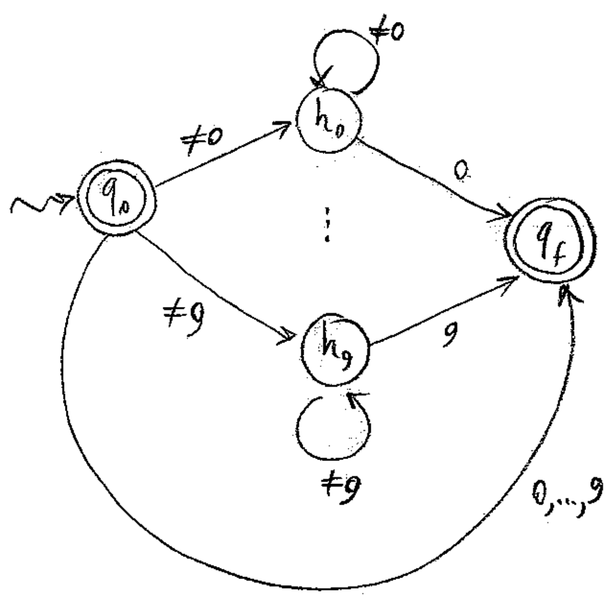
solution

(a)

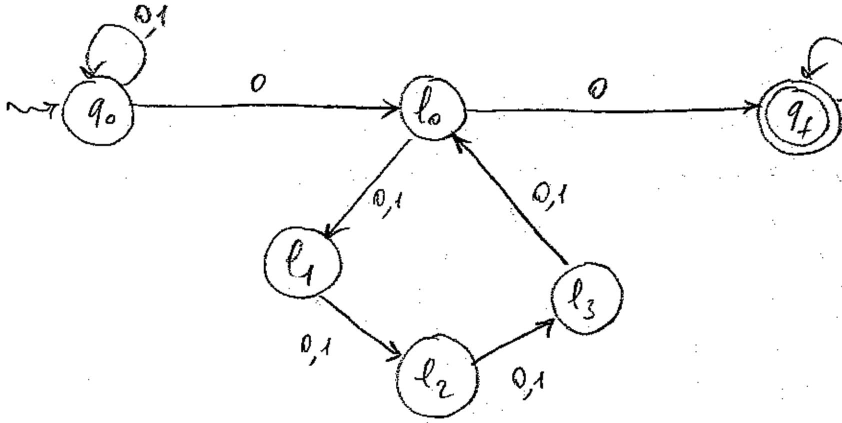


we use states l_i with $0 \leq i \leq 9$ to guess that final digit is i

(b)



E 2.3

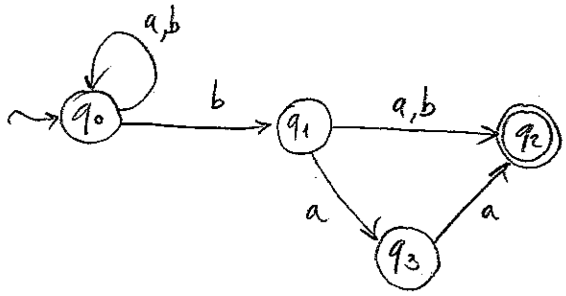


Exercise

E 2.4

Construct a NFA on alphabet $\{a,b\}$ accepting strings that end with ba, bb or baa. Construct a DFA that is equivalent to it.

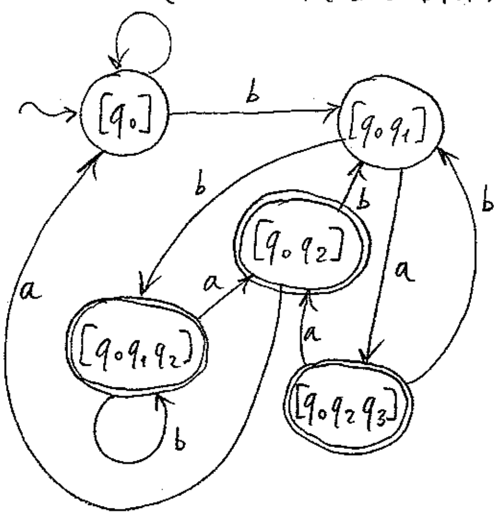
solution



We write the transition function δ of the required DFA directly, in a sort of breadth-first visit of the automaton.

$$\delta([q_0], a) = [q_0]$$
$$\delta([q_0], b) = [q_0 q_1]$$
$$\delta([q_0 q_1], a) = [q_0 q_2 q_3]$$
$$\delta([q_0 q_1], b) = [q_0 q_1 q_2]$$
$$\delta([q_0 q_2 q_3], a) = [q_0 q_2]$$
$$\delta([q_0 q_2 q_3], b) = [q_0 q_1]$$
$$\delta([q_0 q_1 q_2], a) = [q_0 q_2]$$
$$\delta([q_0 q_1 q_2], b) = [q_0 q_1 q_2]$$
$$\delta([q_0 q_2], a) = [q_0]$$
$$\delta([q_0 q_2], b) = [q_0 q_1]$$

Final DFA:



Exercise

Prove that for every regular language E we have $E^* = (E^*)^*$.

solution

We need to prove both inclusions $E^* \subseteq (E^*)^*$ and $(E^*)^* \subseteq E^*$.

$E^* \subseteq (E^*)^*$ trivial

$(E^*)^* \subseteq E^*$

Consider $w \in (E^*)^*$. We want to prove that $w \in E^*$.
We know that

$$w = w_1 \cdot \ldots \cdot w_n \quad , \text{with } w_i \in E^* \ , \ 1 \le i \le n \ , \ n \in \mathbb{N}$$

On the other hand, for all $i \in \{1, \ldots, n\}$:

$$w_i = w_{i1} \cdot \ldots \cdot w_{in_i} \quad , \text{with } w_{ij} \in E \ , \ 1 \le j \le n_i \ , \ n_i \in \mathbb{N}.$$

Therefore $w = (w_{11} \cdot \ldots \cdot w_{1n_1}) \cdot \ \ldots \ \cdot (w_{n1} \cdot \ldots \cdot w_{nn_n})$.

Since w is a concatenation of strings of E, the thesis follows.

Exercise (2.3.5 from textbook)

base step: $|w|=1$

let $w=a$, $a\in\Sigma$;

we have $\hat{\delta}_N(q,w)=\hat{\delta}_N(q,a)=\{\delta_D(q,a)\}=\{p\}$ by construction

inductive step : $|w|>1$

let $w=xa$, $x\in\Sigma^*$, $a\in\Sigma$

We have by definition

$$p=\hat{\delta}_D(q,w)=\hat{\delta}_D(q,xa)=\delta_D(\hat{\delta}_D(q,x),a)=\delta_D(r,a)$$

where we have denoted $r=\hat{\delta}_D(q,x)$

By induction hypothesis we know

$$\hat{\delta}_N(q,x)=\{\hat{\delta}_D(q,x)\}=\{r\}$$

Again, by definition

$$\hat{\delta}_N(q,w)=\hat{\delta}_N(q,xa)=\bigcup_{h\in\hat{\delta}_D(q,x)}\delta_N(h,a)=$$ (by induction hyp.)

$$=\bigcup_{h\in\{r\}}\delta_N(h,a)=\delta_N(r,a)$$

By construction we have

$$\delta_N(r,a)=\{\delta_D(r,a)\}=\{p\}$$ q.e.d.