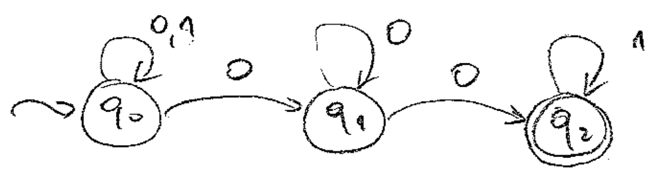
Exercise: E7.1

Consider the following NFA $A = (Q, \Sigma, \delta, q_0, F)$

δ:



$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

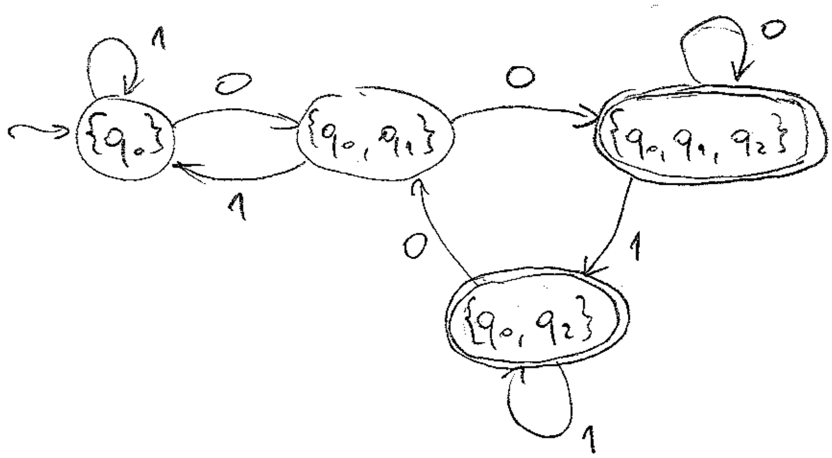
$F = \{q_2\}$

1) Describe the language accepted by this NFA

2) Convert the NFA to a DFA using the subset construction. Keep only the essential states

1) $\mathcal{L}(A) = \{ w \in \{0,1\}^* \mid w$ ends with a sequence of at least two 0's followed by only 1's $\}$

2)



$A_D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$

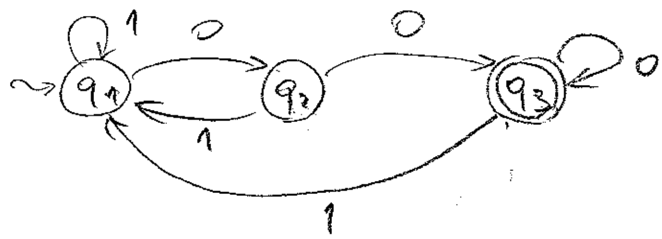
with $Q_D = 2^Q$

$q_{0D} = \{q_0\}$

$F_D = \{ S \subseteq Q \mid F \cap S \neq \emptyset \}$

Exercise:

Consider the following DFA $A = (Q, \Sigma, \delta, q_1, F)$



Construct a regular expression E_A s.t. $\mathcal{L}(E_A) = \mathcal{L}(A)$

by constructing the R.E. E_{ij}^{k} s.t.

$\mathcal{L}(E_{ij}^{k}) = L_{ij}^{k} = \{w \mid A$ goes from q_i to q_j on input w passing only through $q_1, \dots, q_k\}$

k	E_{11}^k	E_{12}^k	E_{13}^k	E_{21}^k	E_{22}^k	E_{23}^k	E_{31}^k	E_{32}^k	E_{33}^k
0	$\varepsilon + 1$	0	$\emptyset$	1	ε	0	1	$\emptyset$	$\varepsilon + 0$
1	1^*	$1^* \cdot 0$	$\emptyset$	$1 \cdot 1^*$	$\varepsilon + 1 \cdot 1^* \cdot 0$	0	$1 \cdot 1^*$	$1 \cdot 1^* \cdot 0$	$\varepsilon + 0$
2	$1^* + 1^* 0 \cdot (1 \cdot 1^* 0)^* \cdot 1 \cdot 1^*$	$1^* \cdot 0 \cdot (1 \cdot 1^* 0)^*$	$1^* 0 \cdot (1 \cdot 1^* 0)^* \cdot 0$	$(1 \cdot 1^* 0)^* \cdot 1 \cdot 1^*$	$(1 \cdot 1^* 0)^*$	$(1 \cdot 1^* 0)^* \cdot 0$	$1 \cdot 1^* \cdot 0 \cdot (1 \cdot 1^* 0)^* \cdot 1 \cdot 1^* =$	$1 \cdot 1^* \cdot 0 \, (1 \cdot 1^* 0)^*$	$1 \cdot 1^* \cdot 0 \cdot (1 \cdot 1^* \cdot 0)^* \cdot 0 + \varepsilon + 0$
3									

$$1^* 0 \cdot (1 \cdot 1^* 0)^* \cdot 0 \cdot \left(\varepsilon + 0 + (1 \cdot 1^* 0)(1 \cdot 1^* 0)^* \cdot 0\right)^*$$

We use $E_{ij}^{k} = E_{ik}^{k-1} \cdot \left(E_{kk}^{k-1}\right)^* \cdot E_{kj}^{k-1} + E_{ij}^{k-1}$

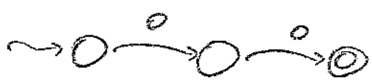
Exercise:

Convert the following regular expression E to an ε-NFA, and then eliminate the ε-transitions:

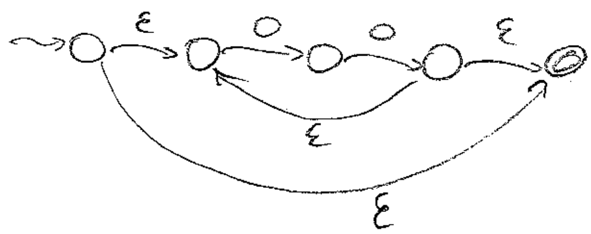
$$E = 1\cdot((0\cdot 0)^* + 1^*)\cdot$$

accepts 0

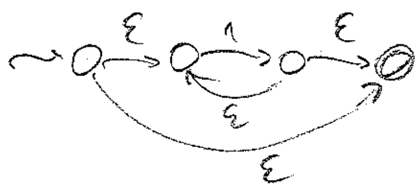
— " — 1



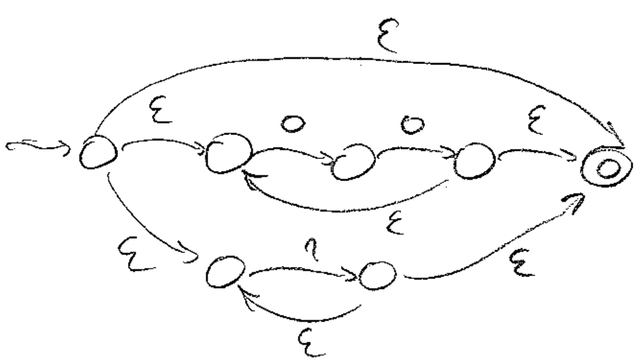
accepts 00 (we have merged initial and final states)



accepts $(00)^*$

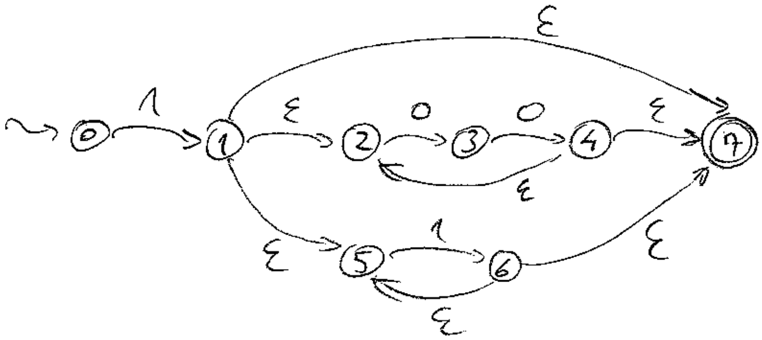


accepts 1^*



accepts $(00)^* + 1^*$

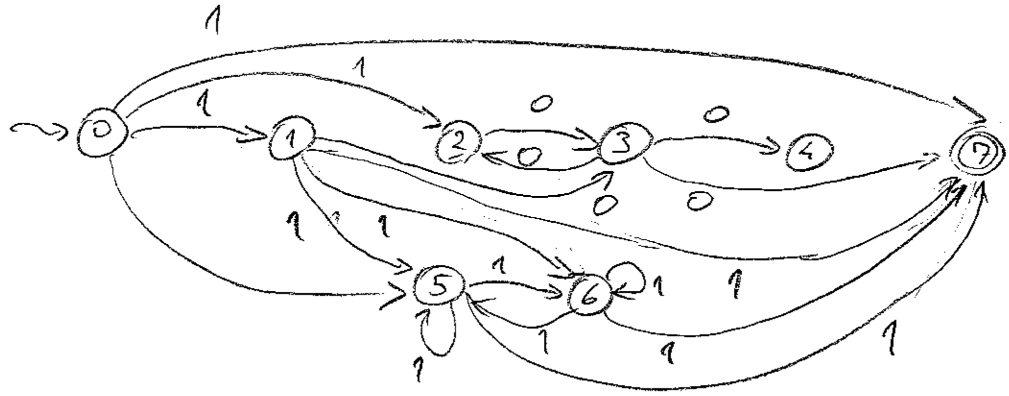
(we have merged initial and final states)



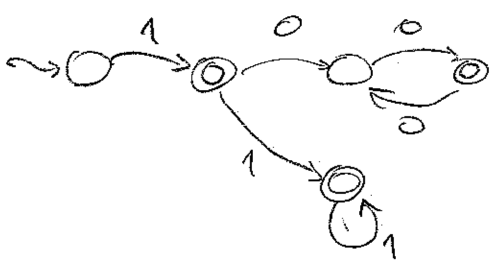
E7.4

To eliminate ε-transitions, remember that

$$\delta_N(q, a) = \text{Eclose}\Big(\bigcup_{p_i \in \text{Eclose}(q)} \delta(p_i, a)\Big)$$



A simpler automaton for $\mathcal{L}(E)$ is the following



Exercise 4.2.3

Let L be a language over Σ, and $a \in \Sigma$

We define $L/a = \{w \in \Sigma^* \mid wa \in L\}$

Example: $L = \{a, aab, baa, bba\}$

$L/a = \{\varepsilon, ba, bb\}$

L/a is called the quotient of L and a.

Prove the following: If L is regular, so is L/a, for $a \in \Sigma$.
(closure under quotient)

Proof: Let $A_L = (Q, \Sigma, \delta, q_0, F)$ be a DFA s.t. $\mathcal{L}(A_L) = L$.

We define $A_{L/a} = (Q, \Sigma, \delta, q_0, F_a)$ with

$F_a = \{q \in Q \mid \delta(q, a) \in F)$

We show that $\mathcal{L}(A_{L/a}) = L/a$

1) $L/a \subseteq \mathcal{L}(A_{L/a})$

Let $w.a \in L$. Then $\hat{\delta}(q_0, w.a) = \delta(\hat{\delta}(q_0, w), a) \in F$

Hence, by definition of F_a, $\hat{\delta}(q_0, w) \in F_a$.

It follows that $w \in \mathcal{L}(A_{L/a})$

2) $\mathcal{L}(A_{L/a}) \subseteq L/a$

Let $w \in \mathcal{L}(A_{L/a})$. Then $\hat{\delta}(q_0, w) = q \in F_a$

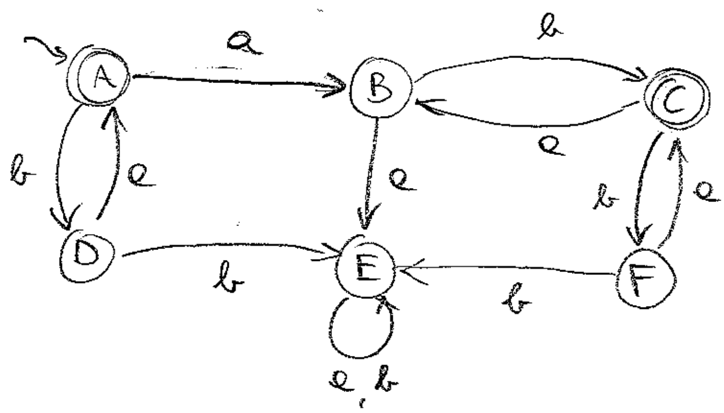
Hence, by definition of F_a, $\delta(q, a) \in F$.

Hence, $\delta(q, a) = \delta(\hat{\delta}(q_0, w), a) = \hat{\delta}(q_0, w.a) \in F$ and $w.a \in L$. It follows that $w \in L/a$.

16/11/2005 E7.6

Exercise:

Minimize the following DFA



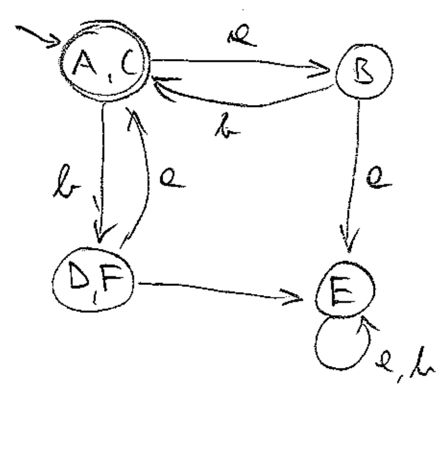
We construct the table of distinguishabilities, which corresponds to determine the equivalence classes wrt $\equiv_i$

	A	B	C	D	E
B	0				
C		0			
D	0	1	0		
E	0	1	0	1	
F	0	1	0		1

$\equiv_0$: $\{A, C\}$ $\{B, D, E, F\}$

$\equiv_1$: $\{A, C\}$ $\{B\}$, $\{E\}$ $\{D, F\}$

$\equiv_2 = \equiv_1$



Exercise 5.11. c

E7.7

Define a context-free grammar that generates the set $L_{\overline{ww}}$ of all strings over $\{a, b\}$ that are not of the form ww (i.e. are not equal to any string repeated)

$G = (V_N, V_T, P, S)$

with $V_N = \{S, A, B, C\}$

$V_T = \{a, b\}$

P :

$$
\begin{aligned}
S &\to A \mid B \mid AB \mid BA \\
A &\to CAC \mid a \\
B &\to CBC \mid b \\
C &\to a \mid b
\end{aligned}
$$

Explanation:

- A generates an arbitrary odd length string with an a in the middle
  B — " — b — "

- Each odd-length string is in $L_{\overline{ww}}$, hence $S \to A \mid B$
- Let w be an even length string in $L_{\overline{ww}}$.
  Then $w = x \cdot y$ with $|x| = |y|$, and x and y differ in at least one character: let $x = c^i a c^j$
  $y = c^i b c^j$ (similarly for $x = c^i b c^j$, $y = c^i a c^j$)
  position where x and y differ
  (each c stands for either a or b)

  Then: $w = x \cdot y = c^i a c^j c^i b c^j = \underbrace{c^i a c^i}_{u} \underbrace{c^j b c^j}_{v} =$

  Hence u is an odd length string with an a in the middle
  v — " — b — " —

  Hence $S \to AB$ (similarly, $S \to BA$)