ES.1

Exercise

Give a grammar for the language $\{a^n b^n \mid n \geq 1\}$.

solution $S \to ab \mid aSb$

Exercise

Give a grammar for the language $\{a^n b^{n+1} \mid n \geq 1\}$

solution

$S \to aSb \mid abb$

This solution can be trivially extended to generate $\{a^n b^{n+k} \mid n \geq 1, k \geq 0\}$

$S \to aSb \mid ab^{k+1}$

Exercise

Give a grammar for palindrome strings on $\Sigma = \{a,b\}$, i.e. for the language $\{w \in \Sigma^* \mid w = w^R\} =$

$= \{ww^R \mid w \in \Sigma^*\} \cup \{wcw^R \mid w \in \Sigma^* \text{ and } c \in \Sigma\}$

solution $S \to aSa \mid bSb \mid a \mid b \mid \epsilon$

Exercise

Give a grammar for the language $\{ww^R \mid w \in \{a,b\}^*\}$ (palindromes on $\Sigma = \{a,b\}$ having even length).

solution $S \to aSa \mid bSb \mid \epsilon$

Similarly, we can generate $\{ww^R \mid w \in \{a,b\}^+\}$ with the grammar

$S \to aSa \mid bSb \mid aa \mid bb$

E S.2

Exercise Give a grammar generating the language

$\{ a^n b^n c^n \mid n \geq 1 \}$.

solution (simpler than the one presented in lectures)

$$S \to aSBc \mid abc$$
$$cB \to Bc$$
$$bB \to bb$$

Exercise Give a grammar for the language

$L = \{ a^n b^n c^n d^n \mid n \geq 1 \}$.

solution

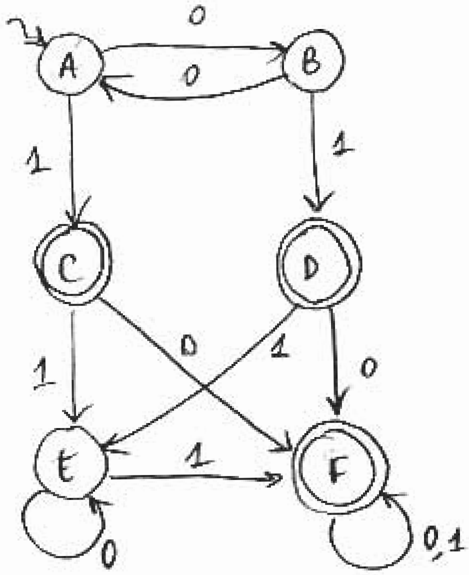
$$S \to aSBCd \mid abcd$$
$$dB \to Bd$$
$$dC \to Cd$$
$$CB \to BC$$
$$bB \to bB$$
$$bC \to bc \qquad (*)$$
$$cC \to cc$$

Notice that the production marked with (*) can be applied earlier than necessary, leading to strings that do not produce any string of $(V_T)^*$; this is fine, since we do not have "spurious" strings with respect to our desired language.

E 5.3

Exercise

Construct the minimum DFA equivalent to the one in the figure below:



solution

We construct the table of distinguishabilities:

	A	B	C	D	E
B					
C	0	0			
D	0	0			
E	2	2	0	0	
F	0	0	1	1	0

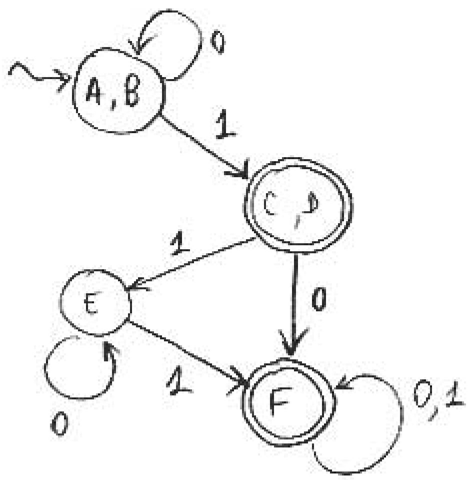
First, we mark immediately all pairs in which C, D or F appear together with some non-final state (we put mark "0" here).
Then we mark pairs $\{C,F\}$ and $\{D,F\}$: in fact for each of such pairs (q_i, q_j) we have that the pair

$\{\delta(q_i, a), \delta(q_j, a)\}$ is a pair marked with "0" at the previous step.

E 5.4

We go on, marking analogously $\{A,E\}$ and $\{B,E\}$ with "2", and realizing that there is no more pair to mark.

The partitions of equivalent blocks are $\{A,B\}$ and $\{D,C\}$.
The minimal equivalent DFA is:



E 5.5

Exercise

Give a grammar for the language $\{ww \mid w \in \{a,b\}^+\}$.

solution

$$S \to aAS \mid bBS \mid aA_0 \mid bB_0$$

(A_0, B_0 mark the end of the string)

$Aa \to aA$
$Ba \to aB$
$Bb \to bB$
$Ba \to aB$

put all A and B at the end of the string

$AA_0 \to A_0a$
$BA_0 \to B_0a$
$AB_0 \to A_0b$
$BB_0 \to B_0b$
$A_0 \to a$
$B_0 \to b$

Note that if we apply the last two productions too early, we get strings that do not produce any string made of terminal symbols only.

ES.6

Exercise

Give a grammar for the language $\{a^{2^n} \mid n \geq 0\}$.

solution

$S \rightarrow IaHF \mid a$

$aH \rightarrow Haa$

$IH \rightarrow IK$

$Ka \rightarrow aaK$

$KF \rightarrow HF$

$KF \rightarrow \epsilon$

$IH \rightarrow \epsilon$

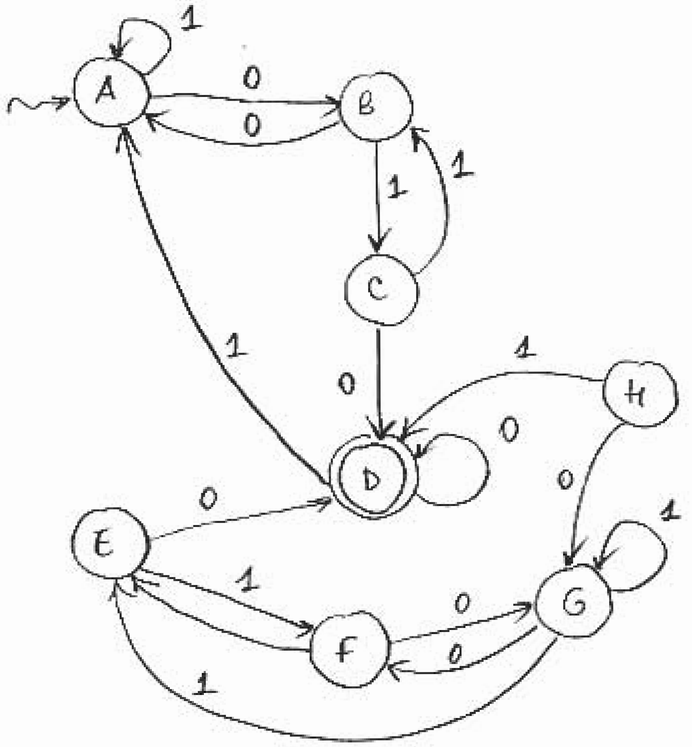
$I \rightarrow \epsilon$

$F \rightarrow \epsilon$

H and K are two markers that multiply the number of a in the middle of the string by two ; I and F mark the beginning and the end of the string respectively.
H goes from right to left and turns itself into K when it reaches I ; K goes from left to right and turns itself into H when it reaches F.

Exercise (4.4.1 from textbook)

Construct the minimal DFA equivalent to the following:



transition table:

	0	1
→ A	B	A
B	A	C
C	D	B
* D	D	A
E	D	F
F	G	E
G	F	G
H	G	D

solution

the distinguishability table is as follows:

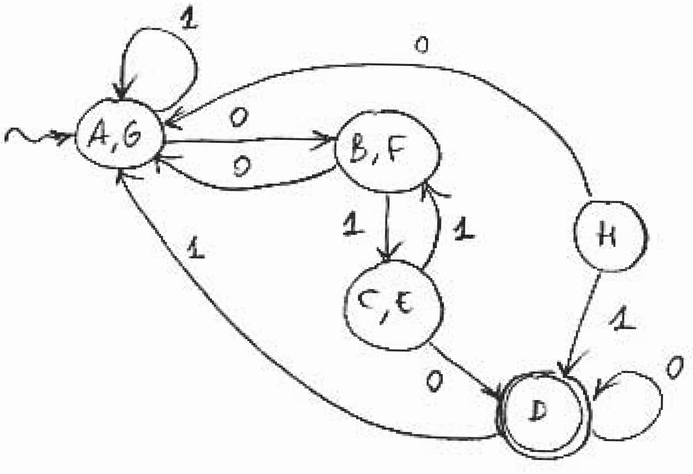
	A	B	C	D	E	F	G
B	2						
C	1	1					
D	0	0	0				
E	1	1	·	0			
F	2	·	1	0	1		
G	·	2	1	0	1	2	
H	1	1	1	0	1	1	1

The equivalence classes are

{A, G} , {B, F} , {C, E} , {D} , {H} .

E 5.8

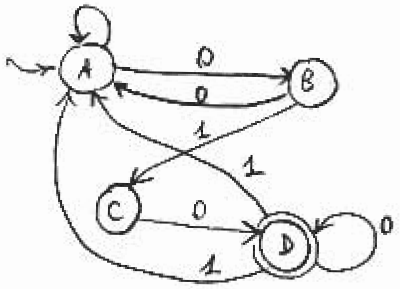
The minimised automaton is as follows



		0	1
→	A,G	B,F	A,G
	B,F	A,G	C,E
	C,E	D	BF
*	D	D	A,G
	H	AG	D

State H is not reachable and needs to be eliminated.

Notice that in the initial automaton is such that states E, F, G and H are not reachable. If we eliminate the non-reachable states we obtain the following automaton:



which is already minimal (verification is left to the reader).