The Actual Weight of Lightweight Description Logics

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[Background images in this presentation were generated using ChatGPT (GPT-5, OpenAI, 2025).]

Outline

Outline


Prehistory
1970's

1970's: Class-based formalisms

Beginning of the 1970's

Many areas of CS independently proposed **class-based formalisms**:

- Semantic Networks (AI)
- Frame Systems (AI)
- Entity-Relationship Schemas (DB)
- Object-Orientation (SE)

By the end of the 1970's

The need for a formal account was evident:

- **William A. Woods – What's in a link?** [Woods, 1975, AAAI]: no clear semantics, reasoning not well understood.

Concept / Terminological Languages

Logic-based formalisms specifically designed to represent class-oriented structured knowledge:

Domain consists of objects organized into:

- Concepts: correspond to classes, denote sets of objects.
- Roles: correspond to (binary) relationships, denote binary relations on objects.

Knowledge asserted through TBox and ABox assertions, i.e., logical axioms.

In 1977

Brachman, Woods, and others developed a precursor of DL-based systems: KL-ONE [Woods and Brachman, 1977].


Ancient Times
1980's

1980's: Description Logics were born

Ron Brachman & Hector Levesque – The Tractability of Subsumption in Frame-Based Description Languages [Brachman and Levesque, 1984, AAAI]:

- Use **logic** to capture class-based formalisms.
- Be **decidable**!
- Use **complexity** to understand the intrinsic computational properties of the language.
- There is a **tradeoff** between expressivity and complexity.
- Focus on **effective/tractable** languages.

These points are still the focus of the research in Description Logics in the current days!

Bell Labs developed the system CLASSIC with tractable subsumption [Borgida *et al.*, 1989, SIGMOD], [Patel-Schneider *et al.*, 1991, SIGART].

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Renaissance
1990’s

1990's: Focus on knowledge bases and expressive DLs

Various research groups worked on semantics, computational properties, and algorithms for **expressive DLs**.

- Tableaux algorithms for satisfiability / subsumption in $\mathcal{ALC}$ and sublanguages [Schmidt-Schauss and Smolka, 1991, AIJ], [Nebel, 1991], [Donini *et al.*, 1991, KR].
- Terminological cycles [Baader, 1990, AAAI] and transitive closure of roles [Baader, 1991, IJCAI].
- Description logics $\equiv$ Modal Logics of programs (with inverses, graded modalities, nominals, fixpoints) [Schild, 1991, IJCAI], [De Giacomo and Lenzerini, 1994a, AAAI], [C. *et al.*, 1995, DOOD].
- Concrete domains [Baader and Hanschke, 1991, IJCAI], [Lutz, 1999, IJCAI].
- Finite model reasoning [C., 1996, KR], [Lutz *et al.*, 2003, CADE].
- ...


Industrial
Revolution
2000's

2000's: Two important developments

On the applied side

Optimized fast tableaux for expressive DLs like $\mathcal{ALCQI}$ [Horrocks, 1998, KR], later $\mathcal{SHIQ}$ [Horrocks *et al.*, 2000, JIGPL].

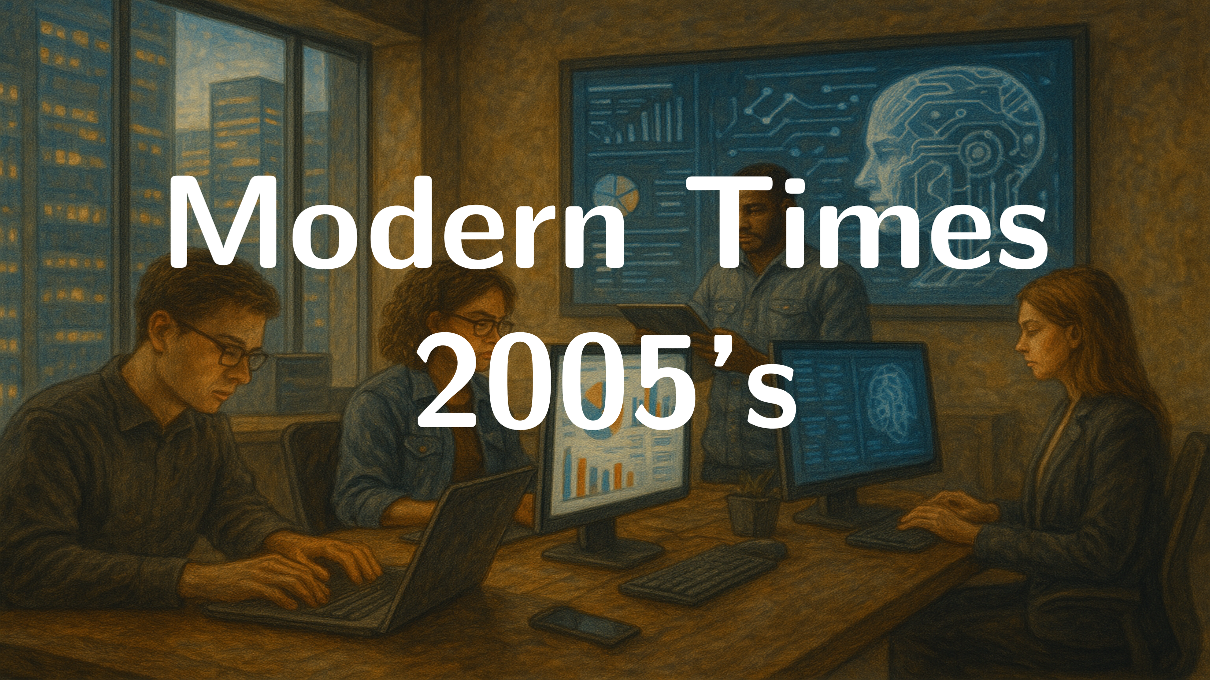
$\leadsto$ *Definition of* ***OWL W3C Standard****, based on scientific grounds!*

OWL 1 DL $\sim \mathcal{SHOIN}(\mathcal{D})$ OWL $\sim \mathcal{SROIQ}(\mathcal{D})$

On the theoretical side

Conjunctive query answering over DL KBs turns out to be decidable [C. *et al.*, 1998, PODS] and becomes a focus of research in DLs and knowledge representation and reasoning.

Modern Times 2005’s



2005's: Lightweight Description Logics

Scalability problems in handling **large ontologies** and **large amounts of data**.

$\rightsquigarrow$ **New kinds of DLs** were needed, with:

- tractable reasoning
- support for efficient query answering

Dresden: $\mathcal{EL}$ [Baader *et al.*, 2005, IJCAI; 2008, OWLED]

Ability to enforce the existence of tree-shaped structures in models (cf. $\mathcal{AL}$ of CLASSIC).

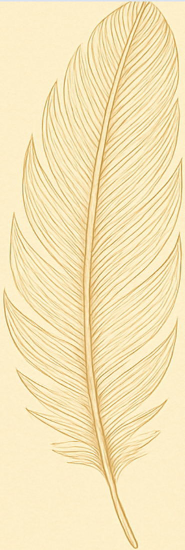
$\rightsquigarrow$ Captures SNOMED CT, a full-fledged medical ontology with 311,000 terms.

Rome + Bolzano: ***DL-Lite*** [C. *et al.*, 2005, AAAI; 2008, JAR]

Relies on dependency theory in Databases and query rewriting, thus scales with data.

$\rightsquigarrow$ Captures conceptual modeling formalisms (UML class diagrams, ER schemas).

Outline



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Knowledge Bases – a.k.a. Ontologies

- We are dealing with description logics, hence the domain is modeled in terms of **concepts** (i.e., classes) and **roles** (i.e., binary relationships).
- Knowledge about the domain is represented in a KB.

A **KB** $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ consists of two components:

- A **TBox** $\mathcal{T}$ modeling intensional (i.e., schema-level) information in the form of universally quantified assertions (logical axioms).
- An **ABox** $\mathcal{A}$ modeling extensional (i.e., data-level) information, through assertions on individuals (facts) of the form:

$$A(c), \quad \text{e.g., } \mathsf{Actor}(\mathsf{Keanu}) \qquad \text{or} \qquad P(c_1, c_2), \quad \text{e.g., } \mathsf{manages}(\mathsf{Bill}, \mathsf{Carrie\text{-}Anne})$$

A KB is interpreted in **standard first-order semantics** (possibly adopting the unique name assumption – UNA).

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Querying KBs – a.k.a. Ontology-mediated query answering

Query answering over KBs has been studied traditionally by considering as query language **conjunctive queries** (CQs) and their union (UCQs), under certain answer semantics.

Conjunctive query over a KB

A **CQ $\boldsymbol{q}$ over a KB $\mathcal{K}$** is a first-order query of the form

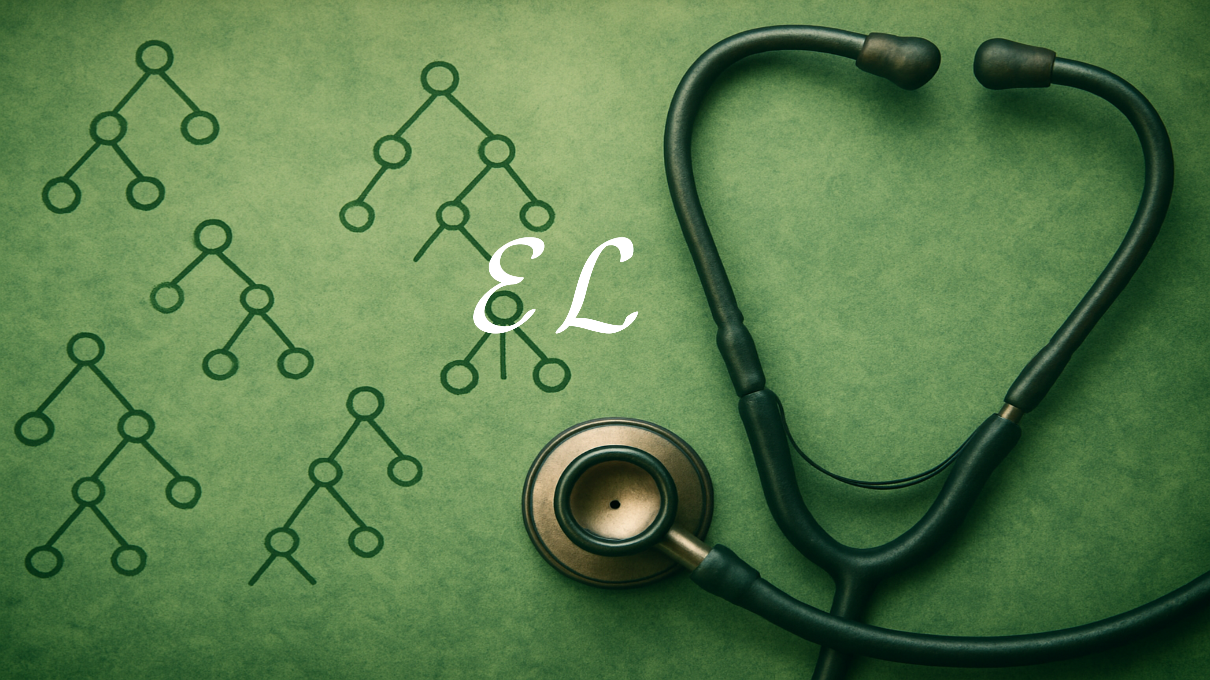
$$\boldsymbol{q}(\vec{x}) \;\leftarrow\; \exists\vec{y}.\, E_1(\vec{x},\vec{y}) \wedge \cdots \wedge E_n(\vec{x},\vec{y})$$

where each $E_i(\vec{x},\vec{y})$ is an atom that:

- has as predicate symbol a concept or role name of $\mathcal{K}$, and
- may use the answer variables $\vec{x}$, the existentially quantified variables $\vec{y}$, and constants.

Certain answer semantics

The **certain answers** $\mathsf{cert}(\boldsymbol{q},\mathcal{O})$ to a query $\boldsymbol{q}(\vec{x})$ over a KB $\mathcal{K}$ are the **tuples $\vec{c}$ of constants** that are logically implied to satisfy the query, i.e., **such that** $\mathcal{K} \models \boldsymbol{q}(\vec{c})$.


EL

The basic $\mathcal{EL}$

Essentially, $\mathcal{EL}$ is half of $\mathcal{ALC}$:

- It supports existential restrictions $\exists P.C$, but not universal ones.
- It supports conjunction $C_1 \sqcap C_2$, but not disjunction.
- Of course, no negation (in some variants $\bot$ is allowed).

$\mathcal{EL}$ concepts and TBoxes

$\mathcal{EL}$ concepts are defined by the grammar: (A denotes a concept name, P a role name)

$$C, C' \longrightarrow A \mid \top \mid C \sqcap C' \mid \exists P.C$$

An $\mathcal{EL}$ TBox consists of concept inclusions $C \sqsubseteq C'$.

Normal form for TBoxes: (A, A_i, B concept names or $\top$)

$$A \sqsubseteq B \qquad A_1 \sqcap A_2 \sqsubseteq B \qquad A \sqsubseteq \exists P.B \qquad \exists P.A \sqsubseteq B$$

Applications of $\mathcal{EL}$

In many applications, existential restrictions and conjunction seem to play a central role.

E.g., medical and Life Sciences KBs / ontologies rely on this kind of axioms.

Example $\mathcal{EL}$ TBox:

$$
\begin{array}{rcl}
\mathsf{ViralPneumonia} & \sqsubseteq & \exists \mathsf{CausativeAgent}.\mathsf{Virus} \\
\mathsf{ViralPneumonia} & \sqsubseteq & \mathsf{InfectiousPneumonia} \\
\mathsf{InfectiousPneumonia} & \sqsubseteq & \mathsf{Pneumonia} \sqcap \mathsf{InfectiousDisease} \\
\mathsf{Pneumonia} & \sqsubseteq & \exists \mathsf{AssociatedMorphology}.\mathsf{Inflammation} \\
\mathsf{Pneumonia} & \sqsubseteq & \exists \mathsf{FindingSite}.\mathsf{Lung}
\end{array}
$$

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Some prominent ontologies in the $\mathcal{EL}$ Family

- SNOMED CT (*Systematized Nomenclature of Medicine – Clinical Terms*) [http://www.ihtsdo.org]
- Large fragments of the GALEN ontology (*Generalized Architecture for Languages, Encyclopedias and Nomenclatures in medicine*) [http://www.openclinical.org/prj_galen.html]
- The Gene Ontology, and ontologies for biology with the aim of "standardizing the representation of gene and gene product attributes across species and databases" [http://www.geneontology.org/]
- Many ontologies in the BioPortal repository [http://bioportal.bioontology.org] and the Open Biomedical Ontologies (OBO) Foundry [http://www.obofoundry.org]

DL-Lite

DL-Lite TBox

DL-Lite allows for **TBox** assertions of the following forms: (A denotes a concept name, P a role name)

DL Syntax	FOL Counterpart	Example	$\mathcal{L}$	Intuition	
$A_1 \sqsubseteq A_2$	$\forall x.A_1(x) \rightarrow A_2(x)$	Actor ⊑ Person	core	ISA on concepts	Inclusion assertions
$\exists P \sqsubseteq A$	$\forall x,y.P(x,y) \rightarrow A(x)$	∃playsIn ⊑ Actor	core	domain of role	Inclusion assertions
$\exists P^- \sqsubseteq A$	$\forall x,y.P(y,x) \rightarrow A(x)$	∃playsIn⁻ ⊑ Movie	core	range of role	Inclusion assertions
$A \sqsubseteq \exists P$	$\forall x.A(x) \rightarrow \exists y.P(x,y)$	Actor ⊑ ∃playsIn	core	mandatory participation	Inclusion assertions
$P_1 \sqsubseteq P_2$	$\forall x,y.P_1(x,y) \rightarrow P_2(x,y)$	hasFather ⊑ hasParent	$\mathcal{R}$	ISA on roles	Inclusion assertions
$P_1 \sqsubseteq P_2^-$	$\forall x,y.P_1(x,y) \rightarrow P_2(y,x)$	hasFather ⊑ hasChild⁻	$\mathcal{R}$	ISA on roles	Inclusion assertions
...	...	...	...	...	Inclusion assertions
$A_1 \sqsubseteq \neg A_2$	$\forall x.A_1(x) \rightarrow \neg A_2(x)$	Manager ⊑ ¬Actor	core	Disjointness assertions	
$A \sqsubseteq \neg\exists P$	$\forall x.A(x) \rightarrow \neg\exists y.P(x,y)$	Manager ⊑ ¬∃playsIn	core	Disjointness assertions	
$P_1 \sqsubseteq \neg P_2$	$\forall x,y.P_1(x,y) \rightarrow \neg P_2(x,y)$	hasParent ⊑ ¬hasSibling	$\mathcal{R}$	Disjointness assertions	
...	...	...	...	Disjointness assertions	
(**funct** P)	$\forall x,y,z.P(x,y) \land P(x,z) \rightarrow y = z$	(**funct** hasFather)	$\mathcal{F}$	Functionality assertions	
(**funct** P^-)	$\forall x,y,z.P(y,x) \land P(z,x) \rightarrow y = z$	(**funct** manages⁻)	$\mathcal{F}$	Functionality assertions	

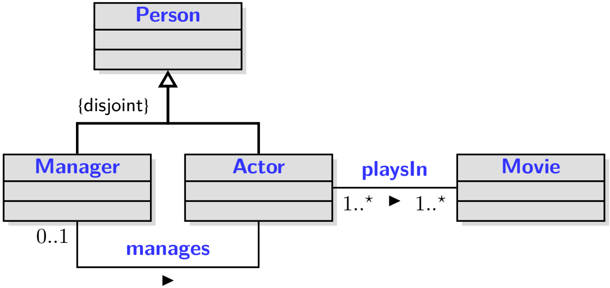
DL-Lite KB: Example and relationship with UML Class Diagrams

TBox $\mathcal{T}$:

$$\begin{array}{rcl}
\text{Manager} & \sqsubseteq & \text{Person} \\
\text{Actor} & \sqsubseteq & \text{Person} \\
\text{Manager} & \sqsubseteq & \neg\text{Actor} \\
\exists\text{playsIn} & \sqsubseteq & \text{Actor} \\
\exists\text{playsIn}^- & \sqsubseteq & \text{Movie} \\
\text{Actor} & \sqsubseteq & \exists\text{playsIn} \\
\text{Movie} & \sqsubseteq & \exists\text{playsIn}^- \\
\exists\text{manages} & \sqsubseteq & \text{Manager} \\
\exists\text{manages}^- & \sqsubseteq & \text{Actor} \\
 & (\textbf{funct}\ \text{manages}^-) &
\end{array}$$

ABox $\mathcal{A}$:

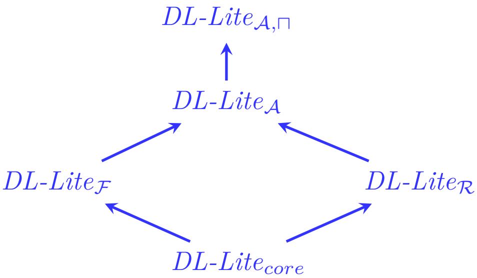
Actor(Keanu)
manages(Bill, Carrie-Anne)



The *DL-Lite* Family

The DLs of the *DL-Lite* Family share the property that:

answering UCQs is FO-rewritable.



From [C. *et al.*, 2007b, JAR].

Note: $DL\text{-}Lite_{\mathcal{A}}$ and $DL\text{-}Lite_{\mathcal{A},\sqcap}$ combine functionality and role inclusions. To guarantee that CQ answering is FO-rewritable, we need to impose that no functional role can have a sub-role.

What makes $\mathcal{EL}$ and $\mathit{DL\text{-}Lite}$ lightweight?

Universal models

For interpretations $\mathcal{I}_1$ and $\mathcal{I}_2$, a homomorphism $\mathcal{I}_1 \rightsquigarrow \mathcal{I}_2$ is a mapping from the domain of $\mathcal{I}_1$ to that of $\mathcal{I}_2$ that preserves predicates and constants.

Universal model (a.k.a. canonical model) of a KB $\mathcal{K}$

Is a model $\mathcal{I}_\mathcal{K}$ s.t. for every model $\mathcal{I}$ of $\mathcal{K}$, there is a homomorphism from $\mathcal{I}_\mathcal{K}$ to $\mathcal{I}$.

- Answers to (U)CQs are preserved under homomorphisms, i.e.,

$$\text{if} \quad \vec{c} \in \text{ans}(\boldsymbol{q}, \mathcal{I}_1) \quad \text{and} \quad \mathcal{I}_1 \rightsquigarrow \mathcal{I}_2, \qquad \text{then} \qquad \vec{c} \in \text{ans}(\boldsymbol{q}, \mathcal{I}_2).$$

- Hence, if $\mathcal{K}$ admits a universal model $\mathcal{I}_\mathcal{K}$, we can "use it" to compute the certain answers:

$$\vec{c} \in \text{cert}(q, \mathcal{K}) \qquad \text{iff} \qquad \vec{c} \in \text{ans}(q, \mathcal{I}) \text{ for every model } \mathcal{I} \text{ of } \mathcal{K} \qquad \text{iff} \qquad \vec{c} \in \text{ans}(q, \mathcal{I}_\mathcal{K})$$

Horn DLs

A DL is **Horn**, if every satisfiable KB $\mathcal{K}$ has a universal model $\mathcal{I}_{\mathcal{K}}$.

Theorem ([Baader *et al.*, 2005, IJCAI; 2008, OWLED])

Every satisfiable $\mathcal{EL}$ KB $\mathcal{K}$ has a universal model. $\leadsto$ $\mathcal{EL}$ is Horn!

Theorem ([C. *et al.*, 2005, AAAI; 2008, JAR])

Every satisfiable *DL-Lite* KB $\mathcal{K}$ has a universal model. $\leadsto$ *DL-Lite* is Horn!

Can we directly exploit the universal model for reasoning / query answering?

Not necessarily so, since **the universal model might be infinite!**

Other features of making a DL lightweight

Being Horn alone does not ensure that the DL is lightweight and that reasoning is efficient.

Further factors that play a role:

- Limited interaction between different constructs.

  E.g., In $DL\text{-}Lite_{\mathcal{A}}$, where both role inclusions and functionality are allowed, we need to ensure that no functional role has a subrole.
  If we relax this restriction, TBox complexity jumps from NLogSpace / PTime to ExpTime, and data complexity from AC^0 to PTime /coNP.

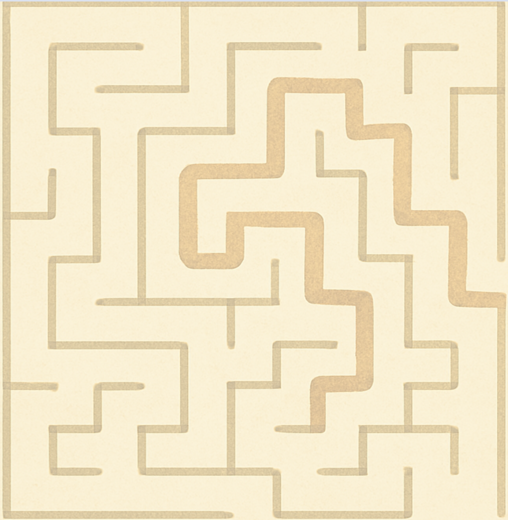
- Limited ability to propagate information along structures.

  E.g., $\mathcal{EL}$ does not allow for inverse roles, and with inverse roles complexity jumps to ExpTime.

- Do you have any additional suggestion, intuition, insight?

Outline

1. A Bit of History
2. Lighweight DLs
3. Reasoning
4. Extensions



Reasoning in $\mathcal{EL}$

Satisfiability in $\mathcal{EL}$

Very easy to see that:

- Every $\mathcal{EL}$ concept C is satisfiable: C induces a description tree, which represents of a model.
- Every $\mathcal{EL}$ concept C is satisfiable w.r.t. every $\mathcal{EL}$ TBox and w.r.t. every KB.

$\leadsto$ We concentrate on deciding subsumption.

Canonical model in $\mathcal{EL}$ – For plain concept subsumption

Let sub(C) be the set of subconcepts of C. We define:

$$\text{ex}(C) = \{C\} \cup \{D \mid \exists P.D \in \text{sub}(C)\}$$

Define the interpretation $\mathcal{I}_C = (\Delta^{\mathcal{I}_C}, \cdot^{\mathcal{I}_C})$ with:

$$\begin{array}{rcl} \Delta^{\mathcal{I}_C} & = & \{d_D \mid D \in \text{ex}(C)\} \\ A^{\mathcal{I}_C} & = & \{d_D \mid D = D' \sqcap A \;\; \text{(i.e., } A \text{ is a conjunct of } D\text{)}\} \\ P^{\mathcal{I}_C} & = & \{(d_D, d_{D'}) \mid D = D'' \sqcap \exists P.D' \;\; \text{(i.e., } \exists P.D' \text{ is a conjunct of } D\text{)}\} \end{array}$$

One can show that $\mathcal{I}_C$ is a **universal model** of C.

We can use this to check plain subsumption $C \sqsubseteq D$. Indeed: $C \sqsubseteq D$ iff $d_C \in D^{\mathcal{I}_C}$

$\leadsto$ PTime **algorithm**:
1. Build $\mathcal{I}_C$ in PTime (because $|\Delta^{\mathcal{I}_C}| \leq |C|$).
2. Test in PTime whether $d_C \in D^{\mathcal{I}_C}$.

Subsumption in $\mathcal{EL}$ w.r.t. TBoxes

We build again a **canonical model** $\mathcal{I}_\mathcal{T}$ of $\mathcal{T}$.

- Start from $\mathcal{I}_0$ that only has $d_A \in A^{\mathcal{I}_\mathcal{T}}$ for all A, and all roles are empty.
- Add objects to concepts and pairs of objects to roles to satisfy the inclusions.
- Always reuse d_A to satisfy $\exists P.A$ $\rightsquigarrow$ Only one witness $d_A \in A^{\mathcal{I}_\mathcal{T}}$ for each concept name A.

$\mathcal{I}_\mathcal{T}$ can be constructed in polytime. $\rightsquigarrow$ **Only polynomially many objects!**

We have that:

- $\mathcal{I}_\mathcal{T}$ is a model of $\mathcal{T}$ – Note: $\mathcal{I}_\mathcal{T}$ is not a universal model, but it suffices for checking subsumption.
- Indeed, for every pair A, B of concept names, $\mathcal{T} \models A \sqsubseteq B$ iff $d_A \in B^{\mathcal{I}_\mathcal{T}}$.

To decide subsumption of arbitrary concepts:

$$\mathcal{T} \models C \sqsubseteq D \quad \text{iff} \quad \mathcal{T} \cup \{A_C \sqsubseteq C,\ D \sqsubseteq A_D\} \models A_C \sqsubseteq A_D$$

Theorem

Subsumption (w.r.t. a TBox) in $\mathcal{EL}$ can be decided in $\textsc{PTime}$.

Example of the canonical model construction (1/2) [Credits to Magdalena & Meghyn]

TBox $\mathcal{T}$:

$$
\begin{array}{rcl}
\text{PenneArrabbiata} & \sqsubseteq & \exists\text{hasIngred}.\text{Penne} \\
\text{Penne} & \sqsubseteq & \text{Pasta} \\
\text{PenneArrabbiata} & \sqsubseteq & \exists\text{hasIngred}.\text{ArrabSauce} \\
\text{ArrabSauce} & \sqsubseteq & \exists\text{hasIngred}.\text{Peperoncino} \\
\text{Peperoncino} & \sqsubseteq & \text{Spicy} \\
\text{PizzaCalabrese} & \sqsubseteq & \exists\text{hasIngred}.\text{Nduia} \\
\text{Nduia} & \sqsubseteq & \text{Spicy}
\end{array}
$$

We saturate the TBox, so that it additionally contains:

$$
\begin{array}{rcl}
\text{PenneArrabbiata} & \sqsubseteq & \exists\text{hasIngred}.(\text{Penne} \sqcap \text{Pasta}) \\
\text{ArrabSauce} & \sqsubseteq & \exists\text{hasIngred}.(\text{Peperoncino} \sqcap \text{Spicy}) \\
\text{PizzaCalabrese} & \sqsubseteq & \exists\text{hasIngred}.(\text{Nduia} \sqcap \text{Spicy})
\end{array}
$$

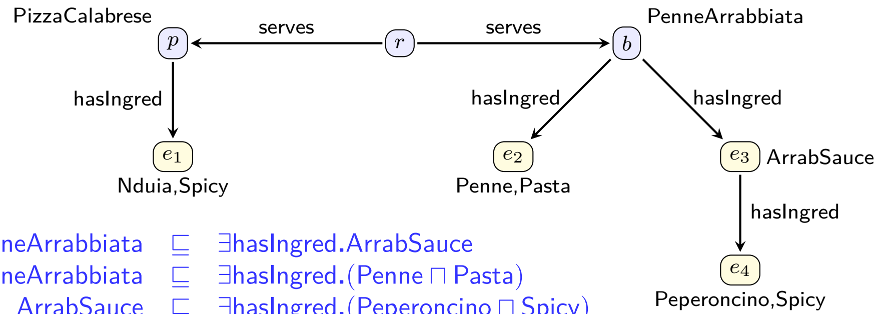
We consider also an ABox $\mathcal{A}$:

$$
\text{PizzaCalabrese}(p) \qquad \text{serves}(r,p) \qquad \text{serves}(r,b) \qquad \text{PenneArrabbiata}(b)
$$

Example of the canonical model construction (2/2) [Credits to Magdalena & Meghyn]

The canonical model $\mathcal{I}_\mathcal{K}$ For a KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ contains the ABox $\mathcal{A}$ and is **closed under inclusions**.

$\mathsf{PizzaCalabrese}(p) \qquad \mathsf{serves}(r,p) \qquad \mathsf{serves}(r,b) \qquad \mathsf{PenneArrabbiata}(b)$



$$
\begin{array}{rcl}
\mathsf{PenneArrabbiata} & \sqsubseteq & \exists \mathsf{hasIngred}.\mathsf{ArrabSauce} \\
\mathsf{PenneArrabbiata} & \sqsubseteq & \exists \mathsf{hasIngred}.(\mathsf{Penne} \sqcap \mathsf{Pasta}) \\
\mathsf{ArrabSauce} & \sqsubseteq & \exists \mathsf{hasIngred}.(\mathsf{Peperoncino} \sqcap \mathsf{Spicy}) \\
\mathsf{PizzaCalabrese} & \sqsubseteq & \exists \mathsf{hasIngred}.(\mathsf{Nduia} \sqcap \mathsf{Spicy})
\end{array}
$$

Notice that the **anonymous objects** witnessing existential concepts form **trees**.

Complexity of $\mathcal{EL}$

Theorem

Concept subsumption w.r.t. $\mathcal{EL}$ TBoxes is PTime-complete.

- Essentially, $\mathcal{EL}$ contains propositional Horn logic:

$$v_1 \wedge \cdots \wedge v_n \rightarrow v \quad \leadsto \quad A_{v_1} \sqcap \cdots \sqcap A_{v_n} \sqsubseteq A_v$$

- Reduction from entailment of a variable from propositional Horn theories is immediate.
- PTime-hard even with no roles / existential concepts.

Data complexity of $\mathcal{EL}$

Theorem

Instance checking in $\mathcal{EL}$ is PTIME-hard, even for a fixed TBox.

Straightforward reduction from Boolean circuit evaluation:

- The circuit is given as an ABox:
  - concept names AndGate and OrGate for the gates
  - role names leftInput and rightInput between gates.
- The input is asserted using a concept name True.
- The fixed TBox contains:

$$
\begin{aligned}
\mathsf{OrGate} \sqcap \exists \mathsf{leftInput}.\mathsf{True} &\sqsubseteq \mathsf{True}\\
\mathsf{OrGate} \sqcap \exists \mathsf{rightInput}.\mathsf{True} &\sqsubseteq \mathsf{True}\\
\mathsf{AndGate} \sqcap \exists \mathsf{leftInput}.\mathsf{True} \sqcap \exists \mathsf{rightInput}.\mathsf{True} &\sqsubseteq \mathsf{True}
\end{aligned}
$$

This means that the **data complexity** of $\mathcal{EL}$ is PTIME-complete.

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Adding $\bot$ to $\mathcal{EL}$ $\leadsto$ $\mathcal{EL}^\bot$

With $\bot$, one can express in $\mathcal{EL}$ concept disjointness and create unsatisfiable concepts and roles:

$$A_1 \sqcap A_2 \sqsubseteq \bot \qquad A_3 \sqsubseteq \bot \qquad \exists P.\top \sqsubseteq \bot$$

Concept satisfiability is not trivial anymore, but can be decided in PTime.

- Simply build the canonical model of C, and return "*unsat*" iff some element must satisfy $\bot$.

In $\mathcal{EL}^\bot$, satisfiability and subsumption are inter-reducible:

- C is satisfiable w.r.t. $\langle \mathcal{T}, \mathcal{A} \rangle$ iff $\langle \mathcal{T}, \mathcal{A} \rangle \not\models C \sqsubseteq \bot$
- $\langle \mathcal{T}, \mathcal{A} \rangle \models C \sqsubseteq D$ iff $C \sqcap A_{\neg D}$ is unsatisfiable w.r.t. $\langle \mathcal{T} \cup \{A_{\neg D} \sqcap D \sqsubseteq \bot\}, \mathcal{A} \rangle$, where $A_{\neg D}$ is a fresh concept name.

Additional polynomial extensions of $\mathcal{EL}$

- Nominals $\{a\}$
- Range restrictions $\top \sqsubseteq \forall P.C$ (also written $\exists P^-.\top \sqsubseteq C$)
  (domain restrictions $\exists P.\top \sqsubseteq C$ are already expressible in $\mathcal{EL}$)
- Complex role inclusions $P_1 \circ \cdots \circ P_n \sqsubseteq P$.
- Concrete domains $p(f_1, \ldots, f_k)$, where the f_is are feature names and p is a k-ary domain predicate.

We can still adapt the canonical model construction to accommodate these features, and reasoning is still feasible in PTime.

The resulting DL is called $\mathcal{EL}^{++}$
[Baader *et al.*, 2008, OWLED]

$\mathcal{EL}^{++}$ is at the basis of the OQL 2 EL profile of OWL 2, standardized by the W3C [Motik *et al.*, 2012, W3C].

Practical Reasoning in $\mathcal{EL}$

A different approach to reasoning in $\mathcal{EL}$

In practice, $\mathcal{EL}$ (and Horn DL) reasoners adopt a different approach:

Consequence Driven Reasoning.

- Make all consequences of the provided information explicit.
- Done by adding implied axioms to the TBox until saturation.
- This is achieved by defining a suitable set of inference rules.
- Mostly used for classification: computing all subsumptions between concept names:
  - no ABox, only TBox saturation
  - sound and complete for atomic entailment, that is:

$$\mathcal{T} \models A \sqsubseteq B \qquad \text{iff} \qquad A \sqsubseteq B \text{ is in the saturated TBox}$$

- The approach can also be extended to instance checking.

Classification for $\mathcal{ELH}$: the ELK reasoner [Kazakov *et al.*, 2014, JAR]

IR1 $\dfrac{}{A \sqsubseteq A}$ **IR2** $\dfrac{}{A \sqsubseteq \top}$

CR1 $\dfrac{A \sqsubseteq B \quad B \sqsubseteq C \in \mathcal{O}}{A \sqsubseteq C}$

CR2 $\dfrac{A \sqsubseteq B \quad A \sqsubseteq C \quad B \sqcap C \sqsubseteq D \in \mathcal{O}}{A \sqsubseteq D}$

CR3 $\dfrac{A \sqsubseteq B \quad B \sqsubseteq \exists r.C \in \mathcal{O}}{A \sqsubseteq \exists r.C}$

CR4 $\dfrac{A \sqsubseteq \exists r.B \quad r \sqsubseteq s \in \mathcal{O}}{A \sqsubseteq \exists s.B}$

CR5 $\dfrac{A \sqsubseteq \exists r.B \quad B \sqsubseteq C \quad \exists r.C \sqsubseteq D \in \mathcal{O}}{A \sqsubseteq D}$

Reasoning in $DL\text{-}Lite$

Reasoning in *DL-Lite*

TBox reasoning is rather straightforward.

- A *DL-Lite* TBox is always satisfiable (possibly in a model where all atomic concepts are empty).
- Concept and role subsumption amounts to computing reachability along the concept / role hierarchy. $\rightsquigarrow$ NLogSpace, or PTime for Horn-variants.

Query answering

- It is the main inference task that is of interest.
- Other TBox+ABox reasoning tasks get reduced to query answering.
- In general we cannot adopt an approach based on deriving all consequences:
  - They might necessarily be infinite – Not all variants of *DL-Lite* have the finite model property.
  - The amount of information to derive might depend on the query and cannot be determined a priory,

$\rightsquigarrow$ **Query answering by query rewriting**

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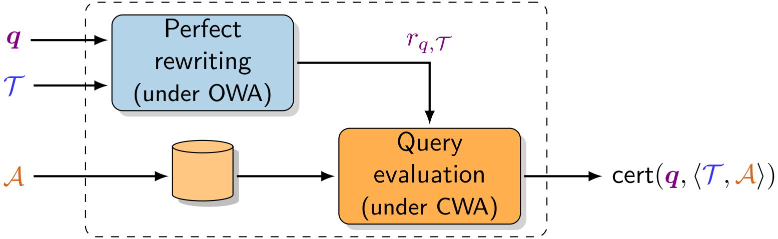
Query answering by query rewriting



To deal with data efficiently, we separate the contribution of $\mathcal{A}$ from the contribution of $\boldsymbol{q}$ and $\mathcal{T}$.

$r_{q,\mathcal{T}}$ is a new query over $\mathcal{T}$, called the **perfect rewriting** of $\boldsymbol{q}$ w.r.t. $\mathcal{T}$

FO-rewritability of conjunctive query answering

In $\mathit{DL\text{-}Lite}$, the perfect rewriting of a UCQ is always a **first-order query** (in fact a union of CQs)! Thus, **answering UCQs in $\mathit{DL\text{-}Lite}$ is in AC^0 in data complexity** (i.e., w.r.t. the ABox only).

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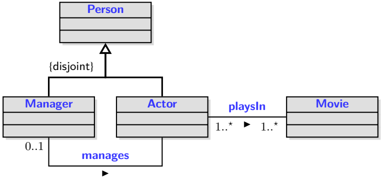
Perfect rewriting: Example

TBox $\mathcal{T}$:

Manager	$\sqsubseteq$	Person	Actor	$\sqsubseteq$	$\exists$playsIn
Actor	$\sqsubseteq$	Person	Movie	$\sqsubseteq$	$\exists$playsIn$^-$
Manager	$\sqsubseteq$	$\neg$Actor	$\exists$manages	$\sqsubseteq$	Manager
$\exists$playsIn	$\sqsubseteq$	Actor	$\exists$manages$^-$	$\sqsubseteq$	Actor
$\exists$playsIn$^-$	$\sqsubseteq$	Movie	(**funct** manages$^-$)		



Query: $q(x) \leftarrow \exists y.\, \text{playsIn}(x, y) \wedge \text{Movie}(y)$

ABox $\mathcal{A}$: Actor(Keanu), manages(Bill, Carrie-Anne)

Perfect rewriting $r_{q,\mathcal{T}}$:
(Algorithm **PerfectRef**)

$q(x) \leftarrow \exists y.\, \text{playsIn}(x, y) \wedge \text{Movie}(y)$

⇊ Use $\exists$playsIn$^-$ $\sqsubseteq$ Movie to rewrite the atom Movie(y).

$q(x) \leftarrow \exists y.\, \text{playsIn}(x, y) \wedge \text{playsIn}(\_, y)$

⇊ Unify the two atoms playsIn(x, y) and playsIn($\_, y$).

$q(x) \leftarrow \exists y.\text{playsIn}(x, y)$

⇊ Use Actor $\sqsubseteq$ $\exists$playsIn to rewrite the atom playsIn(x, y).

$q(x) \leftarrow \text{Actor}(x)$

⇊ Use $\exists$manages$^-$ $\sqsubseteq$ Actor to rewrite the atom Actor(x).

$q(x) \leftarrow \exists y.\text{manages}(y, x)$

⇊ ...

The evaluation of $r_{q,\mathcal{T}}$ over $\mathcal{A}$ returns the set {Keanu, Carrie-Anne} $= \text{cert}(\boldsymbol{q}, \langle \mathcal{T}, \mathcal{A} \rangle)$.

Complexity of query answering in *DL-Lite*

Ontology satisfiability and all classical DL reasoning tasks are:

- PTIME in the size of the TBox.
- In AC^0 the size of the ABox, i.e., in data complexity.

In fact, reasoning can be done by constructing suitable FOL/SQL queries and evaluating them over the ABox (FOL-rewritability).

Query answering for UCQs / SPARQL queries is:

- PTIME in the size of the TBox.
- In AC^0 the size of the ABox, i.e., in data complexity.
- NP-complete in the size of the query, i.e., in combined complexity.

This is precisely the complexity of evaluating CQs in plain relational DBs.

Practical Reasoning in $DL\text{-}Lite$

The *Ontop* system [C. *et al.*, 2017, Semantic Web J.], [Xiao *et al.*, 2020, ISWC]

ontop

https://ontop-vkg.org/

- State-of-the-art VKG system.
- Addresses the key challenges in query answering of scalability and performance.
- Compliant with all relevant Semantic Web standards:
  RDF, RDFS, OWL 2 QL, R2RML, SPARQL, and GeoSPARQL.
- Supports all major relational DBMSs:
  Oracle, DB2, MS SQL Server, Postgres, MySQL, Teiid, Dremio, Denodo, etc.
- Open-source and released under Apache 2 license.

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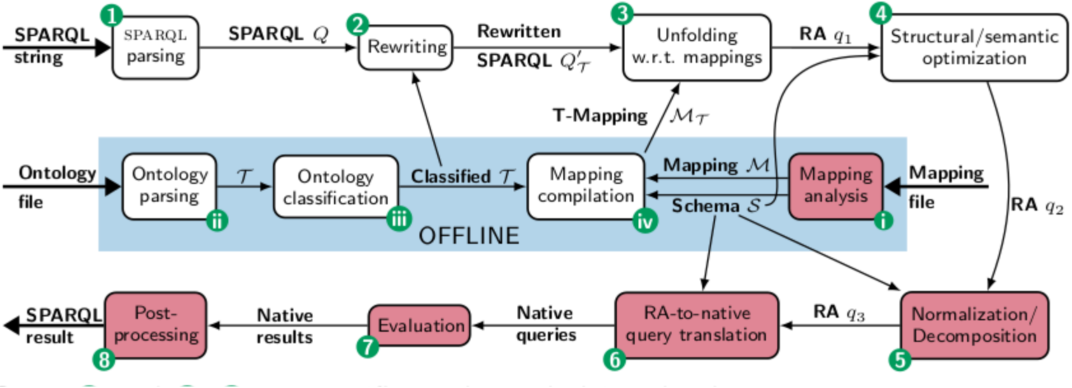
Query answering in *Ontop*



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Outline

Beyond $\mathcal{EL}$ – Adding inverse roles

Existential restrictions with inverses behave like universal restrictions:

$$\exists P^{-}.A \sqsubseteq B \qquad \text{is equivalent to} \qquad A \sqsubseteq \forall P.B$$

- Enables alternation.
- Types of connected objects influence each other.
- Generates exponentially many types.

$\leadsto$ The canonical model construction would fail with inverses.

Theorem

Concept subsumption in $\mathcal{ELI}$ is $\textsc{ExpTime}$-complete.

ExpTime-hard extensions of $\mathcal{EL}$

Reasoning (w.r.t. arbitrary TBoxes) becomes ExpTime-hard for the following extensions of $\mathcal{EL}$:

- $\mathcal{ELU}^\bot$, which extends $\mathcal{EL}^\bot$ with disjunction.
  - We can reduce concept satisfiability w.r.t. to a TBox in $\mathcal{ALC}$ to TBox satisfiability in $\mathcal{ELU}^\bot$.
- $\mathcal{ELU}$, which extends $\mathcal{AL}$ with disjunction.
  - We can reduce concept satisfiability w.r.t. to a TBox in $\mathcal{ELU}^\bot$ to concept subsumption w.r.t. to a TBox in $\mathcal{ELU}$.
- $\mathcal{EL}^\forall$, which extends $\mathcal{EL}$ with universal restrictions $\forall P.C$.
  - We can reduce concept subsumption w.r.t. to a TBox in $\mathcal{ELU}$ to the same problem in $\mathcal{EL}^\forall$

There is no known extension of $\mathcal{EL}$ for which reasoning is between PTime and ExpTime.

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Beyond *DL-Lite*: The border of FO-rewritability

[C. *et al.*, 2006, KR; 2013, AIJ]

	Left-hand side of inclusions	Right-hand side of inclusions	functionalities	Role inclusions	Data complexity of CQ answering
0	*DL-Lite*		$\surd^*$	$\surd^*$	in AC^0
1	$A \mid \exists P.A$	A	–	–	NLogSpace-hard
2	A	$A \mid \forall P.A$	–	–	NLogSpace-hard
3	A	$A \mid \exists P.A$	$\surd$	–	NLogSpace-hard
4	$A \mid \exists P.A \mid A_1 \sqcap A_2$	A	–	–	PTime-hard
5	$A \mid A_1 \sqcap A_2$	$A \mid \forall P.A$	–	–	PTime-hard
6	$A \mid A_1 \sqcap A_2$	$A \mid \exists P.A$	$\surd$	–	PTime-hard
7	$A \mid \exists P.A \mid \exists P^-.A$	$A \mid \exists P$	–	–	PTime-hard
8	$A \mid \exists P \mid \exists P^-$	$A \mid \exists P \mid \exists P^-$	$\surd$	$\surd$	PTime-hard
9	$A \mid \neg A$	A	–	–	coNP-**hard**
10	A	$A \mid A_1 \sqcup A_2$	–	–	coNP-**hard**
11	$A \mid \forall P.A$	A	–	–	coNP-**hard**

* With the “proviso” that functional roles cannot have subroles.

Extensions of *DL-Lite*

Additional constructs and language extensions have been considered, such as:

- Different forms of constraints:
  - identification assertions [C. *et al.*, 2008a, KR]
  - denial assertions [Lembo *et al.*, 2015, JWebSem]
  - epistemic constraints [C. *et al.*, 2007a, IJCAI], [Console and Lenzerini, 2020, AAAI]

  These constraints do not affect query answering, provided the ontology is satisfiable.
  However, the presence of **constraints may cause an ontology to become unsatisfiable**.

- n-ary relationships (as opposed to binary roles only) [C. *et al.*, 2006, KR; 2013, AIJ]

- Attributes and datatypes [Savkovic and C., 2012, ECAI], [Artale *et al.*, 2012, ECAI]:
  - Attributes are used to relate abstract objects to values of datatypes (such as integers, reals, strings).
  - The presence of datatypes has an impact on query answering, and restrictions need to be imposed to ensure FO-rewritability.

Extensions of $DL\text{-}Lite$ (cont'd)

Several works have studied also the **combined complexity** of satisfiability, in addition to data complexity and FO-rewritability for query answering.

Again, various language extensions have been considered.

- Number restrictions, role constructs, different types of concept inclusions, UNA yes/no
  [Artale *et al.*, 2009, JAIR]
  Inference exploits a translation into FOL over unary predicates only.

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Extensions of *DL-Lite* (cont'd)

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  Inference exploits a translatio

Languages	UNA	Complexity		
		Combined complexity	Data complexity	
		Satisfiability	Instance checking	Query answering
$DL\text{-}Lite_{core}^{[\ \|\mathcal{H}]}$	yes/no	NLogSpace $\geq$ [A]	in AC^0	in AC^0
$DL\text{-}Lite_{horn}^{[\ \|\mathcal{H}]}$		P $\leq$ [Th.8.2] $\geq$ [A]	in AC^0	in AC^0 $\leq$ [C]
$DL\text{-}Lite_{krom}^{[\ \|\mathcal{H}]}$		NLogSpace $\leq$ [Th.8.2]	in AC^0	coNP $\geq$ [B]
$DL\text{-}Lite_{bool}^{[\ \|\mathcal{H}]}$		NP $\leq$ [Th.8.2] $\geq$ [A]	in AC^0 $\leq$ [Th.8.3]	coNP
$DL\text{-}Lite_{core}^{[\mathcal{F}\|\mathcal{N}\|(\mathcal{HF})\|(\mathcal{HN})]}$	yes	NLogSpace	in AC^0	in AC^0
$DL\text{-}Lite_{horn}^{[\mathcal{F}\|\mathcal{N}\|(\mathcal{HF})\|(\mathcal{HN})]}$		P $\leq$ [Th.5.8, 5.13]	in AC^0	in AC^0 $\leq$ [Th.7.1]
$DL\text{-}Lite_{krom}^{[\mathcal{F}\|\mathcal{N}\|(\mathcal{HF})\|(\mathcal{HN})]}$		NLogSpace $\leq$ [Th.5.7,5.13]	in AC^0	coNP
$DL\text{-}Lite_{bool}^{[\mathcal{F}\|\mathcal{N}\|(\mathcal{HF})\|(\mathcal{HN})]}$		NP $\leq$ [Th.5.6, 5.13]	in AC^0 $\leq$ [Cor.6.2]	coNP
$DL\text{-}Lite_{core/horn}^{[\mathcal{F}\|(\mathcal{HF})]}$	no	P $\leq$ [Cor.8.8] $\geq$ [Th.8.7]	P $\geq$ [Th.8.7]	P
$DL\text{-}Lite_{krom}^{[\mathcal{F}\|(\mathcal{HF})]}$		P $\leq$ [Cor.8.8]	P	coNP
$DL\text{-}Lite_{bool}^{[\mathcal{F}\|(\mathcal{HF})]}$		NP	P $\leq$ [Cor.8.8]	coNP
$DL\text{-}Lite_{core/horn}^{[\mathcal{N}\|(\mathcal{HN})]}$		NP $\geq$ [Th.8.4]	coNP $\geq$ [Th.8.4]	coNP
$DL\text{-}Lite_{krom/bool}^{[\mathcal{N}\|(\mathcal{HN})]}$		NP $\leq$ [Th.8.5]	coNP	coNP
$DL\text{-}Lite_{core/horn}^{\mathcal{HF}}$	yes/no	ExpTime $\geq$ [Th.5.10]	P $\geq$ [Th.6.7]	P $\leq$ [D]
$DL\text{-}Lite_{krom/bool}^{\mathcal{HF}}$		ExpTime	coNP $\geq$ [Th.6.5]	coNP
$DL\text{-}Lite_{core/horn}^{\mathcal{HN}}$		ExpTime	coNP $\geq$ [Th.6.6]	coNP
$DL\text{-}Lite_{krom/bool}^{\mathcal{HN}}$		ExpTime $\leq$ [F]	coNP	coNP $\leq$ [E]

Extensions of *DL-Lite* (cont'd)

Several works have studied also the **combined complexity** of satisfiability, in addition to data complexity and FO-rewritability for query answering.

Again, various language extensions have been considered.

- Number restrictions, role constructs, different types of concept inclusions, UNA yes/no [Artale *et al.*, 2009, JAIR]
  Inference exploits a translation into FOL over unary predicates only.

- Complex role inclusion axioms [Kontchakov *et al.*, 2019, DL]:
  The complexity of query answering ranges from FO-rewritable to undecidable.

Extensions of *DL-Lite* (cont'd)

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- Number restrictions, role
  [Artale *et al.*, 2009, JAIR]
  Inference exploits a trans

Table 1. Combined complexity of satisfiability checking (all bounds are tight). For the languages $DL\text{-}Lite^{r}_{c}$ in the grey cells, the complexity is the same as for $DL\text{-}Lite^{r}_{r}$ as concept inclusions in $\boldsymbol{c}$ can be expressed by means of role inclusions in $\boldsymbol{r}$.

CIs \ RIs	Bool	guarded Bool	Horn	Krom	core
Bool	NEXPTIME (Th. 3)	EXPTIME (Th. 3)	NP (Th. 6)	NP (Th. 4)	NP [1]
Horn			P (Th. 6)	NP (Th. 4)	P [10]
Krom			P (Th. 6)	NL (Th. 4)	NL [1]
core					NL [9]

- Complex role inclusion axioms [Kontchakov *et al.*, 2019, DL]:
  The complexity of query answering ranges from FO-rewritable to undecidable.

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Extensions of *DL-Lite* (cont'd)

Several works have studied also the **combined complexity** of satisfiability, in addition to data complexity and FO-rewritability for query answering.

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  The complexity of query answering ranges from FO-rewritable to undecidable.

- Temporal extensions [Artale *et al.*, 2007, TIME], [Artale *et al.*, 2013, IJCAI], [Artale *et al.*, 2014, TOCL]
  - The most expressive variants (with temporalized concepts and roles) are typically undecidable.
  - By carefully restricting the temporal operators and the form of inclusions, reasoning and query answering become decidable.

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Reasoning in (lightweight) DLs beyond satisfiability and query answering

- Meta-modeling and meta-reasoning [Lenzerini *et al.*, 2016b, KR, IJCAI], [Lenzerini *et al.*, 2021, AIJ]
- Bag semantics, aggregation operators, and counting [Nikolaou *et al.*, 2019, AIJ], [Bienvenu *et al.*, 2020, IJCAI], [C. *et al.*, 2020, IJCAI]
- Explanation and provenance [Borgida *et al.*, 2008, ODBASE], [C. *et al.*, 2013b, JAIR], [Bourgaux and Ozaki, 2019, AAAI], [C. *et al.*, 2019, IJCAI]
- Inconsistency tolerant reasoning [Lembo and Ruzzi, 2007], [Lembo *et al.*, 2015, JWebSem], [Bienvenu and Bourgaux, 2016, RW], [Bienvenu *et al.*, 2019, JAIR], [Baader *et al.*, 2023, JELIA]
- KB and knowledge graph embeddings [Lacerda *et al.*, 2024, TGDK], [Bourgaux *et al.*, 2024, KR]
- . . . and many others, e.g., finite model reasoning [Rosati, 2008, ESWC], view-based query answering [C. *et al.*, 2008b, KR], query inseparability [Konev *et al.*, 2011, AAAI], [Botoeva *et al.*, 2014, KR], unification [Baader and Gil, 2024, IJCAR], mining KBs [Guimarães *et al.*, 2023, JAIR], . . .

Typical statement that we find in many papers:

We look at problem XYZ, . . . focusing on DLs of the $\mathcal{EL}$ and/or $\mathit{DL\text{-}Lite}$ families.

Semantic Web

- **OWL 2**, the second version of the Web Ontology Language, was developed by the W3C shortly after the introduction of $\mathcal{EL}$ and $\textit{DL-Lite}$.
- OWL 2 comes with three **profiles**[1], i.e., sub-languages tailored to specific needs.

> "The **OWL 2 EL profile** is designed as a subset of OWL 2 that is particularly suitable for applications employing ontologies that define very large numbers of classes and/or properties, captures the expressive power used by many such ontologies, and for which ontology consistency, class expression subsumption, and instance checking can be decided in polynomial time. For example, OWL 2 EL provides class constructors that are sufficient to express the very large biomedical ontology SNOMED CT."

> "The **OWL 2 QL profile** is designed so that sound and complete query answering is in AC^0 with respect to the size of the data, while providing many of the main features necessary to express conceptual models such as UML class diagrams and ER diagrams. [...] It is designed so that data stored in a standard relational database system can be queried through an ontology via a simple rewriting mechanism. [...] $\textit{DL-Lite}_{\mathcal{R}}$ provides the logical underpinning for OWL 2 QL."

- OWL 2 QL/ $\textit{DL-Lite}_{\mathcal{R}}$ is also an extension of the RDF schema language RDFS.

[1] http://www.w3.org/TR/owl2-profiles/

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Conclusions

$\mathcal{EL}$ and $DL\text{-}Lite$ are still the subject of many investigations.

Research:

data privacy, ontology-mediated querying, meta-modeling, abstraction, data quality, virtual knowledge graphs, explainable AI, KB embeddings, ...

Exploitation:

- **ELK Reasoner**, University of Ulm.
- **OBDA Systems**, start-up of Sapienza University of Rome, 2017.
  https://www.obdasystems.com/
- ONTOPIC spin-off of the Free University of Bozen-Bolzano, 2019.
  https://ontopic.ai/

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Thank you!

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