A Logic-Based Approach for
Matching User Profiles

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Abstract. Several applications require the matching of user profiles, e.g., job recruitment or dating systems. In this paper we present a logical framework for specifying user profiles that allows profile description to be incomplete in the parts that are unavailable or are considered irrelevant by the user. We present an algorithm for matching demands and supplies of profiles, taking into account incompleteness of profiles and incompatibility between demand and supply. We specialize our framework to dating services; however, the same techniques can be directly applied to several other contexts.

1 Introduction

The problem of matching demands and supplies of personal profiles arises in the business of recruitment agencies, in firms' internal job assignments, and in the recently emerging dating services. In all scenarios, a list of descriptions of demands is to be matched with a list of descriptions of supplies. In electronic commerce, the general problem is known as *matchmaking*, although here we do not consider any exchange of goods or services.

In matchmaking, finding an exact match of profiles is not the objective; instead, *best possible* matches to be provided; in fact, such a match is very unlikely to be found, and in all cases where an exact match does not exist, a solution to matchmaking must provide one or more *best possible* matches to be explored. Non-exact matches should consider both missing information — details that could be positively assessed in a second phase — and conflicting information — details that should be negotiated if the proposed match is worth enough pursuing. Moreover, when several matches are possible, a matchmaker should list them in a most-promising order, so as to maximize the probability of a

successful match within the first trials. However, such an order should be based on transparent criteria — possibly, logic — in order for the user to trust the system.

Profiles matchmaking can be addressed by a variety of techniques, ranging from simple bipartite graph matching (with or without cost minimization) [9], to vector-based techniques taken from classical Information Retrieval [11, 13, 12], to record matching in databases, among others. We now discuss some drawbacks of these techniques when transferred to solve matchmaking.

Algorithms for bipartite graph matching find optimal solutions when tying to maximize the number of matches [8, 10]. However, such algorithms rely on some way of assigning *costs* to every match between profiles. When costs are assigned manually, knowledge about them is completely implicit (and subjective), and difficult to revise. Moreover, in maximizing the number of matches a system may provide a *bad* service to single end users: for example, person P_1 could have a best match with job profile J_1, but she might be suggested to take job J_2 just because J_1 is the only available job for person P_2. Hence, from end user's viewpoint, maximizing the number of matches is not the feature that a matchmaker should have.

Both Database techniques for record matching (even with null values), and information retrieval techniques using similarity between weighted vectors of stemmed terms, are not suited for dealing with incomplete information usually present in matchmaking scenarios. In fact, information about profiles is almost always incomplete, not only because some information is unavailable, but also because some details are simply considered irrelevant by either the supplier or the demander — and should be left as such. Imposing a system interface for entering profiles with long and tedious forms to be filled in, is the most often adopted "solution" to this incompleteness problem — but we consider this more an escape for constraining real data into an available technique, than a real solution. For example, in a job posting/finding system, the nationality could be considered irrelevant for some profiles (and relevant for others); or in a dating service, some people may find disturbing (or simply inappropriate) the request to specify the kind of preferred music, etc. In such situations, missing information can be assumed as an "any-would-fit" assertion, and the system should cope with this incompleteness as is.

To sum up, we believe that there is a *representation problem* that undermines present solutions to matchmaking: considering how profiles information is represented is a fundamental step to reach an effective solution, and representations that are either too implicit, or overspecified, lead to unsatisfactory solutions.

Therefore, our research starts with proposing a language, borrowed from Artificial Intelligence, that allows for incomplete descriptions of profiles, and both positive and negative information about profiles. In particular, we propose a Description Logic [1] specifically tailored for describing profiles. Then, we model the matching process as a special reasoning service about profiles, along the lines of [5, 6]. Specifically, we consider separately conflicting details and missing details, and evaluate how likely is the match to succeed, given both missing and conflicting details. Our approach makes transparent the way matches are evaluated — allowing end users to request justifications for suggested matches. We devise some special-purpose algorithms to solve the problem

for the language we propose, and evaluate the possible application scenarios of a dating service.

2   A Description Logic for Representing Profiles

We use a restriction of the $\mathcal{ALC}(\mathcal{D})$ Description Logic, that, besides concepts and roles to represent properties of (abstract) objects, also allows one to express quantitative properties of objects, such as weight, length, etc., by means of *concrete domains* [2]. Each concrete domain $\mathcal{D}$, e.g., the real numbers $\mathbb{R}$, has a set of associated predicate names, where each predicate name p denotes a predicate $p^{\mathcal{D}}$ over $\mathcal{D}$. For our purpose, it is sufficient to restrict the attention to unary predicates, and we assume that among such unary predicates we always have a predicate $\top$ denoting the entire domain, and predicates $\geqslant_\ell(\cdot)$ and $\leqslant_\ell(\cdot)$, for arbitrary values ℓ of $\mathcal{D}$. We also assume that the concrete domains we deal with are *admissible*, which is a quite natural assumption, satisfied e.g., by $\mathbb{R}$ (see [2] for the details). Besides roles, the logic makes use of *features*. Each feature has an associated concrete domain $\mathcal{D}$ and represents a (functional) relation between objects and values of $\mathcal{D}$.

Starting from a set of concept names (denoted by the letter A), a set of role names (denoted by R), a set of unary predicate names (denoted by p), and a set of features (denoted by f), we inductively define the set of *concepts* (denoted by C) as follows. Every concept name A is a concept (atomic concept), and for C_1 and C_2 concepts, R a role name, f a feature with associated domain $\mathcal{D}$, and p a unary predicate of $\mathcal{D}$, the following are concepts: $C_1 \sqcap C_2$ (conjunction), $C_1 \sqcup C_2$ (disjunction), and $\neg C$ (negation); $\exists R.C_2$ (existential restriction) and $\forall R.C$ (universal restriction); $p(f)$ (predicate restriction).

To express intentional knowledge about concepts, we make use of a *concept hierarchy*, which is a set of assertions of the form $A_1 \sqsubseteq A_2$ and $A_1 \sqsubseteq \neg A_2$, with A_1 and A_2 concept names. The former assertion expresses an *inclusion*, while the latter expresses a *disjointness*. For example, football $\sqsubseteq$ sport and male $\sqsubseteq$ ¬female could be assertions that are part of a concept hierarchy.

Formally, the semantics of concepts is defined by an *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, consisting of an *abstract domain* $\Delta^{\mathcal{I}}$ and an *interpretation function* $\cdot^{\mathcal{I}}$ that assigns to each concept name A a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$; to each role name R a binary relation $R^{\mathcal{I}}$ over $\Delta^{\mathcal{I}}$, and to each feature name f, associated with the concrete domain $\mathcal{D}$, a partial function $f^{\mathcal{I}} : \Delta^{\mathcal{I}} \to \mathcal{D}$. The interpretation function can be extended to arbitrary concepts as follows:

$$(C_1 \sqcap C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$$
$$(C_1 \sqcup C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$$
$$(\neg C)^{\mathcal{I}} = \neg C^{\mathcal{I}}$$
$$(\exists R.C)^{\mathcal{I}} = \{c \in \Delta^{\mathcal{I}} \mid \text{there exists } d \in \Delta^{\mathcal{I}} \text{ s.t. } (c,d) \in R^{\mathcal{I}} \text{ and } d \in C^{\mathcal{I}}\}$$
$$(\forall R.C)^{\mathcal{I}} = \{c \in \Delta^{\mathcal{I}} \mid \text{for all } d \in \Delta^{\mathcal{I}} \text{ s.t. } (c,d) \in R^{\mathcal{I}} \text{ we have } d \in C^{\mathcal{I}}\}$$
$$(p(f))^{\mathcal{I}} = \{c \in \Delta^{\mathcal{I}} \mid f^{\mathcal{I}}(c) \in p^{\mathcal{D}}\}$$

An assertion $A_1 \sqsubseteq A_2$ is *satisfied* by an interpretation $\mathcal{I}$ if $A_1^{\mathcal{I}} \subseteq A_2^{\mathcal{I}}$. An assertion $A_1 \sqsubseteq \neg A_2$ is satisfied by an interpretation $\mathcal{I}$ if $A_1^{\mathcal{I}} \cap A_2^{\mathcal{I}} = \phi$. We call an interpretation

that satisfies all assertions in a hierarchy $\mathcal{H}$ a *model* of $\mathcal{H}$. A concept C is *satisfiable in* $\mathcal{H}$ if $\mathcal{H}$ admits a model $\mathcal{I}$ such that $C^{\mathcal{I}} \neq \phi$. A hierarchy $\mathcal{H}$ *logically implies* an assertion $C_1 \sqsubseteq C_2$ between arbitrary concepts C_1 and C_2 if $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$, for each model $\mathcal{I}$ of $\mathcal{H}$.

3 Representing User Profiles

We describe how to represent user profiles using the Description Logic presented in Section 2. The user profiles are tailored for dating services, though the same framework can be used, with small modifications, for different applications. We do not use the full expressive power of the Description Logic. In particular, we use a single role hasInterest, to express interest in topics[1], and we make a limited use of the constructs. We assume the set of features to represent physical characteristics such as age, height, etc. Additionally, we use a special feature level that expresses the level of interest in a certain field. The concrete domain associated to level is the interval $\{\ell \in \mathbb{R} \mid 0 < \ell \leqslant 1\}$.

A user profile P consists of the conjunction of the following parts:

- A conjunction of *atomic concepts*, to represent atomic properties associated to the user. We denote the set of such concepts as $Names(P)$.
- A conjunction of concepts of the form $p(f)$, to represent physical characteristics. The (unary) predicate p can be one of the predicates $\geqslant_\ell(\cdot), \leqslant_\ell(\cdot), =_\ell(\cdot)$, where ℓ is a value of the concrete domain associated to f, or any logical conjunction of them. We denote the set of such concepts as $Features(P)$. Since $(p_1 \wedge p_2)(f)$ is equivalent to $p_1(f) \sqcap p_2(f)$, in the following, we can assume w.l.o.g. that $Features(P)$ contains at most one concept of the form $p(f)$ for each feature f.
- A conjunction of concepts of the form $\exists$hasInterest.$(C \sqcap \geqslant_x(\text{level}))$, where C is a conjunction of concept names, and $0 \leqslant x \leqslant 1$. Each such concept represents an interest in a concept C with level at least x. We denote the set of such concepts as $Interests(P)$.
- A conjunction of concepts of the form $\forall$hasInterest.$(\neg C \sqcup \leqslant_x(\text{level}))$, where C is a conjunction of concept names, and $0 \leqslant x \leqslant 1$. Each such concept represents the fact that the interest in a concept C has level at most x. Note that, to represent the complete lack of interest in C, it is sufficient to put $x = 0$. We denote the set of such concepts as $NoInterests(P)$.

Example 1. A supplied profile describing, say, a 35-years-old male, 1.82 cm tall, with strong interests in fantasy novels and japanese comics, fair interest in politics and no interest in football, could be expressed as follows:

$$\text{male} \sqcap\ =_{35}(\text{age}) \sqcap\ =_{1.82}(\text{height}) \sqcap$$
$$\exists\text{hasInterest.}(\text{fantasyNovels} \sqcap \geqslant_{0.8}(\text{level})) \sqcap$$
$$\exists\text{hasInterest.}(\text{japaneseComics} \sqcap \geqslant_{0.8}(\text{level})) \sqcap$$
$$\exists\text{hasInterest.}(\text{politics} \sqcap \geqslant_{0.4}(\text{level})) \sqcap$$
$$\forall\text{hasInterest.}(\neg\text{football} \sqcup \leqslant_{0}(\text{level}))$$

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[1] For modeling profiles in different contexts, additional roles could be added to this language. For example, hasSkill for expressing skills in certain fields.

where we suppose that interests are organized in a hierarchy including fantasyNovels $\sqsubseteq$ novels, japaneseComics $\sqsubseteq$ comics, and male $\sqsubseteq$ ¬female

Observe that, when a profile is demanded, usually features like age and height will be used with range predicates (e.g., $(\geqslant 30 \wedge \leqslant 70)(\mathsf{age})$), instead of equality predicates as in the above example.

The following property follows immediately from the semantics of existential restriction. For every pair of concepts C_1 and C_2, role R, feature f with associated concrete domain $\mathcal{D}$, and p a predicate of $\mathcal{D}$:

$$\text{if } \mathcal{H} \models C_1 \sqsubseteq C_2 \quad \text{then} \quad \mathcal{H} \models \exists R.(C_1 \sqcap \geqslant_\ell (f)) \sqsubseteq \exists R.(C_2 \sqcap \geqslant_\ell (f))$$

For example, if football $\sqsubseteq$ sport, then someone with a level of interest ℓ in football has at least the same level of interest in sport. This property is exploited in the matching algorithm provided in Section 4.

4   The Matching Algorithm

We present the algorithm for matching user profiles. The matching is performed over two profiles: the *demand* profile P_d and the *supply* profile P_s. The algorithm is not symmetric, i.e., it evaluates how P_s is suited for P_d, which is different from how P_d is suited for P_s [7]; of course, in order to determine how P_d is suited for P_s, we can simply exchange the arguments of the algorithm.

From a logical point of view, we extend the non-standard inferences *contraction* and *abduction* defined in [4]. In particular, our contraction either removes or weakens conjuncts from P_d so as to make $P_d \sqcap P_s$ satisfiable in $\mathcal{H}$; abduction, instead, either adds or strengthens conjuncts in P_s so as to make $\mathcal{H} \models P_s \sqsubseteq P_d$. The algorithm is based on structural algorithms for satisfiability and subsumption [3]. Since it is reasonable to assume that users do not enter contradicting information, we assume that the profiles P_d and P_s are consistent.

The result of the match is a *penalty* in $\mathbb{R}$: the larger the penalty, the less P_s is suited for P_d. In particular, partial penalties are added to the overall penalty by matching corresponding conjuncts of the two profiles; this is done in two ways.

Contraction. When a conjunct C_d in P_d is in contrast with some conjunct C_s in P_s, then C_d is removed and a penalty is added. Intuitively, since the supplier has something the demander does not like, in order to make the profiles match the demander *gives up* one of her requests. For example, let $C_d = \forall \mathsf{hasInterest}.(\neg \mathsf{sport} \sqcup \leqslant_{0.2}(\mathsf{level}))$ and $C_s = \exists \mathsf{hasInterest}.(\mathsf{football} \sqcap \geqslant_{0.4}(\mathsf{level}))$, where we have football $\sqsubseteq$ sport in $\mathcal{H}$. In this case the demander looks for someone who does not like sports very much, while the supplier likes football and therefore he likes sports. In this case, pursuing the match would require the demander to give up his/her request about sports, so the algorithm adds a penalty $\Pi_{c\ell}(0.4, 0.2)$ that depends on the gap between the lower bound (0.4) of the supply and the upper bound (0.2) of the demand. Similarly, for a feature f with

contrasting predicates p_d and p_s, a penalty $\Pi_{cf}(p_d(f), p_s(f))$ is added to take into account the removal of $p_d(f)$ from P_d. In case a concept A_d representing an atomic property has to be removed, the algorithm makes use of another penalty function $\Pi_c(\cdot)$, whose argument is the concept A_d.

Abduction. When a conjunct c_d in P_d has no corresponding conjunct in P_s, we add a suitable conjunct c_s in P_s that makes the profiles match, and add a corresponding penalty. Intuitively, the demander wants something which the supplier does not provide explicitly; in this case we assume that the supplier may or may not satisfy the demander's request, and as a consequence of this possibility of conflict we add a penalty. This is done by means of a penalty function $\Pi_a(\cdot)$, whose argument is a concept C, that takes into account the addition of C to P_s. When the level of interest must be strengthened, we use a function $\Pi_{a\ell}(\cdot)$, that takes into account the gap between bounds. Similarly, a penalty function $\Pi_{af}(\cdot)$ takes into account the addition of features.

Algorithm CalculatePenalty
    Input demand profile P_d, supply profile P_s, concept hierarchy $\mathcal{H}$
    Output real value penalty $\geqslant 0$
    penalty := 0;
    // Contraction
    foreach $A_d \in Names(P_d)$ **do**
      if there exists $A_s \in Names(P_s)$
        such that $\mathcal{H} \models A_d \sqsubseteq \neg A_s$
      then remove A_d from P_d
          penalty := penalty + $\Pi_c(A_d)$
    foreach $p_d(f) \in Features(P_d)$ **do**
      if there exists $p_s(f) \in Features(P_s)$
        such that $\exists x.p_d(x) \wedge p_s(x)$ is unsatisfiable in the domain associated to f
      then remove $p_d(f)$ from P_d
          penalty := penalty + $\Pi_{cf}(p_d(f), p_s(f))$
    foreach $\exists \mathsf{hasInterest}.(C_d \sqcap \geqslant_{x_d}(\mathsf{level})) \in Interests(P_d)$ **do**
      foreach $\forall \mathsf{hasInterest}.(\neg C_s \sqcup \leqslant_{x_s}(\mathsf{level})) \in NoInterests(P_s)$ **do**
        if $\mathcal{H} \models C_d \sqsubseteq C_s$ and $x_d \geqslant x_s$
        then replace $\exists \mathsf{hasInterest}.(C_d \sqcap \geqslant_{x_d}(\mathsf{level}))$ in P_d
            with $\exists \mathsf{hasInterest}.(C_d \sqcap \geqslant_{x_s}(\mathsf{level}))$
          penalty := penalty + $\Pi_{c\ell}(x_d, x_s)$
    foreach $\forall \mathsf{hasInterest}.(\neg C_d \sqcup \leqslant_{x_d}(\mathsf{level})) \in NoInterests(P_d)$ **do**
      foreach $\exists \mathsf{hasInterest}.(C_s \sqcap \geqslant_{x_s}(\mathsf{level})) \in Interests(P_s)$ **do**
        if $\mathcal{H} \models C_s \sqsubseteq C_d$ and $x_d \leqslant x_s$
        then replace $\forall \mathsf{hasInterest}.(\neg C_d \sqcup \leqslant_{x_d}(\mathsf{level}))$ in P_d
            with $\forall \mathsf{hasInterest}.(\neg C_d \sqcup \leqslant_{x_s}(\mathsf{level}))$
          penalty := penalty + $\Pi_{c\ell}(x_s, x_d)$

// Abduction

foreach $A_d \in Names(P_d)$ **do**

    if there does not exist $A_s \in Names(P_s)$ such that $\mathcal{H} \models A_s \sqsubseteq A_d$

    then add A_d to P_s

        penalty := penalty + $\Pi_a(A_d)$

foreach $p_d(f) \in Features(P_d)$ **do**

    if there exist $p_s(f) \in Features(P_s)$

    then if $\forall x.p_s(x) \Rightarrow p_d(x)$ is false in the domain associated to f

        then add $p_d(f)$ to P_s

            penalty := penalty + $\Pi_{af}(p_d(f), p_s(f))$

    else add $p_d(f)$ to P_s

        penalty := penalty + $\Pi_{af}(p_d(f), \top(f))$

foreach $\exists\mathsf{hasInterest}.(C_d \sqcap \geqslant_{x_d}(\mathsf{level})) \in Interests(P_d)$ **do**

    if there does not exist $\exists\mathsf{hasInterest}.(C_s \sqcap \geqslant_{x_s}(\mathsf{level})) \in Interests(P_s)$

      such that $\mathcal{H} \models C_s \sqsubseteq C_d$ **and** $x_s \geqslant x_d$

    then if there exists $\exists\mathsf{hasInterest}.(C_s \sqcap \geqslant_{x_s}(\mathsf{level})) \in Interests(P_s)$

        such that $\mathcal{H} \models C_s \sqsubseteq C_d$

        then let $\exists\mathsf{hasInterest}.(C_s \sqcap \geqslant_{x_s}(\mathsf{level}))$ be the concept in $Interests(P_s)$

            with maximum x_s among those for which $\mathcal{H} \models C_s \sqsubseteq C_d$ holds

            penalty := penalty + $\Pi_{a\ell}(x_d, x_s)$

        else penalty := penalty + $\Pi_a(\exists\mathsf{hasInterest}.(C_d \sqcap \geqslant_{x_d}(\mathsf{level})))$

        add $\exists\mathsf{hasInterest}.(C_d \sqcap \geqslant_{x_d}(\mathsf{level}))$ to P_s

foreach $\forall\mathsf{hasInterest}.(\neg C_d \sqcup \leqslant_{x_d}(\mathsf{level})) \in NoInterests(P_d)$

    if there does not exist $\forall\mathsf{hasInterest}.(\neg C_s \sqcup \leqslant_{x_s}(\mathsf{level})) \in NoInterests(P_s)$

      such that $\mathcal{H} \models C_d \sqsubseteq C_s$ **and** $x_d \geqslant x_s$

    then if there exists $\forall\mathsf{hasInterest}.(\neg C_s \sqcup \leqslant_{x_s}(\mathsf{level})) \in NoInterests(P_s)$

        such that $\mathcal{H} \models C_d \sqsubseteq C_s$

        then let $\forall\mathsf{hasInterest}.(\neg C_s \sqcup \leqslant_{x_s}(\mathsf{level}))$ be the concept in $Interests(P_s)$

            with minimum x_s among those for which $\mathcal{H} \models C_d \sqsubseteq C_s$ holds

            penalty := penalty + $\Pi_{a\ell}(x_s, x_d)$

        else penalty := penalty + $\Pi_a(\forall\mathsf{hasInterest}.(\neg C_d \sqcup \leqslant_{x_d}(\mathsf{level})))$

        add $\forall\mathsf{hasInterest}.(\neg C_d \sqcup \leqslant_{x_d}(\mathsf{level}))$ to P_s

return penalty

It is easy to check that all subsumption tests $\mathcal{H} \models C_1 \sqsubseteq C_2$ in the algorithm can be done in polynomial time in the size of $\mathcal{H}$, C_1, and C_2. Hence, it can be straightforwardly proved that the complexity of the algorithm is polynomial w.r.t. the size of the input.

The following theorem establishes the correctness of the above algorithm w.r.t. the computation of contraction and abduction. We denote with P_d^c the profile P_d after contraction, and with P_s^a the profile P_s after abduction.

Theorem 1. *Given a concept hierarchy $\mathcal{H}$, a demand profile P_d, and a supply profile P_s, the following properties hold: (i) $P_d^c \sqcap P_s$ is satisfiable in $\mathcal{H}$; (ii) P_s^a is satisfiable*

in $\mathcal{H}$; (iii) $\mathcal{H} \models P_d^c \sqsubseteq P_s^a$. (iv) there does not exist a profile P_s' more general than P_s^a (i.e., $\mathcal{H} \models P_s^a \sqsubseteq P_s'$ and $\mathcal{H} \not\models P_s' \sqsubseteq P_s$) such that $\mathcal{H} \models P_s' \sqsubseteq P_s$ and $\mathcal{H} \models P_s' \sqsubseteq P_d^c$.

5   Conclusions

In this paper we have addressed the problem of matching user profiles, when the demander's and supplier's profiles can have missing or conflicting information. In such a case, we have to take into account that the demander may need to give up some of her requests, and/or she may need to make assumptions on unspecified properties of the supplier's profile. We have proposed a DL-based framework for expressing user profiles in this setting, and a language suited for dating services. We have proposed an ad-hoc structural algorithm for matching profiles that, given a demander's and a supplier's profile, returns a penalty: the higher the penalty, the less the two profiles are compatible. As a future work, we want to test the algorithm in real cases with a prototype that is currently under development: we believe that promising applications of our techniques can be dating, recruitment, and service discovery systems.

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