FORMAL METHODS

LECTURE II: MODELING SYSTEMS

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M. Benerecetti, A. Cimatti, M. Fisher, F. Giunchiglia, M. Pistore, M. Roveri, R.Sebastiani.

Summary of Lecture II

- Types of Systems.
- Modeling Systems as Kripke Models.
- Languages for Describing Kripke Models.
- Properties of Systems.

Concurrent Reactive Systems

We describe here Concurrent Reactive systems.

- Reactive Systems: Systems that interact with their environment and usually do not terminate (e.g. communication protocols, hardware circuits).
- Concurrent Systems consist of a set of components that execute together.
- We distinguish two types of Concurrent Systems:
  1. *Asynchronous or Interleaved Systems*. Only one component makes a step at a time;
  2. *Synchronous Systems*. All components make a step at the same time.

Modeling Systems

- We need to construct a *Formal Specification* of the system which abstract from irrelevant details.
  - **State**: Snapshot of the system that captures the values of the variables at a particular point in time.
  - **System Transition**: How the state of the system evolves as the result of some action.
  - **Computation**: Infinite sequence of states along the different transitions.

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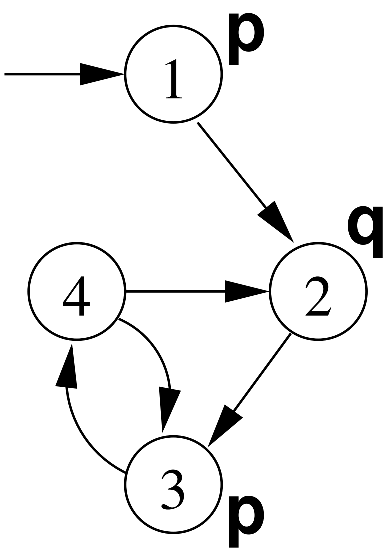
Modeling Systems with Kripke Structures

- Kripke Structures are transition diagrams that represent the dynamic behavior of a reactive system.
- Kripke Structures consist of a set of states, a set of transitions between states, and a set of properties labeling each state.
- A path in a Kripke structure represents a computation of the system.

Kripke model: definition

- Formally, a Kripke model $\langle S, I, R, AP, L \rangle$ consists of
  - a set of states S;
  - a set of initial states $I \subseteq S$;
  - a set of transitions $R \subseteq S \times S$;
  - a set of atomic propositions AP;
  - a labeling function $L : S \mapsto 2^{AP}$.



- A path in a Kripke model M from a state s_0 is an **infinite** sequence of states

$$\pi = s_0, s_1, s_2, \ldots$$

such that $(s_i, s_{i+1}) \in R$, for all $i \geq 0$.

Example: Kripke model for mutual exclusion

- We model two **concurrent asynchronous processes** sharing a resource ensuring they do not access it at the same time.
- Each process has *critical sections* in its code and only one process can be in its critical section at a time.
- We want to find a *protocol* for mutual exclusion which, for example, guarantee the following properties:

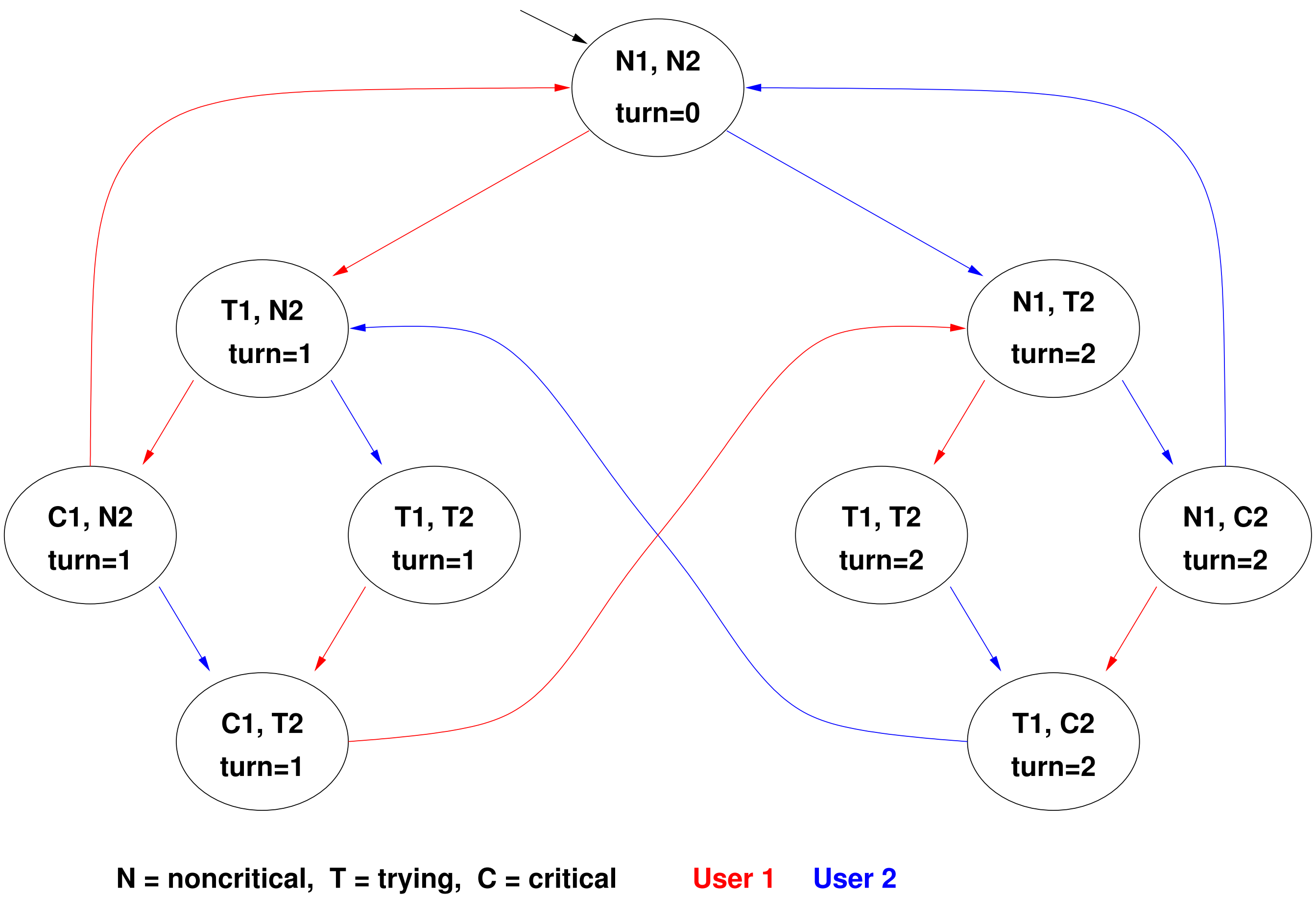
Safety: Only one process is in its critical section at a time.

Liveness: Whenever any process requests to enter its critical section it will *eventually* be permitted to do so.

Non-Blocking: A process can always request to enter its critical section.

Example: a Kripke model for mutual exclusio

Each process can be in its non-critical state (**N**), or trying to enter its critical state (**T**), or in its critical state (**C**). The variable **turn** considers the *first* process that went into its trying state.

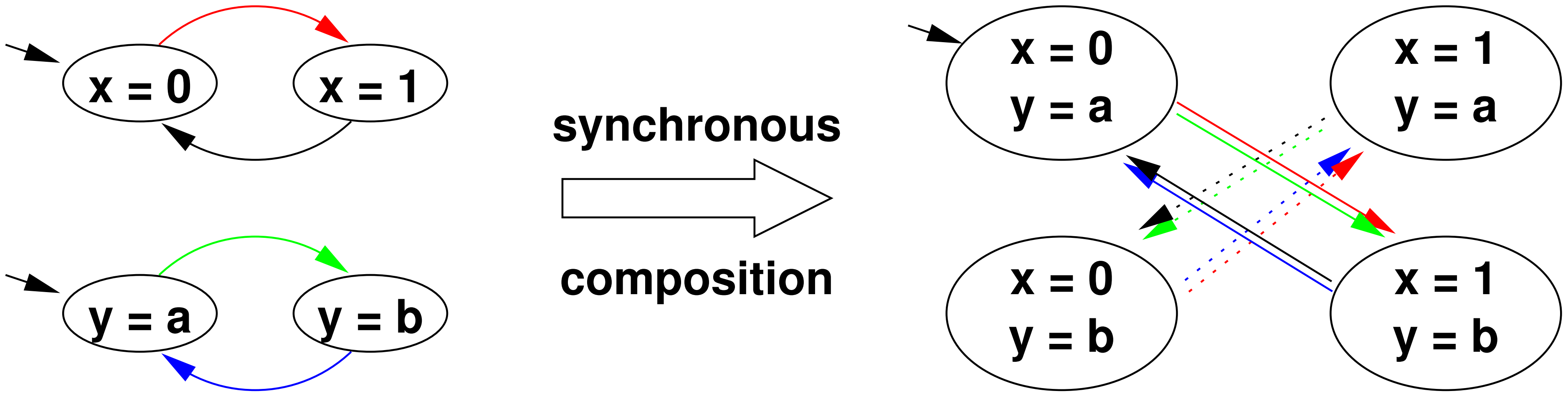


Composing Kripke Models

- Complex Kripke Models are tipically obtained by composition of smaller ones
- Components can be combined via
  - synchronous composition
  - asynchronous composition.

Synchronous Composition

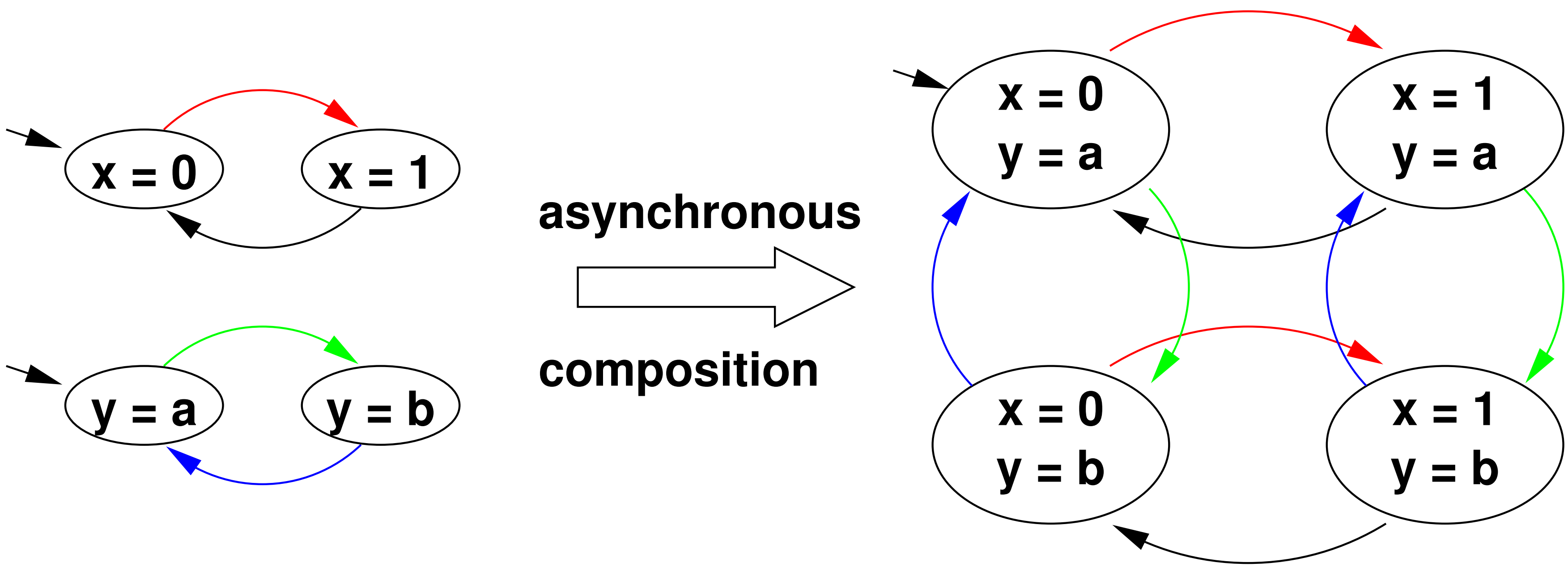
- Components evolve in parallel.
- At each time instant, every component performs a transition.



- Typical example: sequential hardware circuits.

Asynchronous Composition

- Interleaving of evolution of components.
- At each time instant, one component is selected to perform a transition.



- Typical example: communication protocols.

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Description languages for Kripke Model

Tipically a Kripke model is not given explicitly, rather it is usually presented in a structured language (e.g., NuSMV, SDL, PROMELA, StateCharts, VHDL, ...)
Each component is presented by specifying:

- A set of system variables
- Initial values for state variables
- Instructions

Description languages for Kripke Model

The correspondence between a description language and the Kripke Model is the following:

1. States: all possible assignments for system variables;
2. Initial States: Initial values for system variables;
3. Transitions: Instructions;
4. Atomic Propositions: Propositions associated to the values of the system variables;
5. Labeling: Set of atomic propositions true at a state.

The NuSMV language

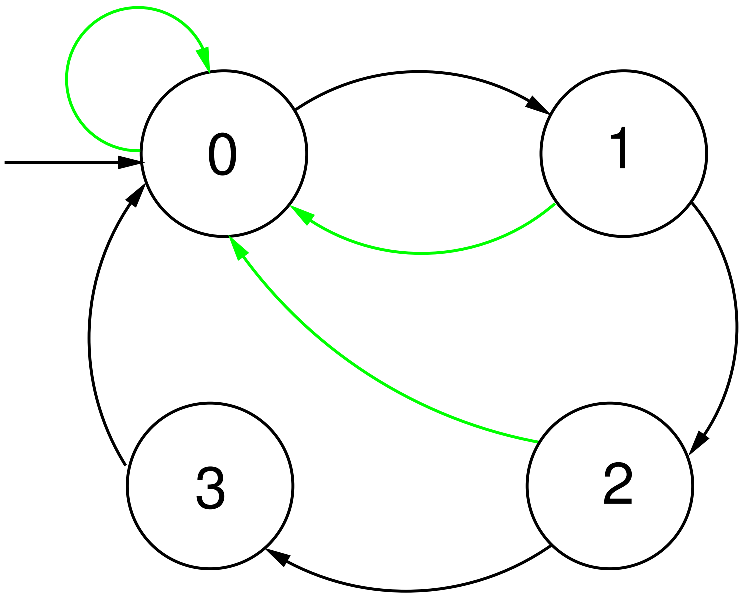
- The NuSMV (New Symbolic Model Verifier) model-checking system is an Open Source product (nusmv.irst.itc.it).
- NuSMV programs consist of:
  - Type declarations of the system variables;
  - Assignments that define the valid initial states (e.g., `init(b0) := 0`).
  - Assignments that define the transition relation (e.g., `next(b0) := !b0`).

NuSMV: The modulo 4 counter with reset

```
MODULE main
 VAR
   b0    : boolean;
   b1    : boolean;
   reset : boolean;
   out   : 0..3;

 ASSIGN
   init(b0) := 0;
   next(b0) := case
                reset = 1: 0;
                reset = 0: !b0;
               esac;

   init(b1) := 0;
   next(b1) := case
                reset: 0;
                1     : ((!b0 & b1)|(b0 & !b1));
               esac;
   out := b0 + 2*b1;
```



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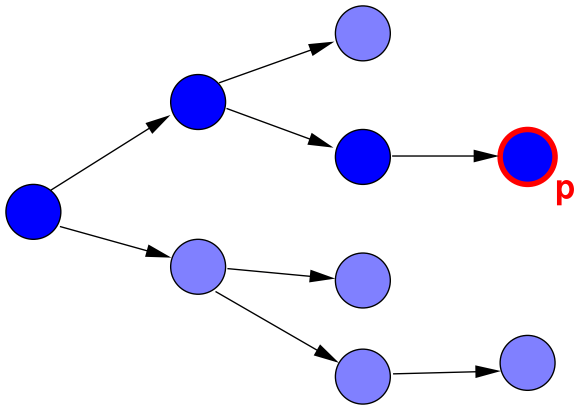
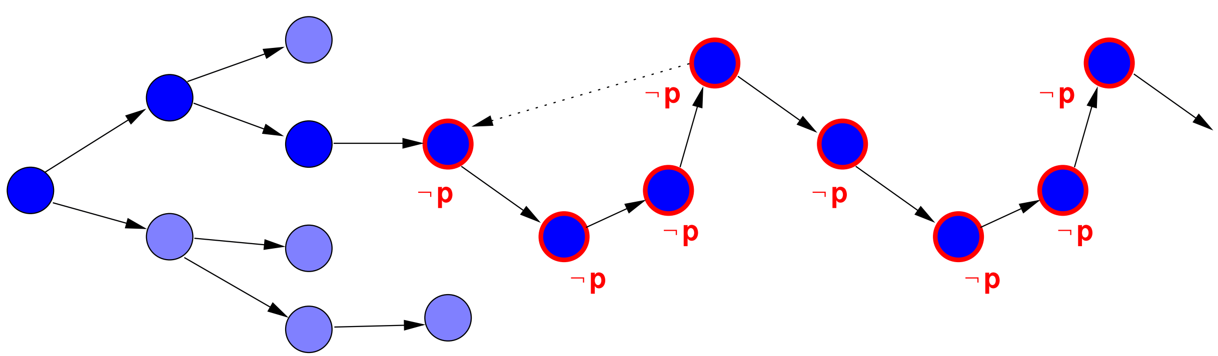
Safety Properties

- Nothing Bad Ever Happens.
  - Deadlock: two processes waiting for input from each other, the system is unable to perform a transition.
  - No reachable state satisfies a "bad" condition, e.g. never two processes in critical section at the same time
- It is expressed by a temporal formula saying that "*it's never the case that p*".



Liveness Properties

- Something Desirable Will Eventually Happen.
  - Whenever a subroutine takes control, it will always return it (sooner or later).
- It is expressed by a temporal formula saying that "*at each state it will be the case that p*".
- Can be refuted by infinite behaviour (represented as a loop)



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