Exercises for Logic 2016–2017

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http://www.inf.unibz.it/ artale/DML/dml.htm

Propositional Logic: Week 7

Computer Science

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Disclaimer. The course exercises are meant as complementary material for the students of the course of Logic at the Free University of Bozen-Bolzano: The notes and exercises are periodically updated and made available in the course web page: http://www.inf.unibz.it/ artale/DML/dml.htm.

Contents

1 Formalisation

1.1 Natural language statements

Formalise the following natural-language sentences in a suitable propositional languages.

- To sneeze or not to sneeze, that yields the question. *(Careful with "that".)*

$$p \vee \neg p \to q$$

- It is sufficient to be a bird in order to fly.

$$BIRD \to FLY$$

- It is necessary to be a bird in order to fly.

$$FLY \to BIRD$$

- It is necessary and sufficient to be a bird in order to fly.

$$BIRD \leftrightarrow FLY$$

- If $x + y = 2$ then $x = 2 - y$.

$$p \to q$$

- If Italy is close to France and France is close to the Netherlands, then Italy is close to the Netherlands.

$$p \wedge q \to r$$

- If Italy is close to France and if France is close to the Netherlands, then Italy is close to the Netherlands.

$$p \wedge q \to r$$

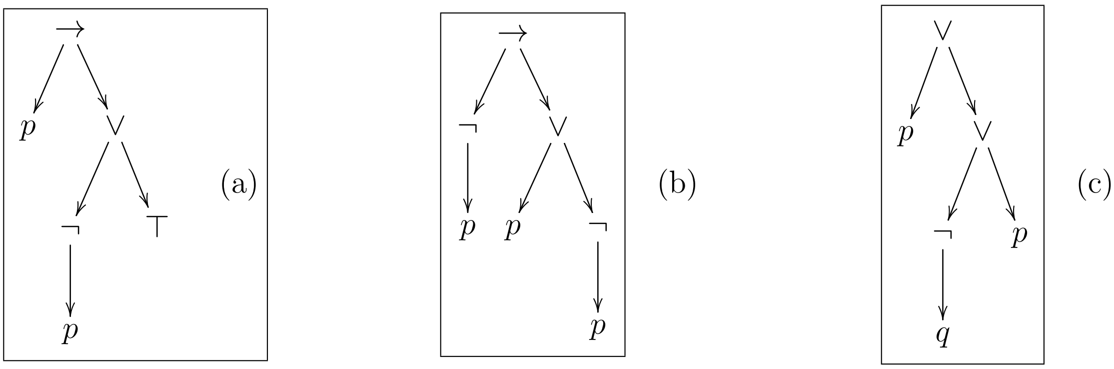

Figure 1: Parsing Trees.

1.2 Asimovland

Consider the following situation.

> The satellite of Asimovland is inhabited by exactly two robots, Al and Bob. The robots are subject to the following laws:
>
> 1. a robot protects the other robot if and only if the former robot does not harm the latter one (*hint*: careful with "*a* robot");
> 2. it is necessary that Al protects itself for Bob to harm Al;
> 3. it is sufficient that Bob protects itself for Al to harm itself;
> 4. Bob does not protect Al.

Formalise the laws in a propositional language, by first rewriting them into statements with only "if-then", "and", "or", "it is not the case that".

2 Subformulae (Homework)

Consider the trees in Fig. 1Parsing Treesfigure.1. For each of them, list the subformulae of the associated formula.

<u>Answer.</u> For (a), we obtain the following set of subformulae (including the entire formula):

$$\{p, \neg p, \top, \neg p \vee \top, p \to \neg p \vee \top\}$$

For (b):

$$\{p, \neg p, p \vee \neg p, \neg p \to p \vee \neg p\}$$

For (c):

$$\{p, q, \neg q, \neg q \vee p, p \vee (\neg q \vee p\}$$

3 Truth tables

3.1 Formulae

Build the truth tables of the following formulae:

1. $(\neg(\neg p))$;
2. $(p \wedge (q \wedge \neg p))$;
3. $((p \rightarrow q) \rightarrow (\neg q \rightarrow \neg q))$.

3.2 Chemical reactions *(Homework)*

Under certain conditions, the following chemical reactions are possible:

$$\begin{array}{lcl} HCl + NaOH & \rightarrow & NaCl + H_2O, \\ C + O_2 & \rightarrow & CO_2, \\ CO_2 + H_2O & \rightarrow & H_2CO_3. \end{array}$$

Formalise the above set of chemical reactions in a suitable propositional language. Then, using truth tables, check whether the set augmented with the negation of the propositional formula for H_2CO_3 is satisfiable.

4 Validity and Satisfiability of a Formula

4.1 Semantic Argument

- Find an interpretation and a formula such that the formula is true under that interpretation (or: the interpretation satisfies the formula; the interpretation is a model for the formula).
- Find an interpretation and a formula such that the formula is not true under that interpretation (or: the interpretation does not satisfy the formula; the interpretation is a counter-model for the formula)).
- Find a formula that cannot be true under any interpretation (or: no interpretation can satisfy the formula).

4.2 Semantic Arguments or Truth Tables

Consider the following problems.

- Which of the following formulae is a tautology (true under all interpretations for its atoms)?
- Which is (only) satisfiable (true under at least one interpretation for its atoms)?
- Which is falsifiable (false under at least one interpretation for its atoms)?
- Which is unsatisfiable (false under all interpretations for its atoms)?

Decide on them by means of a semantic argument, that is, arguing about the definition of interpretation of a formula, or by means of truth tables.

1. $p \to p$;
2. $p \to (q \to p)$;
3. $\neg\neg p \to p$;
4. $\neg\neg p \leftrightarrow p$;
5. $(p \to q) \leftrightarrow (\neg p \vee q)$.