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Formal Languages and Compilers — BSc course

2018/19 – Second Semester
Lexemes and Tokens.

Lexer Representation: Regular Expressions (RE).

Implementing a Recognizer of RE’s: Automata.

Lexical Analyzer Implementation.
  - Breaking the Input in Substrings.
  - Dealing with conflicts.
  - Lexical Analyzer Architecture.
Lexical Analysis is the first phase of a compiler.

**Main Task:** Read the input characters and produce a sequence of **Tokens** that will be processed by the Parser.

Upon receiving a “*get-next-token*” command from the Parser, the input is read to identify the next token.

While looking for next token it eliminates comments and white-spaces.

**Tokens** are treated as **Terminal Symbols** in the Grammar for the source program.
In general, a set of input strings (Lexemes) give rise to the same Token. For example:

<table>
<thead>
<tr>
<th>Token</th>
<th>Lexeme</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>var</td>
<td>var</td>
<td>language keyword</td>
</tr>
<tr>
<td>if</td>
<td>if</td>
<td>language keyword</td>
</tr>
<tr>
<td>relation</td>
<td>&lt;, &lt;=, =, &lt;&gt;, &gt;, &gt;=</td>
<td>comparison operators</td>
</tr>
<tr>
<td>id</td>
<td>position, A1, x</td>
<td>sequence of letters and digits</td>
</tr>
<tr>
<td>num</td>
<td>3.14, 14, 6.02E23</td>
<td>numeric constant</td>
</tr>
</tbody>
</table>
When a Token can be generated by different Lexemes the Lexical Analyzer must transmit also the Lexeme to the subsequent phases of the compiler. Such information is specified as an **Attribute** associated to the Token. Usually, the attribute of a Token is a pointer to the symbol table entry that keeps information about the Token.

**Important!**

- The **Token** influences parsing decisions: Parser relies on the token distinctions, e.g., identifiers are treated differently than keywords;
- The **Attribute** influences the semantic and code generation phase.
Example. Let us consider the following assignment statement:

\[ E := M \times C \times 2 \]

then the following pairs \langle token, attribute \rangle are passed to the Parser:

- \langle id, \text{pointer to symbol-table entry for } E \rangle
- \langle assign-op, \rangle
- \langle id, \text{pointer to symbol-table entry for } M \rangle
- \langle mult-op, \rangle
- \langle id, \text{pointer to symbol-table entry for } C \rangle
- \langle exp-op, \rangle
- \langle num, \text{integer value } 2 \rangle.

- Some Tokens have a null attribute: the Token is sufficient to identify the Lexeme.
- From an implementation point of view, each token is encoded as an integer number.
Programming languages use fixed strings to identify particular **Keywords**—e.g., `if`, `then`, `else`, etc.

Since keywords are just identifiers the Lexical Analyzer must distinguish between these two possibilities.

If keywords are *reserved*—not used as identifiers—we can initialize the symbol-table with all the keywords and mark them as such.

Then, a string is recognized as an identifier only if it is not already in the symbol-table as a keyword.
Lexemes and Tokens.

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**Lexical Analyzer Implementation.**
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- Lexical Analyzer Architecture.
Main Objective

What we want to accomplish:

- Given a way to describe Lexemes of the input language, automatically generate the Lexical Analyzer.

This objective can be split in the following sub-problems:

1. **Lexer specification language**: How to represent Lexemes of the input language.

2. **The lexical analyzer mechanism**: How to generate Tokens starting from Lexeme representations.

3. **Lexical analyzer implementation**: Coding (1) + (2) + breaking inputs into substrings corresponding to the pair Lexeme/Token.
Problem 1. Lexer Specification Language: Regular Expressions

- Regular Expressions are the most popular specification formalisms to describe Lexemes and map them to Tokens.

- **Example.** An identifier is made by a letter followed by zero or more letters or digits:
  \[
  \text{letter} \ (\text{letter} \ | \ \text{digit})^* \\
  \]

  The vertical bar | means “or”; Parentheses are used to group sub-expressions; The * means zero or more occurrences.
Each Regular Expression, say $R$, denotes a Language, $L(R)$. The following are the rules to build them over an alphabet $V$:

1. If $a \in V \cup \{\epsilon\}$ then $a$ is a Regular Expression denoting the language $\{a\}$;
2. If $R, S$ are Regular Expressions denoting the Languages $L(R)$ and $L(S)$ then:
   1. $R \mid S$ is a Regular Expression denoting $L(R) \cup L(S)$;
   2. $RS$ is a Regular Expression denoting the concatenation $L(R)L(S)$, i.e. $L(R)L(S) = \{rs \mid r \in L(R) \text{ and } s \in L(S)\}$;
   3. $R^*$ is a Regular Expression denoting $L(R)^*$, zero or more concatenations of $L(R)$, i.e. $L(R)^* = \bigcup_{i=0}^{\infty} L(R)^i$;
   4. $(R)$ is a Regular Expression denoting $L(R)$. 
Example. Let $V = \{a, b\}$.

1. The Regular Expression $a | b$ denotes the Language $\{a, b\}$.
2. The Regular Expression $(a | b)(a | b)$ denotes the Language $\{aa, ab, ba, bb\}$.
3. The Regular Expression $a^*$ denotes the Language of all strings of zero or more $a$’s, $\{\epsilon, a, aa, aaa, \ldots\}$.
4. The Regular Expression $(a | b)^*$ denotes the Language of all strings of $a$’s and $b$’s.
Notational shorthands are introduced for frequently used constructors.

1. **+, One or more instances.** If $R$ is a Regular Expression then $R^+ \equiv RR^*$.  

2. **?, Zero or one instance.** If $R$ is a Regular Expression then $R? \equiv \epsilon \mid R$.  

3. **Character Classes.** If $a, b, \ldots, z \in \mathbf{V}$ then $[a, b, c] \equiv a \mid b \mid c$, and $[a - z] \equiv a \mid b \mid \ldots \mid z$.  


Regular Definitions are used to give names to regular Expressions and then to re-use these names to build new Regular Expressions.

A Regular Definition is a sequence of definitions of the form:

\[
\begin{align*}
D_1 & \rightarrow R_1 \\
D_2 & \rightarrow R_2 \\
& \quad \ldots \\
D_n & \rightarrow R_n
\end{align*}
\]

Where each \( D_i \) is a distinct name and each \( R_i \) is a Regular Expression over the extended alphabet \( V \cup \{D_1, D_2, \ldots, D_{i-1}\} \).

**Note:** Such names for Regular Expression will be often the Tokens returned by the Lexical Analyzer. As a convention, names are printed in **boldface**.
Example 1. Identifiers are usually strings of letters and digits beginning with a letter:

\[
\begin{align*}
\text{letter} & \rightarrow A \mid B \mid \ldots \mid Z \mid a \mid b \mid \ldots \mid z \\
\text{digit} & \rightarrow 0 \mid 1 \mid \ldots \mid 9 \\
\text{id} & \rightarrow \text{letter(letter | digit)}^* \\
\end{align*}
\]

Using \textit{Character Classes} we can define identifiers as:

\[
\begin{align*}
\text{id} & \rightarrow [A \mid Z a \mid z][A \mid Z a \mid z0 \mid 9]^* \\
\end{align*}
\]
Example 2. Numbers are usually strings such as 5230, 3.14, 6.45E4, 1.84E-4.

\[
\begin{align*}
digit & \rightarrow 0 \mid 1 \mid \cdots \mid 9 \\
digits & \rightarrow digit^+ \\
\text{optional-fraction} & \rightarrow (.digits)\
\text{optional-exponent} & \rightarrow (E(\mid + \mid -)?\text{digits})\
\text{num} & \rightarrow \text{digits optional-fraction optional-exponent}
\end{align*}
\]
Summary

- Lexemes and Tokens.
- Lexer Representation: Regular Expressions (RE).
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Problem 2. The Lexical Analyzer Mechanism. Finite Automata

- We need a mechanism to recognize Regular Expressions and so associating Tokens to Lexemes.
- While Regular Expressions are a *specification language*, Finite Automata are their *implementation*.
  - Given an input string, \( w \), and a Regular Language, \( L \), they answer “yes” if \( w \in L \) and “no” otherwise.
A Deterministic Finite Automata, DFA for short, is a tuple: \( A = (S, \Sigma, \delta, s_0, F) \):

- \( S \) is a finite non empty set of states;
- \( \Sigma \) is the input symbol alphabet;
- \( \delta : S \times \Sigma \rightarrow S \) is a total function called the Transition Function;
- \( s_0 \in S \) is the initial state;
- \( F \subseteq S \) is the set of final states.
To define when an Automaton accepts a string we extend the transition function, $\delta$, to a multiple transition function $\hat{\delta} : S \times V^* \rightarrow S$:

\[
\begin{align*}
\hat{\delta}(s, \epsilon) &= s \\
\hat{\delta}(s, xa) &= \delta(\hat{\delta}(s, x), a); \quad \forall x \in V^*, \forall a \in V
\end{align*}
\]

A DFA accepts an input string, $w$, if starting from the initial state with $w$ as input the Automaton stops in a final state:

\[
\hat{\delta}(s_0, w) = f, \quad \text{and} \quad f \in F.
\]

Language accepted by a DFA, $A = (S, V, \delta, s_0, F)$:

\[
L(A) = \{ w \in V^* \mid \hat{\delta}(s_0, w) \in F \}
\]
A DFA can be represented by Transition Graphs where the nodes are the states and each labeled edge represents the transition function.

The initial state has an input arch marked start. Final states are indicated by double circles.

Example. DFA that accepts strings in the Language $L((a \mid b)^*abb)$.
Transition Tables implement transition graphs, and thus Automata.

A Transition Table has a row for each state and a column for each input symbol.

The value of the cell \((s_i, a_j)\) is the state that can be reached from state \(s_i\) with input \(a_j\).

Example. The table implementing the previous transition graph will have 4 rows and 2 columns, let us call the table \(\delta\), then:

\[
\begin{align*}
\delta(0, a) &= 1 & \delta(0, b) &= 0 \\
\delta(1, a) &= 1 & \delta(1, b) &= 2 \\
\delta(2, a) &= 1 & \delta(2, b) &= 3 \\
\delta(3, a) &= 1 & \delta(3, b) &= 0
\end{align*}
\]
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This objective can be split in the following sub-problems:

1. **Lexer specification language**: How to represent Lexemes of the input language (Regular Expressions).

2. **The lexical analyzer mechanism**: How to generate Tokens starting from Lexeme representations (*Side-Effect of Automata recognition process*).

3. **Lexical analyzer implementation**: Coding (1) + (2) + breaking inputs into substrings corresponding to the pair Lexeme/Token.
Given that the source program is just a single string, the Lexical Analyzer must do two things:

1. **Breaking** the input string by recognizing substrings corresponding to Lexemes;
2. **Return** the pair \( \langle \text{Token, Attribute} \rangle \) for each Lexeme.

Compare this procedure to Automata:

1. Automata accept or reject a string, they do not partition it.
2. We need to do more than just implement an Automaton.
Two important issues:

1. The goal is to partition the source program into Lexemes: This is implemented by reading left-to-right, recognizing one lexeme at a time.

2. Dealing with **conflicts**. E.g., pos Vs. position as identifiers.
Lexemes and Tokens.

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Dealing with Conflicts

There are two possible conflicts:

Case 1. The same Lexeme is recognized by two different RE’s.
Case 2. The same RE can recognize portion of a Lexeme.
Dealing with Conflicts (Case 1)

Let’s assume we have the following RE’s:

\[ R_1 \rightarrow \text{abb} \]
\[ \text{id} \rightarrow \text{letter(letter | digit)*} \]

- The Lexeme “abb” matches both \( R_1 \) and \( \text{id} \).
- **Solution: Ordering between RE’s.** If "\( a_1 \ldots a_n \) \( \in L(R_j) \) and "\( a_1 \ldots a_n \) \( \in L(R_k) \) we use the RE listed first (\( j \), if \( j < k \)).
- **Remark.** To distinguish between keywords and identifiers insert the regular expression for keywords before the one for identifiers.
Let’s consider the RE for Identifiers and the string ”position”:

\[ \text{id} \rightarrow \text{letter(letter | digit)}^* \]

- “p” matches \text{id};
- “po” matches \text{id};
- …;
- “position” matches \text{id}.

**Solution: Maximal Lexemes.** The lexeme for a given token must be maximal.
**Solution 1.** Use Automata with *Lookahead*.

- *Example:* Automaton for *id* with lookahead.

  ![Automaton Diagram]

- The label *other* refers to any character not indicated by any other edge living the node;
- The * indicates that we read an extra character and we must retract the forward input pointer by one character.
Solution 2. Change the response to non-acceptance.

- Don’t stop when reaching an accepting state;
- When failure occurs revert back to the last accepting state.
- This technique is the preferred one—used also by Lex.

Example: Automaton for numbers:

```
0 1 2 3 4 5 6
start digit digit digit digit digit digit
```

Try with the following input: “61.23Express”.

Exercise. Modify the above Automaton to comply with the Lookahead technique.
When several possible lexemes of the input match one or more RE’s:

1. Always prefer a longer lexeme.
2. If the longest lexeme is matched by two or more RE’s prefer the RE listed first.
Lexemes and Tokens.

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Lexical Analyzer Implementation.

- Breaking the Input in Substrings.
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- Lexical Analyzer Architecture.
1. For each Token write the associated Regular Expression;
2. Build the Automaton for each Regular Expression;
3. Combine all the Automata ($A_i$) into a single big Automaton adding a new start state with $\epsilon$-transitions ($\epsilon$-NFA) to each of the start states of the $A_i$ Automata.
4. Read the input to map Lexemes into Tokens till we successfully read the whole input or the Automaton fails without recognizing the input left (case of Lexical error).
Consider the following Regular Expressions:

\[ R_1 \rightarrow a \]
\[ R_2 \rightarrow abb \]
\[ R_3 \rightarrow a^* b^+ \]

Construct the Automaton and consider as input "abb".
Single NFA Automaton.
Reading the Input

- The lexical analysis is the only phase that reads the input.
- Such reading can be time consuming: It is necessary to adopt efficient buffering techniques (see the book, Chapter 3.2, for more details).
- Two pointers, `begin` and `forward`, to the input buffer are maintained.
- The `begin` pointers always points at the beginning of the lexeme to be recognized.
Two pointers, `begin` and `forward`, to the input buffer are maintained:

1. Initially, both pointers point to the first character of the Lexeme to be found;
2. The `forward` pointer scans ahead the input until there are no more next states in the Automaton—we are sure that the longest lexeme has been found.
3. We go back in the sequence of set of states (assuming the Automaton is NFA) till we find a set that contains one or more accepting states, otherwise fail.
4. If there are more accepting states prefer the state associated with the RE listed first.
5. When the Lexeme is successfully processed transmit the token and its attribute to the parser, and set both pointers to the next character immediately past the Lexeme.
6. If there are no more input characters then succeed else go to point 1.
Summary of Lecture VI

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