Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Formal Languages and Compilers– A.Y. 2013/2014 Mid-Term Exam – 11/11/2013
Dr. Alessandro Artale – *Time: 120 minutes*

`This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet.`

Problem 1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) Every subset of a regular language is regular.

(b) If L is regular, then so is $L_1 = \{xy \mid x \in L \text{ and } y \in \overline{L}\}$.

(c) The language $L = \{0^n 1^n \mid n \geq 1\}$ is regular (use the Pumping Lemma).

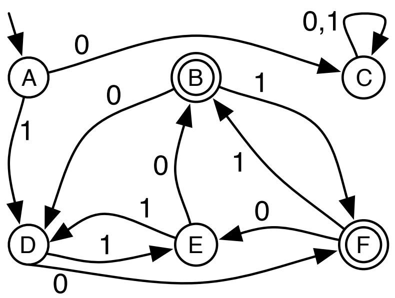
Problem 2 [9 points]

(a) Construct a Context Free Grammar for the language L of Palindroms over the alphabet $\{0, 1\}$. A palindrome is a sequence of symbols (possibly empty) whose meaning may be interpreted the same way in either forward or reverse direction. E.g., $010 \in L, 1001 \in L, 1101011 \in L, \ldots$

(b) Construct a R.E. that accepts the language over the alphabet $\{0, 1, 2\}$ constituted by all strings in which each 1 is immediately followed by a 2. The language includes the empty string. E.g., $012122012 \in \mathcal{L}(R.E.)$, while $001122012 \notin \mathcal{L}(R.E.)$.

(c) Construct an ε-NFA A that accepts the language specified above. Use the Thompson's construction.

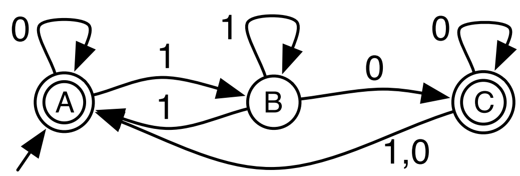
Problem 3 [6 points] Consider the following DFA, A, over $\{0, 1\}$:



(a) Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. The algorithm you have followed to construct A_m should become evident in your construction.

(b) Give 2 strings (of length at least 4) that are in $\mathcal{L}(A)$ and 2 strings (of length at least 4) that are not in $\mathcal{L}(A)$. Provide a description of $\mathcal{L}(A)$ in plain English.

Problem 4 [4 points] Consider the following NFA, N_1, over $\{0, 1\}$:



Construct a DFA D such that $\mathcal{L}(D) = \mathcal{L}(N_1)$. The algorithm you have followed to construct D should become evident in your construction.

Problem 5 [4 points]

(a) Apply the steps that are necessary to Clean-up a context free grammar (Eliminate: ε-productions, unit productions, non-generating and non-reachable symbols) to the context free grammar $G = (\{S, A, B, C\}, \{a, b\}, P, S)$, where P consists of the following productions:

$$
\begin{array}{lcl}
S & \longrightarrow & B \mid BaBb \mid AbB \\
A & \longrightarrow & Ab \mid AC \\
B & \longrightarrow & Bb \mid AC \mid \varepsilon \\
C & \longrightarrow & AC \mid CB \mid Ba
\end{array}
$$

Problem 6 [4 points]

(a) Write a grammar for Boolean Expressions between **identifiers** using the operators **or**, **and**, **not** and parentheses. Be sure to give to both **or** and **and** the same lower precedence, then **not** with the highest precedence. Allow for repeated **not**, as in the Boolean Expression **not not id**. Be sure that the grammar is not ambiguous.

(b) Show the right-most derivation for the string: id_1 **or not** id_2 **and** id_3.