Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Formal Languages and Compilers – A.Y. 2015/2016 Mid-Term Exam – 20/11/2015
Prof. Alessandro Artale – *Time: 120 minutes*

`This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet.`

Problem 1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, if $L_1^* = L_2^*$, then $L_1 = L_2$.

(b) If a language L is constituted by a *finite* set of strings, then L is a regular language.

(c) The language $L = \{ww \mid w \in \{a, b\}^*\}$ is regular (use the Pumping Lemma).

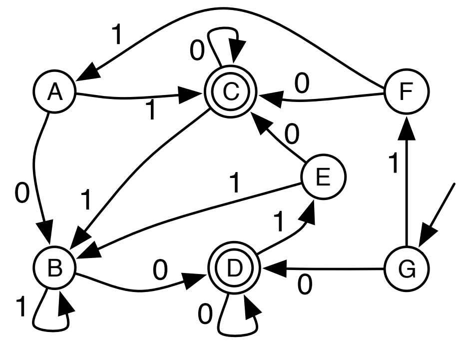
Problem 2 [9 points]

(a) Construct a Context Free Grammar for the language $L = \{0^n 1^m \mid n > m, \text{ and } m \geq 0\}$.

(b) Construct both a regular expression, RE, and the automaton, N_{RE}, for the language over the alphabet $\{a, b\}$ constituted by all strings containing an odd number of a and such that between each pair of consecutive a there is an even number (possibly 0) of b.
E.g., $a \in \mathcal{L}(E)$, $aaa \in \mathcal{L}(E)$, $abbabba \in \mathcal{L}(E)$, $babbaabbb \in \mathcal{L}(E)$, $\varepsilon \notin \mathcal{L}(E)$, $aa \notin \mathcal{L}(E)$, $abaa \notin \mathcal{L}(E)$.

(c) Given two DFA's, said A and B, describe the notion of Product Automaton. Furthermore, show how this notion can be used to prove that regular languages are closed under intersection.

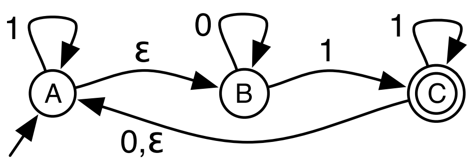
Problem 3 [6 points] Consider the following DFA A over $\{0, 1\}$:



(a) Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. The algorithm you have followed to construct A_m should become evident in your construction.

(b) Give 2 strings (of length at least 4) that are in $\mathcal{L}(A)$ and 2 strings (of length at least 4) that are not in $\mathcal{L}(A)$.

Problem 4 [6 points] Consider the following ε-NFA A_ε over $\{0, 1\}$:



(a) Construct an **NFA** A_n such that $\mathcal{L}(A_n) = \mathcal{L}(A_\varepsilon)$, and show the **$\epsilon$-closure** of each state. The algorithm you have followed to construct A_n should become evident in your construction.

(b) Show all sequences of transitions of A_n that lead to acceptance of the string 11011.

Problem 5 [6 points]

(a) Describe the algorithm to eliminate the ε-transitions from a context free grammar.

(b) Describe the algorithm to eliminate non-generating symbols from a context free grammar.

Apply first algorithm (a) and then algorithm (b) to the grammar $G = (\{S, A, B, C\}, \{a, b\}, P, S)$, where P consists of the following productions:

$$\begin{array}{lcl} S & \longrightarrow & C \mid bAAa \mid CaA \\ A & \longrightarrow & Aa \mid CB \mid \varepsilon \\ B & \longrightarrow & CB \mid BA \mid Ba \mid CD \\ C & \longrightarrow & Ca \mid CB \mid \varepsilon \\ D & \longrightarrow & Da \mid b \end{array}$$