Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Applied Computer Science
Formal Languages – A.Y. 2009/2010
Sample Exam – 20/1/2010

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) If L_1 is a regular language and $L_1 \cup L_2$ is a regular language, then L_2 is a regular language.

(b) For all regular languages L_1, L_2, and L_3, if $L_1 \subseteq L_2$ and $L_2^* \subseteq L_3^*$, then $L_1 \subseteq L_3$.

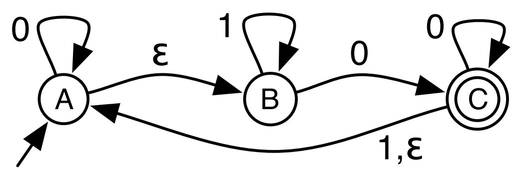
(c) If L is constituted by a *finite* set of strings, then L is a regular language.

Problem 2 [6 points]

(a) Construct a **deterministic** finite automaton (DFA) A that accepts the language over the alphabet $\{a, b, c\}$ constituted by all strings that begin with a sequence of one or more a's, and end with a b.
E.g., $aaabcbccaabb \in \mathcal{L}(A)$, while $aaabcbcc \notin \mathcal{L}(A)$, and $bcbccaabb \notin \mathcal{L}(A)$.

(b) Construct a regular expression E that generates the language over the alphabet $\{x, y, z\}$ constituted by all strings in which each y is immediately followed by a z.
E.g., $xxyzzzzxyzyzzxx \in \mathcal{L}(E)$, while $xxyzzzzxyyzyzzxx \notin \mathcal{L}(E)$.

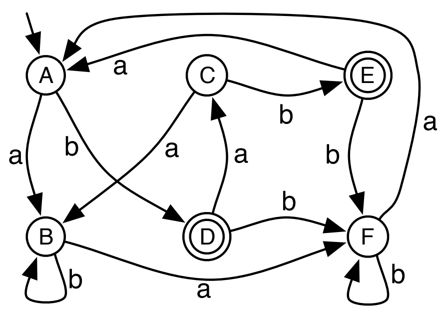
Problem 3 [6 points] Consider the following ε-NFA N_1 over $\{0, 1\}$:



(a) Construct an NFA N_2 such that $\mathcal{L}(N_2) = \mathcal{L}(N_1)$. The algorithm you have followed to construct N_2 should become evident in your construction.

(b) Show **all** sequences of transitions of N_1 and of N_2 that lead to acceptance of 0010.

Problem 4 [6 points] Consider the following DFA A over $\{a, b\}$:



(a) Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. The algorithm you have followed to construct A_m should become evident in your construction.

(b) Give 3 strings (of length at least 4) that are in $\mathcal{L}(A)$ and 3 strings (of length at least 4) that are not in $\mathcal{L}(A)$.

Problem 5 [6 points]

(a) Describe the algorithm to eliminate the unit-productions from a context free grammar.

(b) Describe the algorithm to eliminate the non-generating symbols from a context free grammar.

Apply first algorithm (a) and then algorithm (b) to the grammar $G = (\{S, A, B, C, D\}, \{a, b, c, d\}, P, S)$, where P consists of the following productions:

$$
\begin{array}{lcl}
S & \longrightarrow & A \mid AaBb \mid Da \\
A & \longrightarrow & Aa \mid C \mid D \\
B & \longrightarrow & BD \mid Cd \mid C \\
C & \longrightarrow & AB \mid aB \mid Ca \\
D & \longrightarrow & d \mid dA
\end{array}
$$