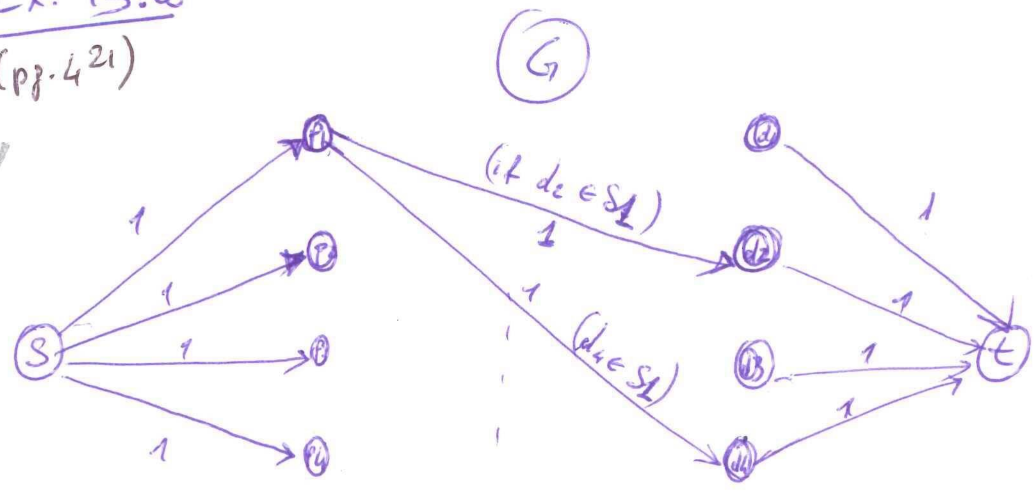
LAB 5

Ex. 15.a

(pg. 421)



n (People) n (dinner)

$S_i \in \{d_1, \dots, d_n\}$: Set of days on which person i can cook.

Feasible dinner shedule:

- Each Person, i, cook on exactly 1 night
- For each night there is someone cooking
- If p_i cook on d_j then $d_j \in S_i$.

Solution: Check if in G there is a Perfect Bipartite Matching.

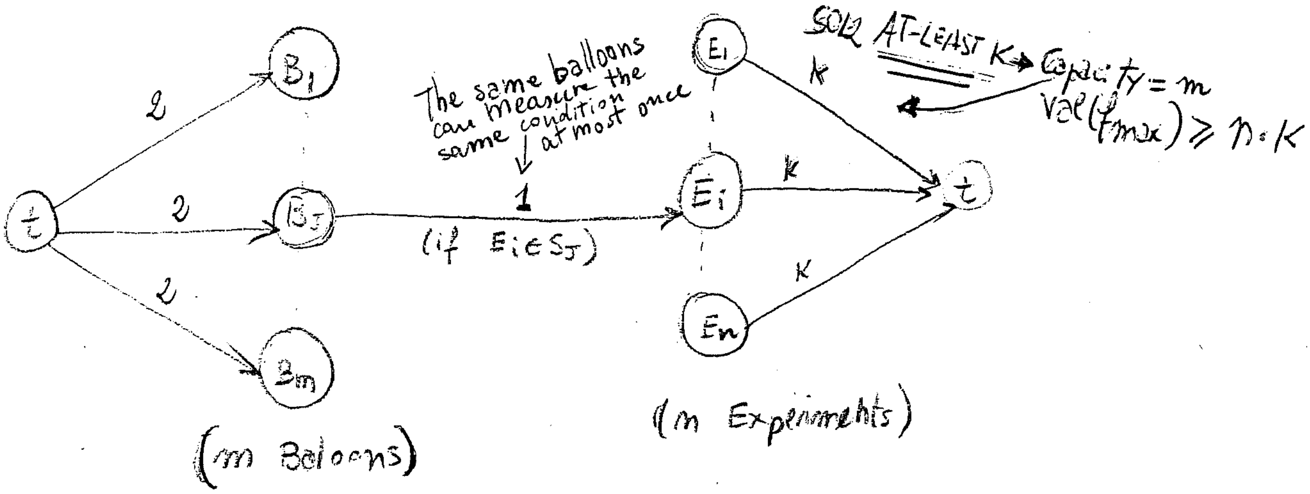
Ex7.19 pg. 425 & Ex7.16 pg 422

LAB-5

Ex. 20 Atmosferic Experiments.
Pg-426

- n Experiments, E_i
- m baloons, B_J
- Each B_J: can make ≤ 2 experiments
  - $S_J \subseteq \{E_1, \dots, E_m\}$ list experiments that B_J can take.
- K: Each experiment must be taken by ~~At least~~ EXATLY / AT-LEAST K balloons.

a)



Sol1: $Val(f_{max}) = n \cdot K$ (EXACTLY K measures for each Experiment)

LAB 5

Ex 7.8

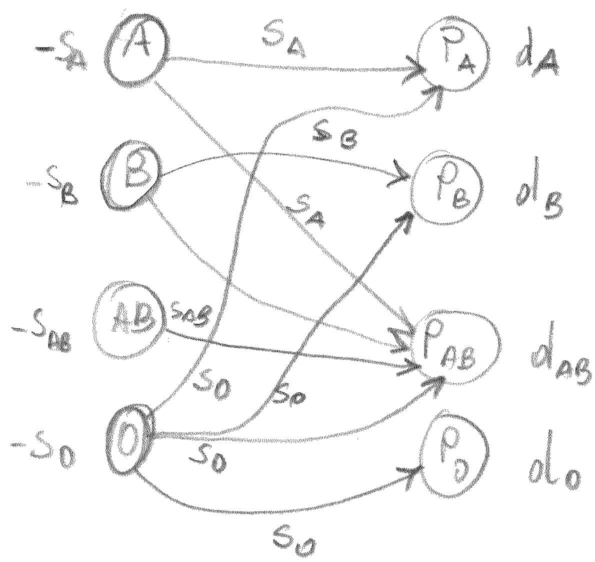
- Blood Types: A, B, AB, O
- Patient with BT: A can recieve A, O
  - B " " B, O
  - AB " " any type
  - O " " O
- In the hospital we have the following Total supply:

S_A, S_B, S_{AB}, S_O

- The demands for the coming week are:

D_A, D_B, D_{AB}, D_O

Problem1: Check if the blood on hand by the hospital is sufficient.



Feasible Circulation

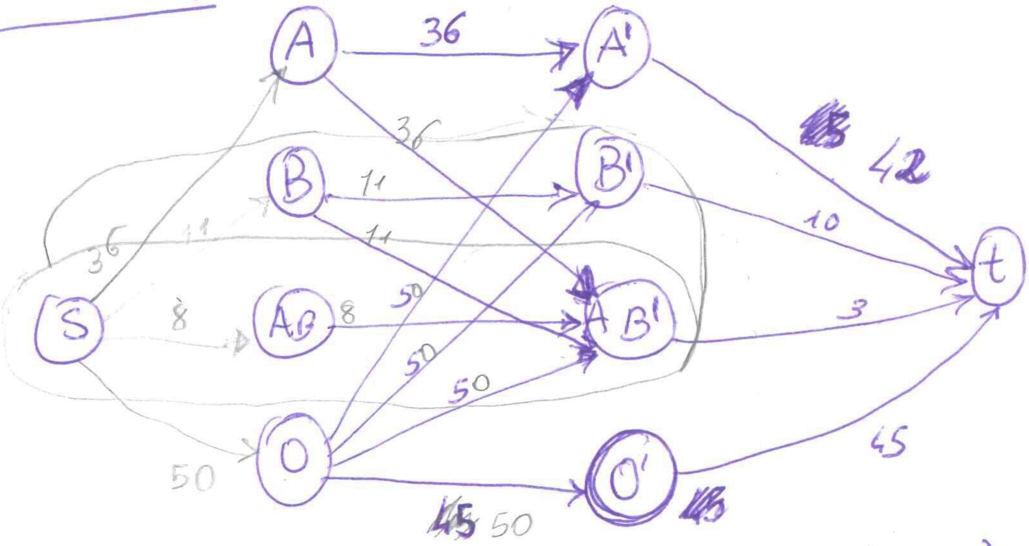
Problem2. Let: $S_A = 36$; $S_B = 11$; $S_{AB} = 8$; $S_O = 50$

$D_A = 42$; $D_B = 10$; $D_{AB} = 3$; $D_O = 45$

Show, using a min-cut argument, that max-Flow < 100

LAB5

Ex 7.8 (book)



min-cut = ({S, B, AB, B', AB'}, {t, A, A', O, O'})

~~min-cut = ({S, A, B, AB, O}, {t, AB'})~~ = (A, T)

$c(S,T) = 36 + 5 + 10 + 3 + 45 = \underline{\underline{99}} < 100$!

36 +
10 +
50 +
3
99

- Find an allocation that satisfies the max number of patients
- Use an argument based on min-cut to show why not all patients can be served.

Ex 19 (book)

Ex. 7.18 (pg 424) **LAB 5**

Coverage Expansion Problem

INPUT: - Bipartite Graph with $V = X \cup Y$
- A matching M; we set $Covered_M = \{ v \in V \mid v \text{ is an endpoint for } e \in M \}$
- $K \geq 1$

OUTPUT: - A Matching M' s.t.: ① $|M'| = |M| + K$, if M' exists.
② $Covered_M \subset Covered_{M'}$

Solution

Build a Net-Flow, G', with lower & upper bounds:

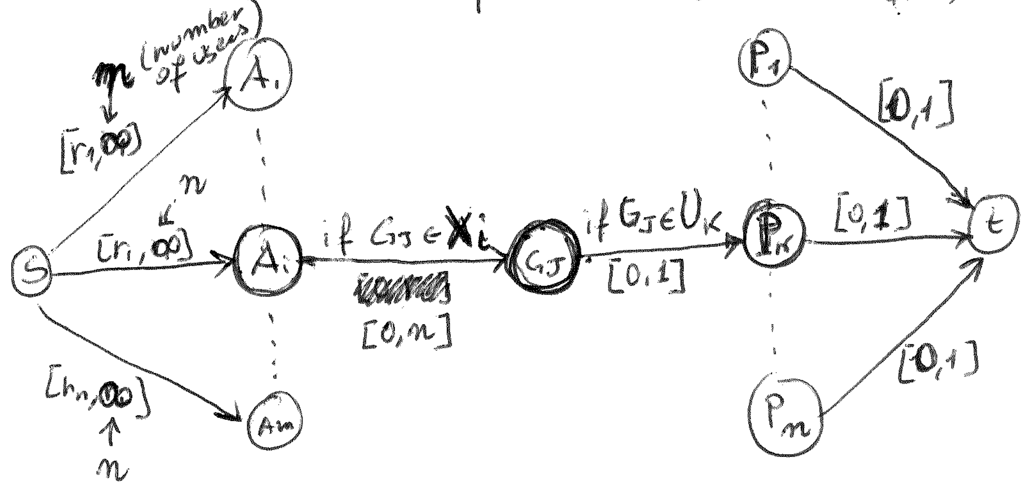
- Edge directed from $X \to Y$ with ~~upper~~ $[0,1]$ as lower/upper capacity
- " " " $s \to x_i$ with: $\begin{cases} [0,1] & \text{if } x_i \notin Covered_M \\ [1,1] & \text{otherwise} \end{cases}$
- " " " $Y_J \to t$ with: $\begin{cases} [0,1] & \text{if } Y_i \notin Covered_M \\ [1,1] & \text{if } Y_i \in Covered_M \end{cases}$

If $val(f_{max}) \geq |M| + K$ then a solution exists. In particular, we can run Max-Flow $(G', |M|+K)$ and stop when there is a flow, f' s.t. a) $val(f') = |M| + K$ or b) $val(f_{max}) < |M| + K$.

In case b) we answer NO SOLUTION. In case a) we can output M':

For each (s, x_i) incident to s
 if $f(s, x_i) = 1$ then found = False
 For each (x_i, y_J) incident in x_i: if $f(x_i, y_J) = 1$ ~~then~~ found = True
 output (x_i, y_J)

- n ~~Web~~ Users, $P_1, \dots, P_n$
- K Demographic Groups, $G_1, \dots, G_K$
- m Advertisers, $A_1, \dots, A_m$
- $X_i \in \{G_1, \dots, G_K\}$, A_i wants to show its ads only to users in the Demographic Group listed in X_i
- P_J belongs to one of: $U_J \subseteq \{G_1, \dots, G_K\}$, $J = 1 \dots n$



Problem: For each Advertiser, $i = 1, \dots, m$, can at least r_i of the n users, **belonging to the demographic group listed in X_i** be shown an ad by A_i?