LAB 4 - MAX-FLOW

Ex.1 Prove the following using the MAX-FLOW/MIN-CUT Theorem.

- Let G be a Net-Flow with max-Flow F_{max}. Consider a Net-Flow G' obtained by increasing by 1 the capacity of an edge of G: $c'(u,v) = c(u,v)+1$. then, the value F'_{max} of max-flow in G' is either $v(f_{max})$ or $v(f_{max})+1$.

Ex.2 Decide whether the following statement is true or false. If it is true, give an explanation, otherwise, provide a counter example.

- If F_{max} is the max-flow then $f_{max}(e) = c(e)$, $\forall e$ out-of s.

Ex.3 Let G' be a net-flow obtained from G by adding 1 to all capacities. Disprove the following:
If (A,B) was a min-cut in G it is also a min-cut in G'

$Sch \sqsubseteq \Diamond_F^{24}\, Sch$ $\quad \psi(x,t) = Sch(x,t) \land \neg Serv(x,t)$

$\mathcal{A} = \{ Sch(a,0),$

$\mathcal{A} \sqsubseteq$ $\quad \mathcal{A} = \{ A(a,0), A(a,1) \}, A(a,2) \}$ $\quad \psi(x,t) = \forall t' > t. A(x,t')$

$ans(\psi, \mathcal{A}) = \{ (a,0), (a,1) \}$

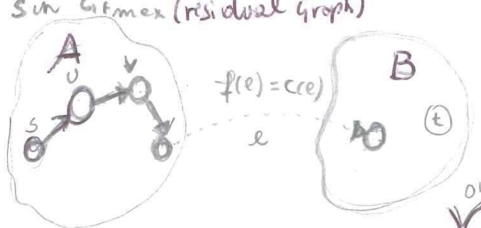
$\psi'(x,t) = \Box_F A(x,t)$

$ans(\psi', \mathcal{A}) = \emptyset$

LAB 4

SOL. LAB4. EX-1 → See book pp. 411

Reachable from s in $G_{f_{max}}$ (residual graph)

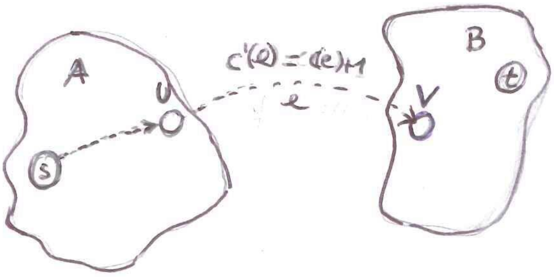


Let min-cut(A,B) then:

$$val(f_{max}) = c(A,B)$$

Two cases

1. If (u,v) is internal to A, or to B, then, $c'(A,B) = c(A,B) \Rightarrow Val(f'_{max}) = Val(f_{max})$
2. if (u,v) is out of A, i.e., $u \in A$ e $v \in B$, then



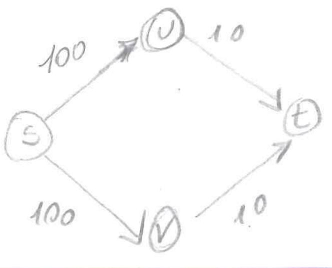
$c'(A,B) = c(A,B) + 1$,

Then since $V(f') \leq c'(A,B)$, and $V(f_m) = c'(A,B) - 1$, then

$$V(f'_m) = \begin{cases} V(f_m) \text{ or} \\ V(f_m)+1 \end{cases}$$

SOL. LAB4-EX-2

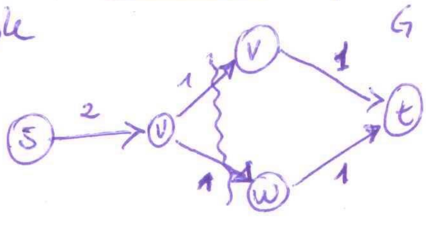
Counter-example

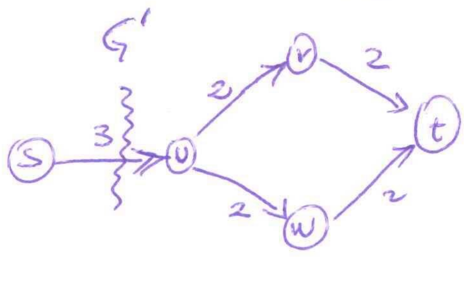


$F_{max} = 20 \neq 200$

Sol. Ex. 3

counter-example





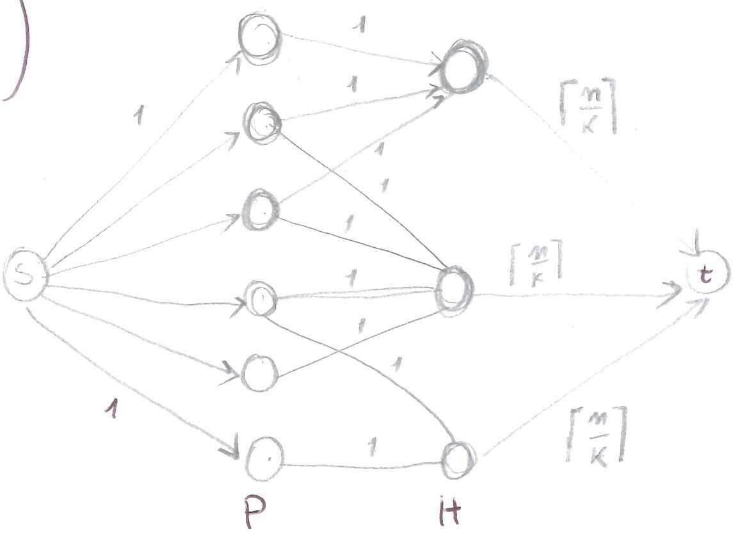
EX. ~~4~~

n = injured people

K = number of hospitals

LAB-4

(Ex 7.9 pg. 419)



$n = 6$, $K = 3$ } $\lceil \frac{n}{K} \rceil = 2$

There are n patients and K hospitals. Each patient has access to those hospitals reachable by 1/2 h driving time. ~~We~~ Show how max-flow can be used to find a solution that mantains balanced the load of each hospitals, i.e., each hospital receives $\leq \lceil \frac{n}{K} \rceil$ patients.

Net-Graph:

1. There is a vertex ~~for~~ each patient (P)
2. There " " " for each hospital (H)
3. Edge (p_i, h_J) iff hospital h_J is reachable by patient p_i
4. capacities:
   - (S, p_i) : $c(S, p_i) = 1$, $\forall p_i$
   - (p_i, h_J) : $c(p_i, h_J) = 1$, $\forall$ edges (p_i, h_J)
   - (h_J, t) : $c(h_J, t) = \lceil \frac{n}{K} \rceil$, $\forall h_J$.