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FREE UNIVERSITY OF BOZEN - BOLZANO



Faculty of Computer Science, Free University of Bozen-Bolzano, Piazza Domenicani 3, 39100 Bolzano, Italy

Tel: +39 04710 16000, fax: +39 04710 16009, <http://www.inf.unibz.it/krdb/>

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On relating heterogeneous elements from different ontologies

Chiara Ghidini¹, Luciano Serafini¹, Sergio Tessaris²

Affiliation	[1] FBK-irst. Via Sommarive 18 Povo, 38050, Trento, Italy [2] Free University of Bozen - Bolzano
Corresponding author	Sergio Tessaris www.inf.unibz.it/~tessariss
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On relating heterogeneous elements from different ontologies

Chiara Ghidini¹ Luciano Serafini¹ Sergio Tessaris²

¹ FBK-irst. Via Sommarive 18 Povo, 38050, Trento, Italy

² Free University of Bozen - Bolzano. Piazza Domenicani 3. 39100 Bolzano, Italy
ghidini@itc.it, serafini@itc.it, tessaris@inf.unibz.it

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Abstract

In the extensive usage of ontologies envisaged by the Semantic Web there is a compelling need for expressing mappings between different elements of heterogeneous ontologies. State of the art languages for ontology mapping enable to express semantic relations between homogeneous components of different ontologies; namely, they allow to map concepts into concepts, individuals into individuals, and properties into properties. In many real world cases this is not enough; for example when relations in an ontology correspond to a class in another ontology (i.e. reification of relations). To support this kind of interoperability we need therefore richer mapping languages, offering constructs for the representation of *heterogeneous mappings*. In this paper, we propose an extension of Distributed Description Logics (DDL) with mappings between concepts and relations. We provide a semantics of the proposed extension and sound and complete characterisation of the effects of these mappings in terms of the new ontological knowledge they entail.

1 Introduction

Most of the formalisms for distributed ontology integration based on the p2p architecture provide a language (hereafter called *mapping language*) able to express semantic relations between concepts belonging to different ontologies. These formalisms can express that a concept C in Ontology 1 is equivalent (less general than, more general than) a concept D in Ontology 2 (see [15] for a survey). Few mapping languages allow also to express semantic relations between properties in different ontologies [8, 9, 4], and thus state that a relation R in Ontology 1 is equivalent (less general than, more general than) a relation S in Ontology 2. These type of mappings are able to cope a large, but not the totality of the heterogeneity between ontologies.

Assume, for instance, that a knowledge engineer builds an ontology of family unions containing the binary relations `marriedWith` and `partnerOf` between two persons. Suppose also that a second ontology engineer, asked to design a ontology for the same purpose, declares a concept `Marriage`, whose instances are the actual civil or religious marriages and the concept `civilUnion`, whose instances are all the civil unions. We can easily see that while the first ontology prefers to model unions as relations, the second represents them as concepts. Despite this difference of style in modelling, the concept `Marriage` and the relation `marriedWith` represent the same (or a very

similar) real world aspect, and similarly with `partnerOf` and `civilUnion`. For instance, we can expect that for all married couples in the first ontology, there is a corresponding marriage element in the second ontology, and similarly for the civil unions. To reconcile the semantic difference between the two heterogeneous representations we need a mapping language that allows to map concept of one ontology to relations of another ontology.

Motivated by these observations, Ghidini and Serafini have illustrated in [11] the need of expressive mapping languages that must incorporate not only *homogeneous mappings*, that is mappings between concepts and mappings between relations of different ontologies, but also *heterogeneous mappings*, that is mappings between concepts and relations in the sense illustrated above. They present a preliminary investigation on how to define such expressive mapping language in the framework of Distributed Description Logics (DDL) [14], a refinement of the multi-context logic presented in [7, 8] to the DL-based framework for the formal representation of ontology, but they do not go beyond preliminary statements and definitions, especially in the case of heterogeneous mappings. In [9] the authors take a step forward and present a proposal and an algorithm for the representation and reasoning with *homogeneous mappings*. In this paper we continue this stream of work by addressing the more complex task of representing and reasoning with *heterogeneous mappings* (as well as *homogeneous mappings*) which represent a specific relation between heterogeneous ontologies, namely the correspondence between a concept and a relation. Thus the goals of this paper are: (i) to extend the framework of DDL, introducing mechanisms for the representation of *heterogeneous mappings* between different ontologies, (ii) to define a clear semantics for the proposed mapping language, and (iii) to investigate the logical properties of the proposed mapping language.

2 A rich language for mappings

Description Logic (DL) has been advocated as the suitable formal tool to represent and reason about ontologies. Distributed Description Logic (DDL) [14] is a *natural* generalisation of the DL framework designed to formalise multiple ontologies *pairwise* linked by semantic mappings. In DDL, ontologies correspond to description logic theories (T-boxes), while semantic mappings correspond to collections of *bridge rules* (\mathfrak{B}).

In the following we recall the basic definitions of DDL as defined in [14, 11], and we provide a new semantics for heterogeneous mappings.

2.1 Distributed Description Logics: the syntax

Given a non empty set I of indexes, used to identify ontologies, let $\{\mathcal{DL}_i\}_{i \in I}$ be a collection of description logics¹. For each $i \in I$ let us denote a T-box of \mathcal{DL}_i as \mathcal{T}_i . In this paper, we assume that each \mathcal{DL}_i is description logic weaker or at most equivalent to \mathcal{ALCQI}_b , which corresponds to \mathcal{ALCQI} with role union, conjunction and difference (see [17]). Because of lack of space, we omit the precise description of \mathcal{ALCQI}_b , and we assume that the reader is familiar with the syntax and semantics of DDLs as described in [14].

We call $\mathbf{T} = \{\mathcal{T}_i\}_{i \in I}$ a family of T-Boxes indexed by I . Intuitively, \mathcal{T}_i is the description logic formalization of the i -th ontology. To make every description distinct, we will prefix it with the index of ontology it belongs to. For instance, the concept C that occurs in the i -th ontology is

¹We assume familiarity with Description Logic and related reasoning systems, described in [1].

denoted as $i : C$. Similarly, $i : C \sqsubseteq D$ denotes the fact that the axiom $C \sqsubseteq D$ is being considered in the i -th ontology.

Semantic mappings between different ontologies are expressed via collections of *bridge rules*. In the following we use A, B, C and D as place-holders for concepts and R, S, P and Q as place-holders for roles. We instead use X and Y to denote both concepts and roles.

Definition 1 (Homogeneous Bridge rules). An homogeneous bridge rule from i to j is an expression defined as follows:

$$i : X \xrightarrow{\sqsubseteq} j : Y \quad (\text{into bridge rule}) \quad (1)$$

$$i : X \xrightarrow{\supseteq} j : Y \quad (\text{onto bridge rule}) \quad (2)$$

where X and Y are either concepts of \mathcal{DL}_i and \mathcal{DL}_j respectively, or roles of \mathcal{DL}_i and \mathcal{DL}_j respectively.

Bridge rules do not represent semantic relations stated from an external *objective* point of view. Indeed, there is no such global view in the web. Instead, bridge rules from i to j express relations between i and j viewed from the *subjective* point of view of the j -th ontology.

Bridge rules (1) and (2) with X and Y instantiated as concepts have been studied in [14]. Hereafter we will call them *concept-into-concept* and *concept-onto-concept* bridge rules. The concept-into-concept bridge rule $i : X \xrightarrow{\sqsubseteq} j : Y$ states that, from the j -th point of view the concept X in i is less general than its local concept Y . Similarly, the concept-onto-concept bridge rule $i : X \xrightarrow{\supseteq} j : Y$ expresses the fact that, according to j , X in i is more general than Y in j . Therefore, bridge rules from i to j provide the possibility of translating into j 's ontology (under some approximation) the concepts of a foreign i 's ontology. Note, that since bridge rules reflect a subjective point of view, bridge rules from j to i are not necessarily the inverse of the rules from i to j , and in fact bridge rules from i to j do not force the existence of bridge rules in the opposite direction. Thus, the bridge rule

$$i : \mathbf{Article} \xrightarrow{\supseteq} j : \mathbf{ConferencePaper}$$

expresses the fact that, according to ontology j , the concept **Article** in ontology i is more general than its local concept **ConferencePapers**, while the bridge rules

$$i : \mathbf{Article} \xrightarrow{\sqsubseteq} j : \mathbf{Article} \quad i : \mathbf{Article} \xrightarrow{\supseteq} j : \mathbf{Article}$$

say that, according to ontology j , the concept **Article** in ontology j is equivalent to its local concept **Article**. Bridge rules (1) and (2) instantiated as bridge rules between roles (hereafter *role-into-role* and *role-onto-role* bridge rules) formalize the analogous intuition for roles. For example, the bridge rule:

$$i : \mathbf{marriedInChurchWith} \xrightarrow{\sqsubseteq} j : \mathbf{marriedWith}$$

says that according to ontology j , the relation **marriedInChurchWith** in ontology i is less general than its own relation **marriedWith**.

Definition 2 (Heterogeneous bridge rule). An heterogeneous bridge rule from i to j is an expression defined as follows:

$$i : R \xrightarrow{\sqsubseteq} j : C \quad (\text{role-into-concept bridge rule}) \quad (3)$$

$$i : R \xrightarrow{\supseteq} j : C \quad (\text{role-onto-concept bridge rule}) \quad (4)$$

$$i : C \xrightarrow{\sqsubseteq} j : R \quad (\text{concept-into-role bridge rule}) \quad (5)$$

$$i : C \xrightarrow{\supseteq} j : R \quad (\text{concept-onto-role bridge rule}) \quad (6)$$

where R is a role and C is a concept.

Bridge rules (3) and (4) state that, from the j -th point of view the role R in i is less general, resp. more general, than its local concept C . Similarly, bridge rules (5) and (6) state that, from the j -th point of view the concept C in i is less general, resp. more general, than its local role R . Thus, the bridge rule

$$i : \text{marriedInChurchWith} \xrightarrow{\sqsubseteq} j : \text{Marriage}$$

expresses the fact that, according to ontology j , the relation `marriedInChurchWith` in ontology i is less general than its local concept `Marriage`, while the bridge rules

$$i : \text{civilUnion} \xrightarrow{\sqsubseteq} j : \text{partnerOf}$$

$$i : \text{civilUnion} \xrightarrow{\supseteq} j : \text{partnerOf}$$

say that, according to ontology j , the concept `civilUnion` in ontology j is equivalent to its local relation `partnerOf`.

Definition 3 (Distributed T-box). A distributed T-box (DTB) $\mathfrak{T} = \langle \mathcal{T}_i, \mathfrak{B} \rangle$ consists of a collection \mathcal{T}_i of T-boxes, and a collection $\mathfrak{B} = \{\mathfrak{B}_{ij}\}_{i \neq j \in I}$ of bridge rules between them.

2.2 Distributed Description Logics: the semantics

The semantic of DDL, which is a refinement of Local Models Semantics [7, 8], assigns to each ontology \mathcal{T}_i a *local interpretation domain*. The first component of an interpretation of a DTB is a family of interpretations $\{\mathcal{I}_i\}_{i \in I}$, one for each T-box \mathcal{T}_i . Each \mathcal{I}_i is called a *local interpretation* and consists of a *possibly empty* domain $\Delta^{\mathcal{I}_i}$ and a valuation function $\cdot^{\mathcal{I}_i}$, which maps every concept to a subset of $\Delta^{\mathcal{I}_i}$, and every role to a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$. The interpretation on the empty domain is used to provide a semantics for distributed T-boxes in which some of the local T-boxes are inconsistent. We do not describe this aspect of DDL further. The interested reader can refer to [14].

The second component of the DDL semantics are families of domain relations. Domain relations define how the different T-box interact and are necessary to define the satisfiability of bridge rules.

Definition 4 (Domain relation). A *domain relation* r_{ij} from i to j is a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$. We use $r_{ij}(d)$ to denote $\{d' \in \Delta^{\mathcal{I}_j} \mid \langle d, d' \rangle \in r_{ij}\}$; for any subset D of $\Delta^{\mathcal{I}_i}$, we use $r_{ij}(D)$ to denote $\bigcup_{d \in D} r_{ij}(d)$.

A domain relation r_{ij} represents a possible way of mapping the elements of $\Delta^{\mathcal{I}_i}$ into its domain $\Delta^{\mathcal{I}_j}$, seen from j 's perspective. For instance, if $\Delta^{\mathcal{I}_1}$ and $\Delta^{\mathcal{I}_2}$ are the representation of time as Rationals and as Naturals, r_{ij} could be the round off function, or some other approximation relation. This function has to be conservative w.r.t., the order relations defined on Rationals and Naturals. Domain relation is used to interpret homogeneous bridge rules according with the following definition.

Definition 5 (Satisfiability of homogeneous bridge rules). The domain relation r_{ij} satisfies a homogeneous bridge rule w.r.t., \mathcal{I}_i and \mathcal{I}_j , in symbols $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j \rangle \models br$, according with the following definition:

1. $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j \rangle \models i : X \xrightarrow{\sqsubseteq} j : Y$, if $r_{ij}(X^{\mathcal{I}_i}) \subseteq Y^{\mathcal{I}_j}$
2. $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j \rangle \models i : X \xrightarrow{\supseteq} j : Y$, if $r_{ij}(X^{\mathcal{I}_i}) \supseteq Y^{\mathcal{I}_j}$

where X and Y are either two concepts or two roles.

Domain relations do not provide sufficient information to evaluate the satisfiability of heterogeneous mappings. Intuitively, an heterogeneous bridge rule between a relation R and a concept C connects a pair of objects related by R with an object which is in C . This suggests that, to evaluate heterogeneous bridge rules from roles in i to concepts in j we need a relation that maps triples of the form $\langle object.1, relation_name, object.2 \rangle$ from ontology i into objects of $\Delta^{\mathcal{I}_j}$. As an example we would like to map a triple $\langle \text{John}, \text{marriedWith}, \text{Mary} \rangle$ of elements from the first ontology into the marriage `m123` of the second ontology, with the intuitive meaning that `m123` is the marriage which correspond to the married couple composed of `John` and `Mary`. We first formally introduce the triples $\langle object.1, relation_name, object.2 \rangle$ for a given ontology i .

Definition 6 (Admissible Triples). Let \mathcal{I}_i be a local interpretation $\langle \Delta^{\mathcal{I}_i}, \mathcal{I}_i \rangle$ for \mathcal{DL}_i . Let \mathcal{R} be the set of all atomic relations relations of \mathcal{DL}_i . We indicate with $\Sigma^{\mathcal{I}_i}$ the set of all triples $\langle x_1, X, x_2 \rangle$ such that $x_1, x_2 \in \Delta^{\mathcal{I}_i}$; $X \subseteq \mathcal{R}$; and $(x_1, x_2) \in \bigcap_{R \in X} R^{\mathcal{I}_i}$.

Intuitively, $\langle \text{John}, \{\text{marriedWith}\}, \text{Mary} \rangle$ is an admissible triple in $\Sigma^{\mathcal{I}_i}$ if `John` is married with `Mary`, or more formally if the pair $(\text{John}, \text{Mary})$ belongs to the interpretation of `marriedWith` in \mathcal{I}_i . Similarly, $\langle \text{John}, \{\text{marriedWith}, \text{loves}\}, \text{Mary} \rangle$ is an admissible triple in $\Sigma^{\mathcal{I}_i}$ if `John` also loves `Mary` in \mathcal{I}_i .

Definition 7 (Concept-role and role-concept domain relation). A *concept-role* domain relation cr_{ij} from i to j is a subset of $\Delta^{\mathcal{I}_i} \times \Sigma^{\mathcal{I}_j}$. A *role-concept* domain relation rc_{ij} from i to j is a subset of $\Sigma^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$.

The domain relation rc_{ij} represents a possible way of mapping pairs of $R^{\mathcal{I}_i}$ into elements of $\Delta^{\mathcal{I}_j}$, seen from j 's perspective. For instance,

$$(\langle \text{John}, \{\text{marriedWith}\}, \text{Mary} \rangle, \text{m123}) \in rc_{ij} \quad (7)$$

represents the fact that `m123` is an object in ontology j corresponding to the marriage between `John` and `Mary` in ontology i , while

$$(\langle \text{John}, \{\text{dancePartnerOf}\}, \text{Mary} \rangle, \text{couple124}) \in rc_{ij} \quad (8)$$

represents the fact that `couple124` is an object in ontology j corresponding to the pair of dancers composed of `John` and `Mary` (e.g., used to record results for dance competitions). This example emphasises one of the main characteristics of the concept-role and role-concept domain relations, that is the possibility for the same pair of objects in an ontology to correspond to different elements in another ontology because they belong to different relations. As shown in the example above we want to be able to “reify” the fact that `John` is married with `Mary` in the element `m123`, and the fact that `John` dances with `Mary` in the different object `couple124`.

Given a set of roles X in the language of \mathcal{DL}_i , we use $X^{\mathcal{I}_i}$ to denote the set $\bigcap_{R \in X} R^{\mathcal{I}_i}$.

Definition 8 (Satisfiability of heterogeneous bridge rules). The role-concept domain relation rc_{ij} satisfies a role-(into/onto)-concept bridge rule w.r.t., \mathcal{I}_i and \mathcal{I}_j , in symbols $\langle \mathcal{I}_i, rc_{ij}, \mathcal{I}_j \rangle \models br$, according with the following definition:

1. $(\mathcal{I}_i, rc_{ij}, \mathcal{I}_j) \models i : R \xrightarrow{\sqsubseteq} j : C$ if for all $(x_1, x_2) \in R^{\mathcal{I}_i}$ and for all pairs $((x_1, X, x_2), x) \in rc_{ij}$ with $X^{\mathcal{I}_i} \subseteq R^{\mathcal{I}_i}$, we have that $x \in C^{\mathcal{I}_j}$
2. $(\mathcal{I}_i, rc_{ij}, \mathcal{I}_j) \models i : R \xrightarrow{\sqsupseteq} j : C$ if for all $x \in C^{\mathcal{I}_j}$ there is a pair $((x_1, X, x_2), x) \in rc_{ij}$, such that $X^{\mathcal{I}_i} \subseteq R^{\mathcal{I}_i}$.

The concept-role domain relation cr_{ij} satisfies a concept-(into/onto)-role bridge rule w.r.t., \mathcal{I}_i and \mathcal{I}_j , in symbols $\langle \mathcal{I}_i, cr_{ij}, \mathcal{I}_j \rangle \models br$, according with the following definition:

3. $(\mathcal{I}_i, cr_{ij}, \mathcal{I}_j) \models i : C \xrightarrow{\sqsubseteq} j : R$ if for all $x \in C^{\mathcal{I}_i}$, and for all pairs $(x, \langle x_1, X, x_2 \rangle) \in cr_{ij}$, it is true that $X^{\mathcal{I}_j} \subseteq R^{\mathcal{I}_j}$;
4. $(\mathcal{I}_i, cr_{ij}, \mathcal{I}_j) \models i : C \xrightarrow{\sqsupseteq} j : R$ if for all $(x_1, x_2) \in R^{\mathcal{I}_j}$ there is a pair $(x, \langle x_1, X, x_2 \rangle) \in cr_{ij}$, such that $X^{\mathcal{I}_j} \subseteq R^{\mathcal{I}_j}$ and $x \in C^{\mathcal{I}_i}$.

Satisfiability of a role-into-concept bridge rule forces the role-concept domain relation cr_{ij} to map pair of elements (x_1, x_2) which belong to $R^{\mathcal{I}_i}$ into elements x in $C^{\mathcal{I}_j}$. Note that, from the definition of role-concept domain relation two arbitrary objects y_1 and y_2 could occur in a pair $(\langle y_1, X, y_2 \rangle, y)$ with X different from $\{R\}$ itself but such that $X^{\mathcal{I}_i} \subseteq R^{\mathcal{I}_i}$. Thus also this pair (y_1, y_2) belongs to $R^{\mathcal{I}_i}$ and we have to force also y to be in $C^{\mathcal{I}_j}$. In other words, we can say that satisfiability of a role-into-concept bridge rule forces the role-concept domain relation to map pairs of elements (x_1, x_2) which belong to R , or to any of its subroles X , into elements x in $C^{\mathcal{I}_i}$.

Consider the bridge rule

$$i : \text{marriedWith} \xrightarrow{\sqsubseteq} j : \text{Marriage} \quad (9)$$

Let $(\text{John}, \text{Mary})$ and $(\text{Philip}, \text{Joanna})$ be two married couples such that $\mathcal{I}_i \models \text{marriedWith}(\text{John}, \text{Mary})$ and $\mathcal{I}_i \models \text{marriedInChurchWith}(\text{Philip}, \text{Joanna})$, with $\mathcal{I}_i \models \text{marriedInChurchWith} \sqsubseteq \text{marriedWith}$. Let the concept-role domain relation cr_{ji} contain (only) the two pairs

$$(\langle \text{John}, \{\text{marriedWith}\}, \text{Mary} \rangle, \text{m123}) \quad (10)$$

$$(\langle \text{Philip}, \{\text{marriedInChurchWith}\}, \text{Joanna} \rangle, \text{e345}) \quad (11)$$

Bridge rule (9) is satisfied if both `m123` and `e345` are instances of `Marriage`.

Satisfiability of a role-onto-concept bridge rule forces the role-concept domain relation cr_{ij} to identify a corresponding pre-image (x_1, x_2) in $R^{\mathcal{I}_i}$ (or in any of its sub-roles) for all x in $C^{\mathcal{I}_j}$. Thus the bridge rule

$$i : \text{marriedWith} \xrightarrow{\exists} j : \text{Marriage} \quad (12)$$

is satisfied by the pairs (10) and (10) above, under the assumption that **m123** and **e345** are the only elements in $\text{Marriage}^{\mathcal{I}_j}$. The satisfiability of concept-into/onto-roles bridge rules is analogous.

The effects of all the bridge rules introduced in this Section are studied in detail in Section 3. We only want to emphasise here one of the interesting characteristics of the heterogeneous mappings by means of an example. Assume we have an ontology i containing three relations **marriedWith**, **marriedInChurchWith** and **dancePartnerOf**. Assume also that there is an ontology j containing three concepts **Marriage**, **ReligiousMarriage**, and **DanceCouple**, which intuitively describe the same “real word entities” of the three relations of ontology i . Assume we want to capture this correspondence by means of the following heterogeneous bridge rules:

$$i : \text{marriedWith} \xrightarrow{\equiv} j : \text{Marriage} \quad (13)$$

$$i : \text{marriedInChurchWith} \xrightarrow{\equiv} j : \text{ReligiousMarriage} \quad (14)$$

$$i : \text{dancePartnerOf} \xrightarrow{\equiv} j : \text{DanceCouple} \quad (15)$$

Assume also that $\mathcal{I}_i \models \text{marriedInChurchWith} \sqsubseteq \text{marriedWith}$. Then we would like to propagate the hierarchical relation of subsumption between these two roles into the analogous hierarchical relation between the corresponding concepts, that is $\mathcal{I}_j \models \text{ReligiousMarriage} \sqsubseteq \text{Marriage}$. This fact is guaranteed by applying the rule (19) at page 9. On the contrary, assume that **marriedWith** and **dancePartnerOf** are not related by any subsumption relation (as we do not want to impose that all married couple dance together or that all dancing couples are married to each other) but assume they only have a non-empty intersection. In this case we do not want to propagate this information by inferring that **Marriage** has a non empty intersection with **DanceCouple**, as intuitively an identifier of a **Marriage** is never an identifier of a **DanceCouple**, even if they concern the same pair of persons. The usage of admissible triples in the role-concept domain relation gives us the possibility to obtain this by allowing the same pair of objects in an ontology to correspond to different elements in another ontology because they belong to different roles, as shown in (7) and (8).

Definition 9 (Distributed interpretation). A *distributed interpretation* \mathfrak{J} of a DTB \mathfrak{X} consists of the 4-tuple $\mathfrak{J} = \langle \{\mathcal{I}_i\}_{i \in I}, \{r_{ij}\}_{i \neq j \in I}, \{cr_{ij}\}_{i \neq j \in I}, \{rc_{ij}\}_{i \neq j \in I} \rangle$

Definition 10 (Satisfiability of a Distributed T-box). A distributed interpretation \mathfrak{J} satisfies the elements of a DTB \mathfrak{X} according to the following clauses: for every $i, j \in I$

1. $\mathfrak{J} \models i : A \sqsubseteq B$, if $\mathcal{I}_i \models A \sqsubseteq B$
2. $\mathfrak{J} \models \mathcal{I}_i$, if $\mathfrak{J} \models i : A \sqsubseteq B$ for all $A \sqsubseteq B$ in \mathcal{I}_i
3. $\mathfrak{J} \models \mathfrak{B}_{ij}$, if
 - $\langle \mathcal{I}_i, r_{ij}, \mathcal{I}_j \rangle$ satisfies all the homogeneous bridge rules in \mathfrak{B}_{ij} ,
 - $\langle \mathcal{I}_i, cr_{ij}, \mathcal{I}_j \rangle$ satisfies all the concept-to-role bridge rules in \mathfrak{B}_{ij} ,

- $\langle \mathcal{I}_i, rc_{ij}, \mathcal{I}_j \rangle$ satisfies all the role-to-concept bridge rules in \mathfrak{B}_{ij}

4. $\mathfrak{J} \models \mathfrak{T}$, if for every $i, j \in I$, $\mathfrak{J} \models \mathcal{T}_i$ and $\mathfrak{J} \models \mathfrak{B}_{ij}$

Definition 11 (Distributed Entailment and Satisfiability). $\mathfrak{T} \models i : A \sqsubseteq B$ (read as “ \mathfrak{T} entails $i : A \sqsubseteq B$ ”) if for every \mathfrak{J} , $\mathfrak{J} \models \mathfrak{T}$ implies $\mathfrak{J} \models_d i : A \sqsubseteq B$. \mathfrak{T} is *satisfiable* if there exists a \mathfrak{J} such that $\mathfrak{J} \models \mathfrak{T}$. Concept $i : A$ is *satisfiable* with respect to \mathfrak{T} if there is a \mathfrak{J} such that $\mathfrak{J} \models \mathfrak{T}$ and $A^{\mathcal{I}_i} \neq \emptyset$.

3 The effects of mappings

In the previous section we have defined a declarative language (of mappings) which allows to state set-theoretic relations between the extensions of the concepts and roles in different ontologies (T-boxes). Mappings can be thought of as inter-theory axioms, which constrain the possible models of the theories representing the different ontologies. Thus, a set of mappings between two ontologies allows to combine the knowledge contained in the two ontologies in order to derive new knowledge. In this section we discuss the main effects of mapping in terms of the new ontological knowledge they allow to infer. In particular, we characterise the mappings by means of sound and complete inference rules from i to j as the one illustrated by Equation (16). For the sake of presentation and lack of space we describe (and prove the statements of soundness for) simple versions of the inference rules. The general versions of the rules are given in Figure 1, and the proofs of soundness and completeness for the general version of the rules is given in [10].

3.1 Propagation of the concept hierarchy

The propagation of the concept hierarchy forced by mappings between concepts and is widely described in [14]. The simplest version of this effect is described by the following rule:

$$\frac{\begin{array}{l} i : A \sqsubseteq B \\ i : A \xrightarrow{\exists} j : C \\ i : B \xrightarrow{\sqsubseteq} j : D \end{array}}{j : C \sqsubseteq D} \quad (16)$$

where A, B, C and D are concepts. The general version of this rule is shown in Equation (21) of Figure 1.

Proposition 1 (Concept into/onto concept). Rule (16) is sound.

Proof. Let $y \in C^{\mathcal{I}_j}$. From the satisfiability of $i : A \xrightarrow{\exists} j : C$ there is an object $x \in A^{\mathcal{I}_i}$ such that $(x, y) \in r_{ij}$. From the hypothesis we know that $\mathcal{T}_i \models A \sqsubseteq B$, and thus $x \in B^{\mathcal{I}_i}$, and from the satisfiability of $i : B \xrightarrow{\sqsubseteq} j : D$ we have that $y \in D^{\mathcal{I}_j}$. Thus $\mathcal{T}_j \models C \sqsubseteq D$. \square

3.2 Propagation of the role hierarchy

The first effect of mappings between roles concern the propagation of the role hierarchy across ontologies. If $P \sqsubseteq Q$ is a fact of the T-box \mathcal{T}_i , then the effect of the bridge rules $i : P \xrightarrow{\exists} j : R$ and $i : Q \xrightarrow{\sqsubseteq} j : S$ is that $R \sqsubseteq S$ is a fact in \mathcal{T}_j .

Formally, we describe this effect by means of the following rule:

$$\begin{array}{c}
i : P \sqsubseteq Q, \\
i : P \xrightarrow{\exists} j : R \\
i : Q \xrightarrow{\sqsubseteq} j : S \\
\hline
j : R \sqsubseteq S
\end{array} \tag{17}$$

where each P, Q, R , and S is either a role or an inverse role. This rule can be obtained from Equation (22) in Figure 1 by setting $l = 1, p = 0, m = 0$.

Proposition 2 (Role into/onto role). Rule (17) is sound.

Proof. Let $(y_1, y_2) \in R^{\mathcal{I}_j}$. From the satisfiability of $i : P \xrightarrow{\exists} j : R$ there is a pair $(x_1, x_2) \in P^{\mathcal{I}_i}$ such that $(x_1, y_1) \in r_{ij}$ and $(x_2, y_2) \in r_{ij}$. From the hypothesis we know that $\mathcal{T}_i \models P \sqsubseteq Q$, and thus $(x_1, x_2) \in Q^{\mathcal{I}_i}$, and from the satisfiability of $i : Q \xrightarrow{\sqsubseteq} j : S$ we have that $(y_1, y_2) \in S^{\mathcal{I}_j}$. Thus $\mathcal{T}_j \models R \sqsubseteq S$. \square

Remark 1. An open point concerns the extension of our framework in order to account for transitive roles. It is well known that the unrestricted interaction between number restriction and transitivity is a source of undecidability; moreover, the bridge rules as the one above may infer additional subsumption relations among the roles. Therefore, guaranteeing appropriate restrictions to ensure decidability is no longer a matter of analysing the “static” role hierarchy (e.g., a in the case of \mathcal{SHIQ}).

3.3 Propagation of the role domain and of the range restriction

The effect of the combination of mappings between roles and mappings between concepts is the propagation of domain and range among relations linked by role-onto-role mappings. The simplest version of this rule is the following:

$$\begin{array}{c}
i : \exists P.\top \sqsubseteq B \\
i : P \xrightarrow{\exists} j : R \\
i : B \xrightarrow{\sqsubseteq} j : D \\
\hline
j : \exists R.\top \sqsubseteq D
\end{array} \tag{18}$$

where P, R are roles and B, D are concepts.

The rule above says that if the domain of P is contained in B and the appropriate bridge rules hold, then we can infer that the domain of R is contained in D . A similar rule allows to obtain $j : \exists R^-\top \sqsubseteq D$ from $i : \exists P^-\top \sqsubseteq B$ with the same bridge rules, thus expressing the propagation of the range restriction. Rule (18) can be obtained from Equation (22) in Figure 1 by setting $l = 0, p = 0, m = 1$. Analogously the rule for range restriction can be obtained by setting $l = 0, p = 1, m = 0$.

Proposition 3 (Role domain and range restriction). Rule (18) is sound.

Proof. Let $y_1 \in \exists R.\top^{\mathcal{I}_j}$. Thus there is an object $y_2 \in \Delta^{\mathcal{I}_j}$ such that $(y_1, y_2) \in R^{\mathcal{I}_j}$. From the satisfiability of $i : P \xrightarrow{\exists} j : R$ there is a $(x_1, x_2) \in P^{\mathcal{I}_i}$ such that $(x_1, y_1) \in r_{ij}$ and $(x_2, y_2) \in r_{ij}$. From the hypothesis we know that $\mathcal{T}_i \models \exists P.\top \sqsubseteq B$, and thus $x_1 \in B^{\mathcal{I}_i}$, and from the satisfiability of $i : B \xrightarrow{\sqsubseteq} j : D$ we have that $y_1 \in D^{\mathcal{I}_j}$. Thus $\mathcal{T}_j \models \exists R.\top \sqsubseteq D$. Similarly for the range restriction. \square

3.4 Propagation of role hierarchy into concept hierarchy

The first effect of the heterogeneous bridge rules mapping roles into/onto corresponding concepts is the propagation of the subsumption relations between these role into subsumption relations between the corresponding concepts. The simplest form of this rule is:

$$\begin{array}{l}
 i : P \sqsubseteq Q \\
 i : P \xrightarrow{\exists} j : C \\
 i : Q \xrightarrow{\sqsubseteq} j : D \\
 \hline
 j : C \sqsubseteq D
 \end{array} \tag{19}$$

The general version of this rule is presented in Equation (25).

Proposition 4 (Role hierarchy into concept hierarchy). Rule (19) is sound.

Proof. Let $x \in C^{\mathcal{I}_j}$. From the satisfiability of $i : P \xrightarrow{\exists} j : C$, there is a triple (x_1, X, x_2) in $\Sigma_{\mathcal{I}_i}$ such that $(x, \langle x_1, X, x_2 \rangle) \in rc_{ij}$ with $X^{\mathcal{I}_i} \sqsubseteq P^{\mathcal{I}_i}$. $(x_1, x_2) \in X^{\mathcal{I}_i}$ from the definition of admissible triple. Since $\mathcal{I}_i \models P \sqsubseteq Q$, we have that $(x_1, x_2) \in Q^{\mathcal{I}_i}$. From the satisfiability of the into-bridge rule $i : Q \xrightarrow{\sqsubseteq} j : D$, we can conclude that $x \in D^{\mathcal{I}_j}$. \square

3.5 Propagation of concept hierarchy into role hierarchy

An effect analogous to the one above is the propagation of the concept hierarchy into the role hierarchy. The simplest form of this rule is:

$$\begin{array}{l}
 i : A \sqsubseteq B \\
 i : A \xrightarrow{\exists} j : R \\
 i : B \xrightarrow{\sqsubseteq} j : S \\
 \hline
 j : R \sqsubseteq S
 \end{array} \tag{20}$$

The general version of this rule is presented in Equations (25) and (24).

Proposition 5 (Concept hierarchy into role hierarchy). Rule (20) is sound.

Proof. Let $(x_1, x_2) \in R^{\mathcal{I}_j}$. From the satisfiability of the onto-bridge rule $i : A \xrightarrow{\exists} j : R$ there must be a triple $\langle x_1, X, x_2 \rangle$ of $\Sigma^{\mathcal{I}_j}$ and an $x \in A^{\mathcal{I}_i}$, such that $X^{\mathcal{I}_j} \sqsubseteq R^{\mathcal{I}_j}$, and $(x, \langle x_1, X, x_2 \rangle) \in cr_{ij}$. The fact $\mathcal{I}_i \models A \sqsubseteq B$ implies that $x \in B^{\mathcal{I}_i}$ and, from the satisfiability of the into-bridge rule $i : B \xrightarrow{\sqsubseteq} j : S$, we can conclude that $X^{\mathcal{I}_j} \sqsubseteq S^{\mathcal{I}_j}$, and therefore that $(x_1, x_2) \in S^{\mathcal{I}_j}$. \square

Remark 2. As for the propagation of the role hierarchy due to role into/onto bridge rules, we need to be careful with the modification of the role hierarchy due to bridge rules even in this case (see Remark 1). In addition, the expression $\bigsqcup_{k=1}^n S_k$ with $n = 0$ represents the empty role R_{\perp} . The introduction of the empty role can be easily obtained with the axiom $\top \sqsubseteq \forall R_{\perp} \perp$.

$$\begin{array}{l}
i : A \sqsubseteq \bigsqcup_{k=1}^n B_k \\
i : A \xrightarrow{\exists} j : C \\
i : B_k \xrightarrow{\sqsubseteq} j : D_k, \text{ for } 1 \leq k \leq n \\
\hline
j : C \sqsubseteq \bigsqcup_{k=1}^n D_k
\end{array} \tag{21}$$

$$\begin{array}{l}
i : \exists (P \sqcap \neg(\bigsqcup_{h=1}^l Q_h)) \cdot (\neg \bigsqcup_{h=1}^p A_h) \sqsubseteq (\bigsqcup_{h=1}^m B_h) \\
i : P \xrightarrow{\exists} j : R \\
i : Q_h \xrightarrow{\sqsubseteq} j : S_h, \text{ for } 1 \leq h \leq l \\
i : A_h \xrightarrow{\sqsubseteq} j : C_h, \text{ for } 1 \leq h \leq p \\
i : B_h \xrightarrow{\sqsubseteq} j : D_h, \text{ for } 1 \leq h \leq m \\
\hline
j : \exists (R \sqcap \neg(\bigsqcup_{h=1}^l S_h)) \cdot (\neg \bigsqcup_{h=1}^p C_h) \sqsubseteq (\bigsqcup_{k=1}^m D_k)
\end{array} \tag{22}$$

$$\begin{array}{l}
i : P \sqsubseteq Q \\
i : P \xrightarrow{\exists} j : C \\
i : Q \xrightarrow{\sqsubseteq} j : D \\
\hline
j : C \sqsubseteq D
\end{array} \tag{23}$$

$$\begin{array}{l}
i : P \sqsubseteq \perp_R \\
i : P \xrightarrow{\exists} j : C \\
\hline
j : C \sqsubseteq \perp
\end{array} \tag{24}$$

$$\begin{array}{l}
i : A \sqsubseteq \bigsqcup_{k=1}^n B_k \\
i : A \xrightarrow{\exists} j : R \\
i : B_k \xrightarrow{\sqsubseteq} j : S_k \text{ for } 1 \leq k \leq n \\
\hline
j : R \sqsubseteq \bigsqcup_{k=1}^n S_k
\end{array} \tag{25}$$

Figure 1: Sound and complete set of inference rules

4 Soundness and Completeness

An operator \mathfrak{B}_{ij} can be defined on top of the general inference rules shown in Figure 1, similarly to what happens in [14]. Given a DTB $\mathfrak{T} = \langle \mathcal{T}_i, \mathcal{T}_j, \mathfrak{B}_{ij} \rangle$, we can define $\mathfrak{B}_{ij}(\mathcal{T}_i)$ as an operator which enriches the T-box \mathcal{T}_j with all the conclusions of the rules in Figure 1 provided that \mathcal{T}_i satisfies the premises (in i) and that the bridge rules in the premise of the rule are among the bridge rules of \mathfrak{T} .

Roughly speaking, given a set of bridge rules \mathfrak{B}_{ij} from \mathcal{DL}_i to \mathcal{DL}_j , the operator $\mathfrak{B}_{ij}(\cdot)$ takes as input a T-box in \mathcal{DL}_i and produces a T-box in \mathcal{DL}_j , in accordance with the inference rules.

Theorem 1 (Soundness and Completeness). *Let $\mathfrak{T}_{12} = \langle \mathcal{T}_1, \mathcal{T}_2, \mathfrak{B}_{12} \rangle$ be a distributed T-box. Then:*

$$\mathfrak{T}_{12} \models 2 : X \sqsubseteq Y \iff \mathcal{T}_2 \cup \mathfrak{B}_{12}(\mathcal{T}_1) \models X \sqsubseteq Y \quad (26)$$

4.1 Soundness

As far as the soundness, it is enough to prove that if $\mathfrak{T} \models \mathfrak{T} = \langle \mathcal{T}_i, \mathcal{T}_j, \mathfrak{B}_{ij} \rangle$, then $\mathcal{T}_j \models \mathfrak{B}_{ij}(\mathcal{T}_i)$. $\mathfrak{B}_{ij}(\mathcal{T}_i)$ contains four types of formulae, obtained with the application of the rules in 1. We analyse these four rules one by one:

- Soundness of rule (21) is shown in [14].
- Let us consider rule (22). Suppose that $j : \exists(R \sqcap \neg(\bigsqcup_{h=1}^l S_h)) \cdot (\neg \bigsqcup_{h=1}^p C_h) \sqsubseteq (\bigsqcup_{k=1}^m D_k) \in \mathfrak{B}_{ij}(\mathcal{T}_i)$. Then the mappings $i : P \xrightarrow{\exists} j : R, i : Q_h \xrightarrow{\exists} j : S_h$, for $1 \leq h \leq l, i : A_h \xrightarrow{\exists} j : C_h$, for $1 \leq h \leq p$, and $i : B_h \xrightarrow{\exists} j : D_h$, for $1 \leq h \leq m$ belong to three bridge rules of \mathfrak{T} . Let x_j be an object in $\exists(R \sqcap \neg(\bigsqcup_{h=1}^l S_h)) \cdot (\neg \bigsqcup_{h=1}^p C_h)^{\mathcal{T}_j}$. Then there is a y_j such that the pair $\langle x, y \rangle \in R \sqcap \neg(\bigsqcup_{h=1}^l S_h)^{\mathcal{T}_j}$ and $y_j \in \neg(\bigsqcup_{h=1}^p C_h)^{\mathcal{T}_j}$. From the satisfiability of the role-onto-role bridge rule $i : P \xrightarrow{\exists} j : R$, there is a $\langle x_i, y_i \rangle$ such that $\langle x_i, y_i \rangle \in P^{\mathcal{T}_i}$. From the satisfiability of all the role-into-role bridge rules $i : Q_h \xrightarrow{\exists} j : S_h$, for $1 \leq h \leq l$ we also have that $\langle x_i, y_i \rangle \in \neg Q_h^{\mathcal{T}_i}$, for all h and from the satisfiability of all the concept-into-concept bridge rules $i : A_h \xrightarrow{\exists} j : C_h$, for $1 \leq h \leq p$ we have that $y_i \in \neg A_h^{\mathcal{T}_i}$ for all h . Thus we can conclude that x_i belongs to $\exists(P \sqcap \neg(\bigsqcup_{h=1}^l Q_h)) \cdot (\neg \bigsqcup_{h=1}^p A_h)^{\mathcal{T}_i}$, and since \mathcal{T}_i satisfies all the premises of the rule, then $x_i \in (\bigsqcup_{h=1}^m B_h)^{\mathcal{T}_i}$. We can now use the concept-into-concept bridge rules $i : B_h \xrightarrow{\exists} j : D_h$, for $1 \leq h \leq m$ to conclude that x_j belongs to $\bigsqcup_{k=1}^m D_k^{\mathcal{T}_j}$, and the proof is done.
- Let us consider rule (23). Suppose that $j : C \sqsubseteq D \in \mathfrak{B}_{ij}(\mathcal{T}_i)$ and that $x \in C^{\mathcal{T}_j}$. From the satisfiability of $i : P \xrightarrow{\exists} j : C$, there is a triple (x_1, X, x_2) in $\Sigma_{\mathcal{T}_i}$ such that $(x, \langle x_1, X, x_2 \rangle) \in rc_{ij}$ with $X^{\mathcal{T}_i} \subseteq P^{\mathcal{T}_i}$. $(x_1, x_2) \in X^{\mathcal{T}_i}$ from the definition of admissible triple. Since $\mathcal{T}_i \models P \sqsubseteq Q$, we have that $(x_1, x_2) \in Q^{\mathcal{T}_i}$. From the satisfiability of the into-bridge rule $i : Q \xrightarrow{\exists} j : D$, we can conclude that $x \in D^{\mathcal{T}_j}$.²
- Let us now consider rule (24). Suppose that $j : C \sqsubseteq \perp \in \mathfrak{B}_{ij}(\mathcal{T}_i)$ and that $x \in C^{\mathcal{T}_j}$. From the satisfiability of $i : P \xrightarrow{\exists} j : C$, there is a triple (x_1, X, x_2) in $\Sigma_{\mathcal{T}_i}$ such that

²This proof is the same proof of Proposition 4.

$(x, \langle x_1, X, x_2 \rangle) \in rc_{ij}$ with $X^{\mathcal{I}_i} \subseteq P^{\mathcal{I}_i}$. Thus, $(x_1, x_2) \in P^{\mathcal{I}_i}$, but since $\mathcal{I}_i \models P \sqsubseteq \perp_R$ this is impossible. Thus there is no $x \in C^{\mathcal{I}_j}$ and $\mathcal{I}_j \models C \sqsubseteq \perp$.

- Let us consider rule (25). Suppose that $j : R \sqsubseteq \bigsqcup_{k=1}^n S_k \in \mathfrak{B}_{ij}(\mathcal{T}_i)$. Suppose also that $(x_1, x_2) \in R^{\mathcal{I}_j}$. Then from the satisfiability of concept-onto-role $i : A \xrightarrow{\exists} j : R$ we know that there is a pair $(x, \langle x_1, X, x_2 \rangle)$ with $X^{\mathcal{I}_j} \subseteq R^{\mathcal{I}_j}$ and $x \in A^{\mathcal{I}_i}$. Thus $x \in \bigsqcup_{k=1}^n B_k$ from the fact that \mathcal{T}_i satisfies $A \sqsubseteq \bigsqcup_{k=1}^n B_k$. Let $x \in B_k$. From the satisfiability of the concept-into-role bridge rule $i : B_k \xrightarrow{\exists} j : S_k$ we have that $Xj \subseteq S_k^{\mathcal{I}_j}$. Thus, the pair $(x_1, x_2) \in S_k^{\mathcal{I}_j}$ from the definition of admissible triple. Therefore $(x_1, x_2) \in \bigsqcup_{k=1}^n S_k^{\mathcal{I}_j}$ and the proof is done.

4.2 Completeness

Lemma 1. If \mathcal{I}_j is an interpretation of $\mathcal{T}_j \cup \mathfrak{B}_{ij}(\mathcal{T}_i)$, then there is a model $\langle \mathcal{I}_i, \mathcal{I}_j, r_{ij}, cr_{ij}, rc_{ij} \rangle$ of $\langle \mathcal{T}_i, \mathcal{T}_j, \mathfrak{B}_{ij} \rangle$.

Proof. We organize the proof in five steps.

1. In the first step we build an interpretation \mathcal{I}_i^{cc} for \mathcal{T}_i and a domain relation r_{ij}^C , and show that $\langle \mathcal{I}_i^{cc}, r_{ij}^C, \mathcal{I}_j \rangle$ satisfies all the homogeneous bridge rules between concepts.
2. In the second step we build an interpretation \mathcal{I}_i^{rr} for \mathcal{T}_i and a domain relation r_{ij}^{rr} and show that $\langle \mathcal{I}_i^{rr}, r_{ij}^{rr}, \mathcal{I}_j \rangle$ satisfies all the homogeneous bridge rules between roles.
3. In the third step we build the interpretation \mathcal{I}_i^{RC} and the role-to-concept relation rc_{ij} , and show that $\langle \mathcal{I}_i^{RC}, rc_{ij}, \mathcal{I}_j \rangle$ satisfies all the role-to-concepts bridge rules.
4. In the fourth step we build the interpretation \mathcal{I}_i^{CR} and the concept-to-role relation rc_{ij} , and show that $\langle \mathcal{I}_i^{CR}, rc_{ij}, \mathcal{I}_j \rangle$ satisfies all the concept-to-role bridge rules.
5. In the last step, we combine \mathcal{I}_i^C , \mathcal{I}_i^R , \mathcal{I}_i^{CR} , and \mathcal{I}_i^{RC} into an interpretation \mathcal{I}_i of \mathcal{T}_i , the relations r_{ij}^C and r_{ij}^R into a domain relation r_{ij} , showing that $\langle \mathcal{I}_i, \mathcal{I}_j, r_{ij}, cr_{ij}, rc_{ij} \rangle$ satisfies all the bridge rules in \mathfrak{B}_{ij} .

Building \mathcal{I}_i^C and r_{ij}^C To build \mathcal{I}_i^{cc} and r_{ij}^C , we report the proof described in [14].

For every onto-bridge rule between concepts $i : A \xrightarrow{\exists} j : C$ in \mathfrak{B}_{ij} , and for every $x \in C^{\mathcal{I}_j}$ let D_1, \dots, D_n be the consequences of all the into-bridge rules between concepts, such that $x \notin D_k$ for $1 \leq k \leq n$. Let $\mathcal{I}_i^{C(x)}$ be an interpretation of \mathcal{T}_i , and v be an elements of its domain, called the *preimages of x* in $\mathcal{I}_i^{C(x)}$, such that the following conditions hold:

1. $v \in A^{\mathcal{I}_i^{C(x)}}$
2. $v \notin B_1^{\mathcal{I}_i^{C(x)}} \cup \dots \cup B_m^{\mathcal{I}_i^{C(x)}}$;

where each B_k is the left hand side of the into bridge rule with consequence D_k . Let us prove the existence of $\mathcal{I}_i^{C(x)}$ and of the preimage of x in $\mathcal{I}_i^{C(x)}$. Assume by contradiction that there is no such

an interpretation, i.e., that for all interpretation \mathcal{I}_i of \mathcal{T}_i one of the above 2 conditions is false. This implies that the following condition is true.

$$\text{For all interpretations } \mathcal{I}_i \text{ of } \mathcal{T}_i \text{ and for all } v \in \Delta^{\mathcal{I}_i}, \text{ if } v \in A^{\mathcal{I}_i} \text{ then } v \in B_k^{\mathcal{I}_i} \text{ for some } 1 \leq k \leq m \quad (27)$$

Condition (27) can be expressed with the \mathcal{ALCQI}_b -axiom

$$A \sqsubseteq B_1 \sqcup \dots \sqcup B_n \quad (28)$$

This implies that if $\mathcal{I}_i^{C(x)}$ does not exist, then $\mathcal{T}_i \models (28)$. By the generalization of rule (16), we have that

$$C \sqsubseteq D_1 \sqcup \dots \sqcup D_n \quad (29)$$

is contained in $\mathfrak{B}_{ij}(\mathcal{T}_i)$. But this contradicts the initial hypothesis that $x \in C^{\mathcal{I}_j}$, and $x \notin D_k^{\mathcal{I}_j}$, for all $1 \leq k \leq n$. This implies the existence of $\mathcal{I}_i^{C(x)}$ and of the preimage of x in $\mathcal{I}_i^{C(x)}$.

We repeat this construction for every x in C , and for all onto-bridge rules between concepts. With no loss of generality, we can assume that the domains of the $\mathcal{I}_i^{C(x)}$'s is disjoint from all the others. We define \mathcal{I}_i^C as the union of all of them. In symbols:

$$\mathcal{I}_i^{cc} = \bigcup_{\substack{i : A \xrightarrow{\exists} j : C \in \mathfrak{B}_{ij} \\ x \in C^{\mathcal{I}_j}}} \mathcal{I}_i^{C(x)}$$

The domain relation r_{ij}^C is defined as the set of pairs (v, x) where v is the selected elements of each $\mathcal{I}_i^{C(x)}$.

Building \mathcal{I}_i^R and r_{ij}^R As far as the homogeneous bridge rule between roles, we proceed in an analogous way. For every onto-bridge rule $i : P \xrightarrow{\exists} j : R$ in \mathfrak{B}_{ij} , and for every $(x, y) \in R^{\mathcal{I}_j}$ let S_1, \dots, S_n be the consequences of all the into-bridge rules between roles, such that $(x, y) \notin S_k$ for $1 \leq k \leq n$. Let C_1, \dots, C_m and D_1, \dots, D_l be the consequences of the into-bridge rules between concepts, such that $x \notin C_h$, for $1 \leq h \leq m$, and $y \notin D_k$ for $1 \leq k \leq l$. Let $\mathcal{I}_i^{R(x,y)}$ be an interpretation of \mathcal{T}_i , and v, w be two elements of its domain relation, called the *preimages of x and y* in $\mathcal{I}_i^{R(x,y)}$, such that the following conditions holds:

1. $(v, w) \in P^{\mathcal{I}_i^{R(x,y)}}$
2. $(v, w) \notin Q_1^{\mathcal{I}_i^{R(x,y)}} \cup \dots \cup Q_n^{\mathcal{I}_i^{R(x,y)}}$;
3. $v \notin A_1^{\mathcal{I}_i^{R(x,y)}} \cup \dots \cup A_l^{\mathcal{I}_i^{R(x,y)}}$;
4. $w \notin B_1^{\mathcal{I}_i^{R(x,y)}} \cup \dots \cup B_m^{\mathcal{I}_i^{R(x,y)}}$;

where Q_k is the left hand side of the into bridge rule between roles with consequence S_k ; A_k and B_k are the left hand side of the bridge rule between concepts with consequence C_k and D_k respectively.

Let us prove the existence of $\mathcal{I}_i^{R(x,y)}$ and v, w . Assume by contradiction that there is no such an interpretation, i.e., that for all interpretation \mathcal{I}_i that satisfies \mathcal{T}_i one of the above 4 conditions are false. This implies that the following condition is true.

$$\text{For all interpretations } \mathcal{I}_i \text{ of } \mathcal{T}_i \text{ and for all } v, w \in \Delta^{\mathcal{I}_i}, \text{ if (i) } (v, w) \in P^{\mathcal{I}_i}, \text{ (ii) } (v, w) \notin Q_k \quad (30) \\ \text{for } 1 \leq k \leq n \text{ and (iii) } w \notin A_k^{\mathcal{I}_i} \text{ for } 1 \leq k \leq l, \text{ then } v \in B_k^{\mathcal{I}_i} \text{ for some } 1 \leq k \leq m$$

Condition (30) can be expressed with the \mathcal{ALCCQI}_b -axiom

$$\exists [P \setminus (Q_1 \sqcup \dots \sqcup Q_n)]. \neg (A_1 \sqcup \dots \sqcup A_m) \sqsubseteq B_1 \sqcup \dots \sqcup B_n \quad (31)$$

This implies that if $\mathcal{I}_i^{R(x,y)}$ does not exist $\mathcal{T}_i \models (31)$. By the generalization of rules (17) and (18), we can infer that

$$\exists [R \setminus (S_1 \sqcup \dots \sqcup S_n)]. \neg (C_1 \sqcup \dots \sqcup C_m) \sqsubseteq D_1 \sqcup \dots \sqcup D_l \quad (32)$$

is contained in $\mathfrak{B}_{ij}(\mathcal{I}_i)$. But this contradicts the initial hypothesis that $(x, y) \in R^{\mathcal{I}_j}$, $(x, y) \notin S_k^{\mathcal{I}_j}$, $x \notin C_{j_k}$ and $y \notin D_k^{\mathcal{I}_j}$. This implies that there must be an interpretation $\mathcal{I}_i^{R(x,y)}$ and a pair (v, w) that satisfies conditions 1–4.

We repeat this construction for every (x, y) that belongs to the interpretation of any R which is the consequence of an onto bridge rule. Again with no loss of generality we can assume that the domain $\mathcal{I}^{R(x,y)}$ is defined as:

$$\mathcal{I}_i^R = \bigcup_{\substack{i : P \xrightarrow{\exists} j : R \in \mathfrak{B}_{ij} \\ (x, y) \in R^{\mathcal{I}_j}}} \mathcal{I}_i^{R(x,y)}$$

The domain relation r_{ij}^R contains all the pairs (v, x) and (w, y) where v and w are the preimages of x and y w.r.t. $\mathcal{I}^{R(x,y)}$.

Building \mathcal{I}_i^{CR} and cr_{ij} Now we apply the very same method to build the role to concept relation and the concept to role relations. For every concept-to-role onto bridge rule of the form $i : A \xrightarrow{\exists} j : R$, and for every $(x, y) \in R^{\mathcal{I}_j}$ let S_1, \dots, S_n be the consequence of the concept-to-role into bridge rules, such that $(x, y) \notin S_k^{\mathcal{I}_j}$ for $1 \leq k \leq n$. Let $\mathcal{I}_i^{R(x,y)}$ be an interpretation of \mathcal{T}_i and v an element of the domain of $\mathcal{I}_i^{R(x,y)}$, called the *preimage* of $(x, \{R\}, y)$ in $\mathcal{I}_i^{R(x,y)}$. such that the following conditions hold:

1. $v \in A^{\mathcal{I}_i^{R(x,y)}}$
2. $v \notin B_k^{\mathcal{I}_i^{R(x,y)}}$.

where B_k is the premise of the concept-to-role into bridge rule with consequence S_k . Let us prove that $\mathcal{I}_i^{R(x,y)}$ exists. Assume by contradiction that there is no such an interpretation, i.e., that for all interpretation \mathcal{I}_i of \mathcal{T}_i one of the above 2 conditions is false. This implies that the following condition is true.

$$\text{For all interpretations } \mathcal{I}_i \text{ of } \mathcal{T}_i \text{ and for all } v \in \Delta^{\mathcal{I}_i}, \text{ if } v \in A^{\mathcal{I}_i} \text{ then } v \in B_k^{\mathcal{I}_i} \text{ for some } \quad (33) \\ 1 \leq k \leq n$$

Condition (33) can be expressed with the \mathcal{ALCQI}_b -axiom

$$A \sqsubseteq B_1 \sqcup \dots \sqcup B_n \quad (34)$$

This implies that, if $\mathcal{I}_i^{R(x,y)}$ does not exist, then $\mathcal{T}_i \models (34)$. By the generalization of rule (20) we can infer that

$$R \sqsubseteq S_1 \sqcup \dots \sqcup S_n \quad (35)$$

is in $\mathfrak{B}_{ij}(\mathcal{T}_i)$. But this contradicts the initial hypothesis that $(x, y) \in R^{\mathcal{I}_j}$ and $(x, y) \notin S_k^{\mathcal{I}_j}$ for $1 \leq k \leq n$. We repeat this construction for every (x, y) that belongs to $R^{\mathcal{I}_j}$ and for any concept-to-role onto bridge rule. Again, with no loss of generality, we can assume that the domain of each $\mathcal{I}^{R(x,y)}$ is disjoint from the domains of the others. We define the interpretation \mathcal{I}_i^{CR} as follows:

$$\mathcal{I}_i^{CR} = \bigcup_{\substack{i : A \xrightarrow{\exists} j : R \in \mathfrak{B}_{ij} \\ (x, y) \in R^{\mathcal{I}_j}}} \mathcal{I}_i^{R(x,y)}$$

and cr_{ij} contains all the pairs $(v, (x, \{R\}, y))$ where v is the preimage of $(x, \{R\}, y)$ in $\mathcal{I}_i^{R(x,y)}$.

Building \mathcal{I}_i^{RC} and rc_{ij} Consider a role-to-concept onto bridge rule $i : P \xrightarrow{\exists} j : C \in \mathfrak{B}_{ij}$. For every $x \in C^{\mathcal{I}_j}$ let's consider two cases. (i) there is at least a role-to-concept into bridge rule $i : Q \xrightarrow{\exists} j : D$ with $x \notin D^{\mathcal{I}_j}$ (ii) there is no such a into bridge rule. In case (i) for all role-to-concept into bridge rule $i : Q \xrightarrow{\exists} j : D$, let $\mathcal{I}_i^{C(x) \wedge \neg D(x)}$ be an interpretation of \mathcal{T}_i and $(v, \{P\}, w)$ be an admissible triple of $\mathcal{I}_i^{C(x) \wedge \neg D(x)}$, called the *preimage of x in $\mathcal{I}_i^{C(x) \wedge \neg D(x)}$* such that the following conditions hold:

1. $(v, w) \in P^{\mathcal{I}_i^{C(x) \wedge \neg D(x)}}$
2. $(v, w) \notin Q^{\mathcal{I}_i^{C(x) \wedge \neg D(x)}}$

Let us prove that $\mathcal{I}_i^{C(x) \wedge \neg D(x)}$ exists. Assume by contradiction that there is no such an interpretation, i.e., that for all interpretation \mathcal{I}_i of \mathcal{T}_i one of the above 2 conditions is false. This implies that the following condition is true.

$$\text{For all interpretations } \mathcal{I}_i \text{ of } \mathcal{T}_i \text{ and for all } v, w \in \Delta^{\mathcal{I}_i}, \text{ if } (v, w) \in P^{\mathcal{I}_i} \text{ then } (v, w) \in Q^{\mathcal{I}_i} \quad (36)$$

Condition (36) can be expressed with the \mathcal{ALCQI}_b -axiom

$$P \sqsubseteq Q \quad (37)$$

This implies that if $\mathcal{I}_i^{C(x) \wedge \neg D(x)}$ does not exist, then $\mathcal{T}_i \models (37)$. By the rule (19) we can infer that

$$C \sqsubseteq D \quad (38)$$

is in $\mathfrak{B}_{ij}(\mathcal{T}_i)$. But this contradicts the initial hypothesis that $x \in C^{\mathcal{I}_j}$ and $x \notin D^{\mathcal{I}_j}$. We repeat this process for all the into bridge rules $i : Q \xrightarrow{\exists} j : D$ with $x \notin D^{\mathcal{I}_j}$, and define

$$\mathcal{I}_i^{C(x)} = \bigcup_{\substack{i : Q \xrightarrow{\exists} j : D \in \mathfrak{B}_{ij} \\ x \notin D^{\mathcal{I}_j}}} \mathcal{I}_i^{C(x) \wedge \neg D(x)}$$

Notice that, since we are in the case in which there is at least a bridge rule $i : Q \xrightarrow{\sqsubseteq} j : D$ with $x \notin D^{\mathcal{I}_j}$ we have that $\mathcal{I}_i^{C(x)}$ is not the empty model.

Consider now the case (ii) in which x belongs to all the consequences of the role-to-concept into bridge rules. In this case let $\mathcal{I}_i^{C(x)}$ be any model of \mathcal{T}_i and $(v, \{P\}, w)$ be an admissible triple of $\mathcal{I}_i^{C(x)}$, called the preimage of x in $\mathcal{I}_i^{C(x)}$. $\mathcal{I}_i^{C(x)}$ must exist otherwise $\mathcal{T}_i \models P \sqsubseteq \perp_R$ and by bridge rule (24), $C \sqsubseteq \perp \in \mathfrak{B}_{ij}(\mathcal{T}_i)$ contradicting the fact that $x \in C^{\mathcal{I}_j}$, i.e., that $C^{\mathcal{I}_j}$ is not empty.

We repeat the process for every $x \in C^{\mathcal{I}_j}$ and role-to-concept onto bridge rule. With no loss of generality, we can assume that the domains the interpretations $\mathcal{I}^{C(x)}$'s are disjoint. So we define \mathcal{I}_i^{RC} as follows:

$$\mathcal{I}_i^{RC} = \bigcup_{\substack{i : P \xrightarrow{\sqsupseteq} j : C \in \mathfrak{B}_{ij} \\ x \in C^{\mathcal{I}_j}}} \mathcal{I}_i^{C(x)}$$

and cr_{ij} contains all the pairs $((v, \{P\}, w), x)$ where $(v, \{P\}, w)$ is the preimage of x in $\mathcal{I}_i^{C(x)}$.

With no loss of generality we can again assume that the domain of interpretation of \mathcal{I}_i^C , \mathcal{I}_i^R , \mathcal{I}_i^{RC} and \mathcal{I}_i^{CR} are disjoint. Let define $\mathcal{I}_i = \mathcal{I}_i^C \cup \mathcal{I}_i^R \cup \mathcal{I}_i^{RC} \cup \mathcal{I}_i^{CR}$. We also define $r_{ij} = r_{ij}^C \cup r_{ij}^R$. Let us show that $\mathfrak{J} = \langle \mathcal{I}_i, \mathcal{I}_j, r_{ij}, cr_{ij}, rc_{ij} \rangle$ satisfies all the bridge rules in \mathfrak{B}_{ij} .

- $\mathfrak{J} \models i : B \xrightarrow{\sqsubseteq} j : D$, Let $(v, x) \in r_{ij}$ and $v \in B^{\mathcal{I}_i}$. Then v is the preimage of x either in $\mathcal{I}_i^{C(x)}$ for some onto-bridge rule $i : A \xrightarrow{\sqsupseteq} j : C$, or in $\mathcal{I}_i^{R(x,y)}$ for some onto-bridge rule $i : P \xrightarrow{\sqsupseteq} j : R$, or in $\mathcal{I}_i^{R(y,x)}$ for some onto-bridge rule $i : P \xrightarrow{\sqsupseteq} j : R$. In all the cases we have that $v \in D^{\mathcal{I}_j}$. Indeed if $v \notin D^{\mathcal{I}_j}$, by construction, v should not be the preimage of x in $\mathcal{I}^{C(x)}$.
- $\mathfrak{J} \models i : A \xrightarrow{\sqsupseteq} j : C$. For every element of $x \in C^{\mathcal{I}_j}$ by construction there is a preimage $v \in A^{\mathcal{I}_i^{C(x)}} \subseteq A^{\mathcal{I}_i}$, such that $(v, x) \in r_{ij}^C \subseteq r_{ij}$.
- $\mathfrak{J} \models i : P \xrightarrow{\sqsubseteq} j : R$, Let $(v, w) \in P^{\mathcal{I}_i}$ and $(x, y) \in r(v, w)$. If this is the case than v and w are the preimages of x and y in $\mathcal{I}^{R(x,y)}$. Notice that if $(v, w) \in P^{\mathcal{I}_i}$, they must be the preimages of some x and y for the same sub-model of \mathcal{I} , as each sub-model of \mathcal{I}_i are not crossed.
- $\mathfrak{J} \models i : Q \xrightarrow{\sqsupseteq} j : S$, Let $(x, y) \in S^{\mathcal{I}_j}$, then by construction there are two preimages v and w of such that $(v, w) \in Q^{\mathcal{I}_i^{R(x,y)}}$, and such that $(x, y) \in r_{ij}(v, w)$.
- $\mathfrak{J} \models i : Q \xrightarrow{\sqsubseteq} j : D$, Suppose that $((v, X, w), x) \in rc_{ij}$, we have to prove that

$$X^{\mathcal{I}_i} \subseteq R^{\mathcal{I}_i} \implies x \in D^{\mathcal{I}_j} \quad (39)$$

If $((v, X, w), x) \in rc_{ij}$, then (v, X, w) is a preimage of x in some $\mathcal{I}_i^{C(x)}$, associated to an role-to-concept onto bridge rule $i : P \xrightarrow{\sqsupseteq} j : C$, and $X = \{P\}$. If $x \in D^{\mathcal{I}_j}$, then condition (39) is satisfied. If, instead $x \notin D^{\mathcal{I}_j}$, then by construction there is an interpretation $\mathcal{I}_i^{C(x) \wedge \neg D(x)}$ such that $P^{\mathcal{I}_i^{C(x) \wedge \neg D(x)}} \not\subseteq Q^{\mathcal{I}_i^{C(x) \wedge \neg D(x)}}$. Since $\mathcal{I}_i^{C(x) \wedge \neg D(x)}$ in $\mathcal{I}^C(x)$ and therefore in \mathcal{I}_i , then $P^{\mathcal{I}_i} = X^{\mathcal{I}_i} \not\subseteq Q^{\mathcal{I}_i}$, which implies that condition (39) is true.

□

5 Related Work and Concluding Remarks

Recently, several proposals go in the direction of providing semantic mapping among different ontologies (e.g. [16, 14, 3, 8]). However, to the best of our knowledge there is no specific work on heterogeneous mappings as described in this paper. This in spite of the fact that there are several attempts at providing some sort of mappings relating non-homogeneous elements. For example in [4], it is possible to express the mapping

$$\forall x.(\exists y.R(x, y) \rightarrow C(x)) \quad (40)$$

while, in the original version of DDL (see [14]), an analogous mappings can be established by means of the formula

$$1 : \exists R.\top \xrightarrow{\sqsubseteq} 2 : C \quad (41)$$

Note that both cases cannot be considered heterogeneous mappings because they relates the domain of the relation R with the concept C . The limits of these approaches can be highlighted by the following example.

Assume we want to impose that the relation `marriedWithin` ontology i is equivalent to the concept `Marriage` in ontology j , and we only have mappings as in Equation (41). Then, we can only state expressions of the form:

$$i : \exists \text{marriedWith}.\top \xrightarrow{\sqsubseteq} j : \text{Marriage} \quad i : \exists \text{marriedWith}.\top \xrightarrow{\supseteq} j : \text{Marriage}$$

But these mappings express something rather different from our initial goal as they map single elements of a couple into marriages. Moreover, assume we also have a bridge rule mappings wives in ontology i into women in ontology j as follows:

$$i : \text{Wife} \xrightarrow{\sqsubseteq} j : \text{Woman}$$

together with the axiom `Wife` \sqsubseteq `existsMarried`. \top in ontology i stating that a wife is a married entity. From all this we can infer in ontology j that a wife is a marriage, i.e., `Wife` \sqsubseteq `Marriage`. The problem in this approach lies in the fact that in mapping the two ontologies, we have identified the participants of a relation, (the married person) with the relation itself (the marriage).

In the same spirit of the above cited approaches, but in the area of federated databases, the work described in [2] provides a formalisation of heterogeneous mappings between concepts and relations. In this work the authors define five types of correspondences between concepts and properties, and provide the semantics of these correspondences as follows, where A is a concept and R is a property (i.e. binary relation);

Relation	Semantics
A is equivalent to R	$\forall x.(A(x) \leftrightarrow \exists y.R(y, x))$
A is more general to R	$\forall x.(\exists y.R(y, x) \rightarrow A(x))$
A is less general to R	$\forall x.(A(x) \rightarrow \exists y.R(y, x))$
A and R do overlap	$\exists x.(A(x) \wedge \exists y.R(y, x))$
A and R do not overlap	$\forall x.(A(x) \rightarrow \neg \exists y.R(y, x))$

This semantics is similar to the encoding described in Equation (40). The only difference is that they considers the range of the relation instead of the domain. Therefore, this approach suffers of the same limitations described early on.

The work presented in this paper is clearly connected to the well known modelling process of *reification* (aka *objectification*) adopted in UML or ORM (see [12, 13]). As described in [12], this corresponds to think of certain relationship instances as objects. In UML this is supported by means of *association classes*, while in Entity-Relationship diagram this is often mediated by means of *weak entities*. Note that these modelling paradigms do not support rich inter-schema axioms in the spirit of ontology mappings as described in [16].

There are other modelling formalisms which enable the bridging between relations and classes in the context of Description Logics. In particular, the work on \mathcal{DLR} (see [5]), specifically w.r.t. the technique for encoding n-ary relations within a standard Description Logic, and [6]. The advantage of our approach lies in the fact that the local semantics (i.e. the underlying semantics of the single ontology languages) does not need to be modified in order to consider the global semantics of the system. Specifically, there is no need to provide an explicit reification of relations since this is incorporated into the global semantics.

The language and the semantics presented in this paper constitute a genuine contribution in the direction of the integration of heterogeneous ontologies. The language proposed in this paper makes it possible to directly bind a concept with a relation in a different ontology, and vice-versa. At the semantic level we have introduced a domain relation that maps pairs of object appearing in a relation into objects and vice-versa. This also constitute a novelty in the semantics of knowledge integration. Finally we have shown soundness and completeness of the effects of the mappings and we leave the study of decidability and the definition of a reasoning algorithm for future work.

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