Propositions

- An interpretation is an assignment of values to all variables.
- A model is an interpretation that satisfies the constraints.
- Often we don't want to just find a model, but want to know what is true in all models.
- A proposition is statement that is true or false in each interpretation.

Why propositions?

- Specifying logical formulae is often more natural than filling in tables
- It is easier to check correctness and debug formulae than tables
- We can exploit the Boolean nature for efficient reasoning
- We need a language for asking queries (of what follows in all models) that may be more complicated than asking for the value of a variable
- It is easy to incrementally add formulae
- It can be extended to infinitely many variables with infinite domains (using logical quantification)

Human's view of semantics

- Step 1 Begin with a task domain.
- Step 2 Choose atoms in the computer to denote propositions. These atoms have meaning to the KB designer.
- Step 3 Tell the system knowledge about the domain.
- Step 4 Ask the system questions.

— the system can tell you whether the question is a logical consequence.
— You can interpret the answer with the meaning associated with the atoms.

Role of semantics

In computer:

$$light1\_broken \leftarrow sw\_up \wedge power \wedge unlit\_light1.$$

$sw\_up.$

$power \leftarrow lit\_light2.$

$unlit\_light1.$

$lit\_light2.$

In user's mind:

- *light1_broken*: light #1 is broken
- *sw_up*: switch is up
- *power*: there is power in the building
- *unlit_light1*: light #1 isn't lit
- *lit_light2*: light #2 is lit

Conclusion: *light1_broken*

- The computer doesn't know the meaning of the symbols
- The user can interpret the symbol using their meaning

Simple language: propositional definite clauses

- An **atom** is a symbol starting with a lower case letter
- A **body** is an atom or is of the form $b_1 \wedge b_2$ where b_1 and b_2 are bodies.
- A **definite clause** is an atom or is a rule of the form $h \leftarrow b$ where h is an atom and b is a body.
- A **knowledge base** is a set of definite clauses

Semantics

- An interpretation I assigns a truth value to each atom.
- A body $b_1 \wedge b_2$ is true in I if b_1 is true in I and b_2 is true in I.
- A rule $h \leftarrow b$ is false in I if b is true in I and h is false in I. The rule is true otherwise.
- A knowledge base KB is true in I if and only if every clause in KB is true in I.

Models and Logical Consequence

- A model of a set of clauses is an interpretation in which all the clauses are *true*.
- If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	model?
I_1	*true*	*true*	*true*	*true*	
I_2	*false*	*false*	*false*	*false*	
I_3	*true*	*true*	*false*	*false*	
I_4	*true*	*true*	*true*	*false*	
I_5	*true*	*true*	*false*	*true*	

Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	model?
I_1	*true*	*true*	*true*	*true*	is a model of KB
I_2	*false*	*false*	*false*	*false*	not a model of KB
I_3	*true*	*true*	*false*	*false*	is a model of KB
I_4	*true*	*true*	*true*	*false*	is a model of KB
I_5	*true*	*true*	*false*	*true*	not a model of KB

Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	model?
I_1	*true*	*true*	*true*	*true*	is a model of KB
I_2	*false*	*false*	*false*	*false*	not a model of KB
I_3	*true*	*true*	*false*	*false*	is a model of KB
I_4	*true*	*true*	*true*	*false*	is a model of KB
I_5	*true*	*true*	*false*	*true*	not a model of KB

Which of p, q, r, q logically follow from KB?

Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	model?
I_1	*true*	*true*	*true*	*true*	is a model of KB
I_2	*false*	*false*	*false*	*false*	not a model of KB
I_3	*true*	*true*	*false*	*false*	is a model of KB
I_4	*true*	*true*	*true*	*false*	is a model of KB
I_5	*true*	*true*	*false*	*true*	not a model of KB

Which of p, q, r, q logically follow from KB?
$KB \models p$, $KB \models q$, $KB \not\models r$, $KB \not\models s$

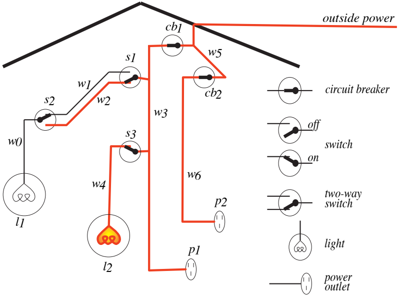
User's view of Semantics

1. Choose a task domain: intended interpretation.
2. Associate an atom with each proposition you want to represent.
3. Tell the system clauses that are true in the intended interpretation: axiomatizing the domain.
4. Ask questions about the intended interpretation.
5. If $KB \models g$, then g must be true in the intended interpretation.
6. Users can interpret the answer using their intended interpretation of the symbols.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Electrical Environment



$light\_l_1.$

$light\_l_2.$

$down\_s_1.$

$up\_s_2.$

$up\_s_3.$

$ok\_l_1.$

$ok\_l_2.$

$ok\_cb_1.$

$ok\_cb_2.$

$live\_outside.$

$lit\_l_1 \leftarrow live\_w_0 \wedge ok\_l_1$

$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2.$

$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2.$

$live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1.$

$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1.$

$lit\_l_2 \leftarrow live\_w_4 \wedge ok\_l_2.$

$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$