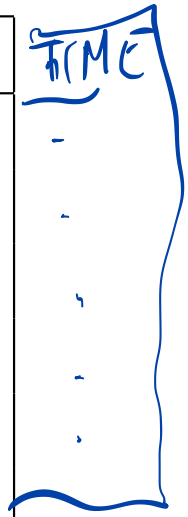
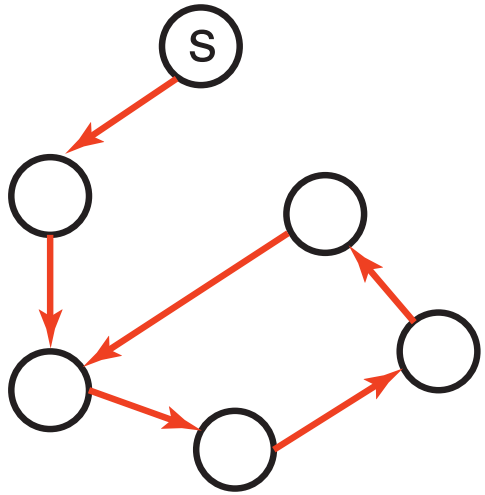
Summary of Search Strategies

Strategy	Frontier Selection	Halts?	Space
Depth-first	Last node added	No	Linear
Breadth-first	First node added	Yes	Exp
Heuristic depth-first	Local min $h(n)$	No	Linear
Best-first	Global min $h(n)$	No	Exp
Lowest-cost-first	Minimal $cost(n)$	Yes	Exp
A^*	Minimal $f(n)$	Yes	Exp



Cycle Checking

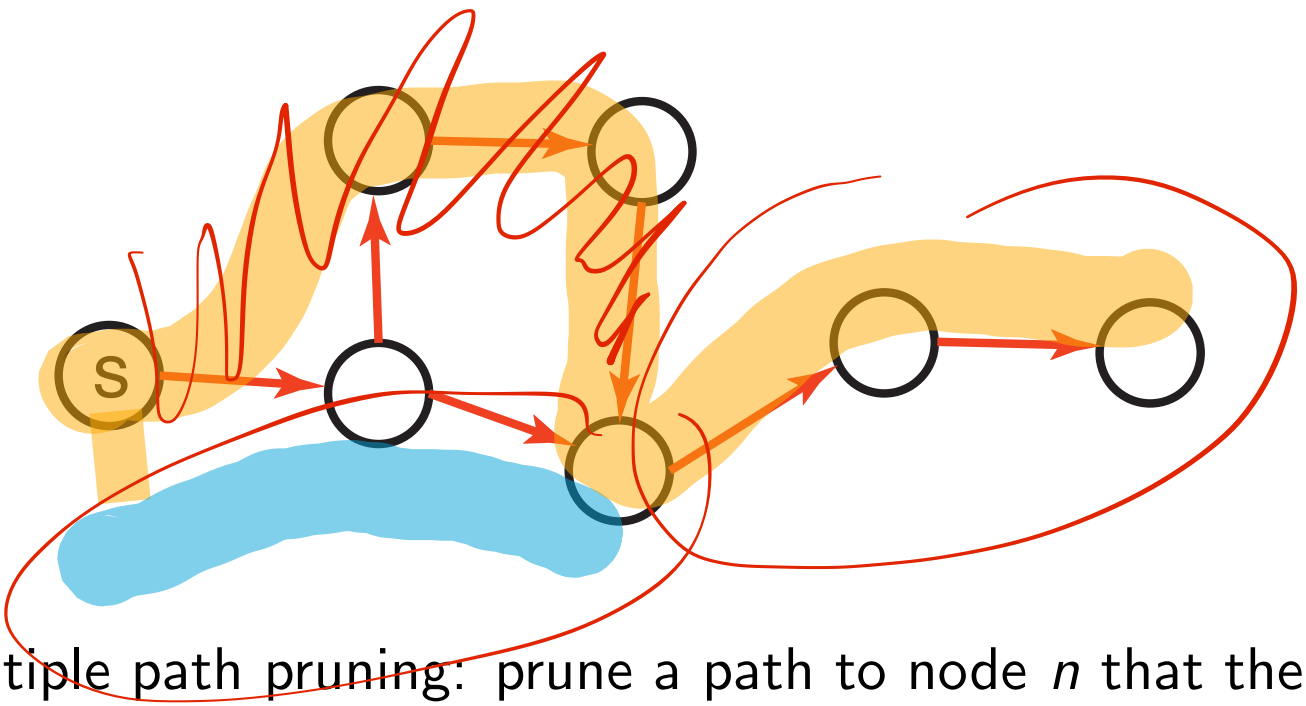


- A searcher can prune a path that ends in a node already on the path, without removing an optimal solution.

Graph searching with cycle pruning

Input: a graph,
a set of start nodes,
Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s\rangle : s \text{ is a start node}\}$
while *frontier* is not empty:
select and **remove** path $\langle n_0, \ldots, n_k\rangle$ from *frontier*
if $n_k \notin \{n_0, \ldots, n_{k-1}\}$:
if $goal(n_k)$:
return $\langle n_0, \ldots, n_k\rangle$
$Frontier \leftarrow Frontier \cup \{\langle n_0, \ldots, n_k, n\rangle : \langle n_k, n\rangle \in A\}$

Multiple-Path Pruning



- Multiple path pruning: prune a path to node n that the searcher has already found a path to.
- Multiple-path pruning subsumes a cycle check.
- This entails storing all nodes it has found paths to.
- Want to guarantee that an optimal solution can still be found.

Graph searching with multiple-path pruning

Input: a graph,
    a set of start nodes,
    Boolean procedure $goal(n)$ that tests if n is a goal node.
$frontier := \{\langle s\rangle : s \text{ is a start node}\}$
$expanded := \{\}$
while *frontier* is not empty:
    select and **remove** path $\langle n_0, \ldots, n_k\rangle$ from *frontier*
    if $n_k \notin expanded$:
        add n_k to *expanded*
        if $goal(n_k)$:
            return $\langle n_0, \ldots, n_k\rangle$
        $Frontier \leftarrow Frontier \cup \{\langle n_0, \ldots, n_k, n\rangle : \langle n_k, n\rangle \in A\}$

Multiple-Path Pruning & Optimal Solutions

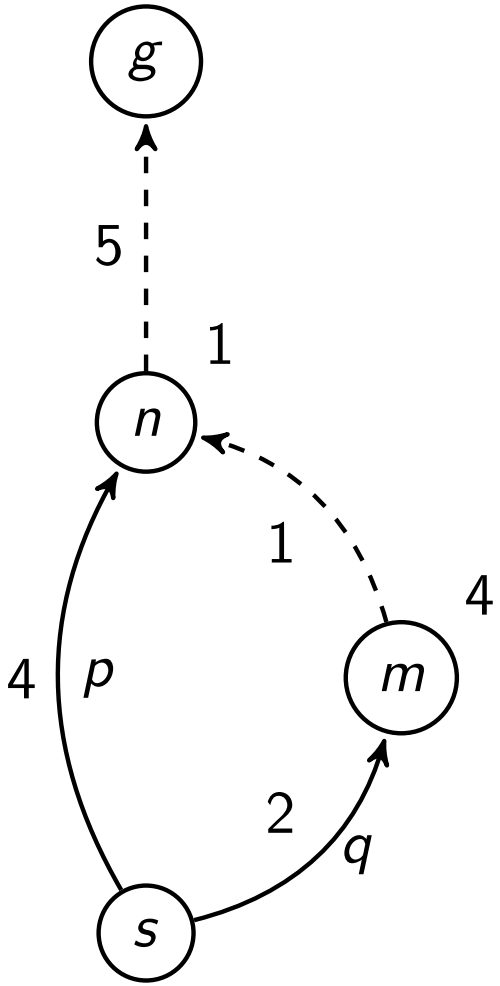
Problem:

what if a subsequent path to n is shorter than the first path to n?

Do any of the following:

- remove all paths from the frontier that use the longer path.
- change the initial segment of the paths on the frontier to use the shorter path.
- ensure this doesn't happen. Make sure that the shortest path to a node is found first.

Multiple-Path Pruning & A^*



A^* does not guarantee that when a path to a node is selected for the first time it is the lowest cost path to that node.

- Suppose path p to n was selected, but there is a shorter path to n. Suppose this shorter path is via path q on the frontier.
- Suppose path q ends at node m.
- $cost(p) + h(n) \leq cost(q) + h(m)$ because p was selected before q.
- $cost(q) + cost(m, n) < cost(p)$ because the path to n via q is shorter.
- Therefore:

$$cost(m, n) < cost(p) - cost(q) \leq h(m) - h(n).$$

Monotone Restriction

- You can ensure that

$$cost(m, n) < cost(p) - cost(q) \leq h(m) - h(n).$$

  doesn't occur if

$$|h(m) - h(n)| \leq cost(m, n)$$

- Heuristic function h satisfies the monotone restriction if $|h(m) - h(n)| \leq cost(m, n)$ for every arc $\langle m, n \rangle$.
- If h satisfies the monotone restriction, A^* with multiple path pruning always finds the shortest path to a goal.
- This is a strengthening of the admissibility criterion.

Monotone Restriction

- The difference in the heuristic values for two nodes must be less than or equal to the actual cost of the lowest-cost path between the nodes.
- Example: the heuristic function of Euclidean distance (the straight-line distance in an n-dimensional Euclidean space) between two points when the cost function is distance.
- It is also typically applicable when the heuristic function is a solution to a simplified problem that has shorter solutions.
- With the monotone restriction, the f-values on the frontier are monotonically non-decreasing:
  - when the frontier is expanded, the f-values do not get smaller;
  - thus, with the monotone restriction, subsequent paths to any node can be pruned in A* search.

Monotone Restriction cost

- Multiple-path pruning can be done in constant time, if the graph is explicitly stored, by setting a bit on each node to which a path has been found.
- It can be done in logarithmic time (in the number of nodes expanded, as long as it is indexed appropriately), if the graph is dynamically generated, by storing the closed list of all of the nodes that have been expanded.
- Multiple-path pruning is preferred over cycle checking for breadth-first methods where virtually all of the nodes considered have to be stored anyway.
- For depth-first search strategies, however, the algorithm does not otherwise have to store all of the nodes already considered. Storing them makes the method exponential in space. Therefore, cycle checking is preferred over multiple-path checking for depth-first methods.

Dijkstra's algorithm

- Lowest-cost-first search with multiple-path pruning is **Dijkstra's algorithm**, and is the same as A^* with multiple-path pruning and a heuristic function of 0.

Iterative Deepening

- So far all search strategies that are guaranteed to halt use exponential space.
- Idea: let's recompute elements of the frontier rather than saving them.
- Look for paths of depth 0, then 1, then 2, then 3, etc.
- You need a depth-bounded depth-first searcher.
- If a path cannot be found at depth B, look for a path at depth $B + 1$. Increase the depth-bound when the search fails unnaturally (depth-bound was reached).

Iterative-deepening search

Boolean natural_failure;
Procedure $dbsearch(\langle n_0, \ldots, n_k \rangle : path, bound : int)$:
    if $goal(n_k)$ and $bound = 0$ report path $\langle n_0, \ldots, n_k \rangle$;
    if $bound > 0$
      for each neighbor n of n_k
            $dbsearch(\langle n_0, \ldots, n_k, n \rangle, bound - 1)$;
    else if n_k has a neighbor then $natural\_failure := false$;
end procedure *dbsearch*;
Procedure $idsearch(S : node)$:
    Integer $bound := 0$;
    repeat
      $natural\_failure := true$;
      $dbsearch(\langle s \rangle, bound)$;
      $bound := bound + 1$;
    until $natural\_failure$;
end procedure *idsearch*

Depth-first Branch-and-Bound

- Way to combine depth-first search with heuristic information.
- Finds optimal solution.
- Most useful when there are multiple solutions, and we want an optimal one.
- Uses the space of depth-first search.

Depth-first Branch-and-Bound

- Idea: maintain the cost of the lowest-cost path found to a goal so far, call this *bound*.
- If the search encounters a path p such that $cost(p) + h(p) \geq bound$, path p can be pruned.
- If a non-pruned path to a goal is found, it must be better than the previous best path. This new solution is remembered and *bound* is set to its cost.
- The search can be a depth-first search to save space.

Depth-first Branch-and-Bound

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- If the search encounters a path p such that $cost(p) + h(p) \geq bound$, path p can be pruned.
- If a non-pruned path to a goal is found, it must be better than the previous best path. This new solution is remembered and *bound* is set to its cost.
- The search can be a depth-first search to save space.
- How should the bound be initialized?

Depth-first Branch-and-Bound: Initializing Bound

- The bound can be initialized to ∞.
- The bound can be set to an estimate of the optimal path cost. After depth-first search terminates either:
  - A solution was found.
  - No solution was found, and no path was pruned
  - No solution was found, and a path was pruned.

Direction of Search

- The definition of searching is symmetric: find path from start nodes to goal node or from goal node to start nodes.
- Forward branching factor: number of arcs out of a node.
- Backward branching factor: number of arcs into a node.
- Search complexity is b^n. Should use forward search if forward branching factor is less than backward branching factor, and vice versa.
- Note: sometimes when graph is dynamically constructed, you may not be able to construct the backwards graph.

Bidirectional Search

- You can search backward from the goal and forward from the start simultaneously.
- This wins as $2b^{k/2} \ll b^k$. This can result in an exponential saving in time and space.
- The main problem is making sure the frontiers meet.
- This is often used with one breadth-first method that builds a set of locations that can lead to the goal. In the other direction another method can be used to find a path to these interesting locations.

Island Driven Search

- Idea: find a set of islands between s and g.

$$s \longrightarrow i_1 \longrightarrow i_2 \longrightarrow \ldots \longrightarrow i_{m-1} \longrightarrow g$$

  There are m smaller problems rather than 1 big problem.
- This can win as $mb^{k/m} \ll b^k$.
- The problem is to identify the islands that the path must pass through. It is difficult to guarantee optimality.
- You can solve the subproblems using islands $\Longrightarrow$ hierarchy of abstractions.

Dynamic Programming

Idea: for statically stored graphs, build a table of $dist(n)$ the actual distance of the shortest path from node n to a goal.
This can be built backwards from the goal:

$$dist(n) = \begin{cases} 0 & \text{if } is\_goal(n), \\ \min_{\langle n,m\rangle \in A}(|\langle n, m\rangle| + dist(m)) & \text{otherwise.} \end{cases}$$

This can be used locally to determine what to do.
There are two main problems:

- You need enough space to store the graph.
- The *dist* function needs to be recomputed for each goal.