Search Strategies

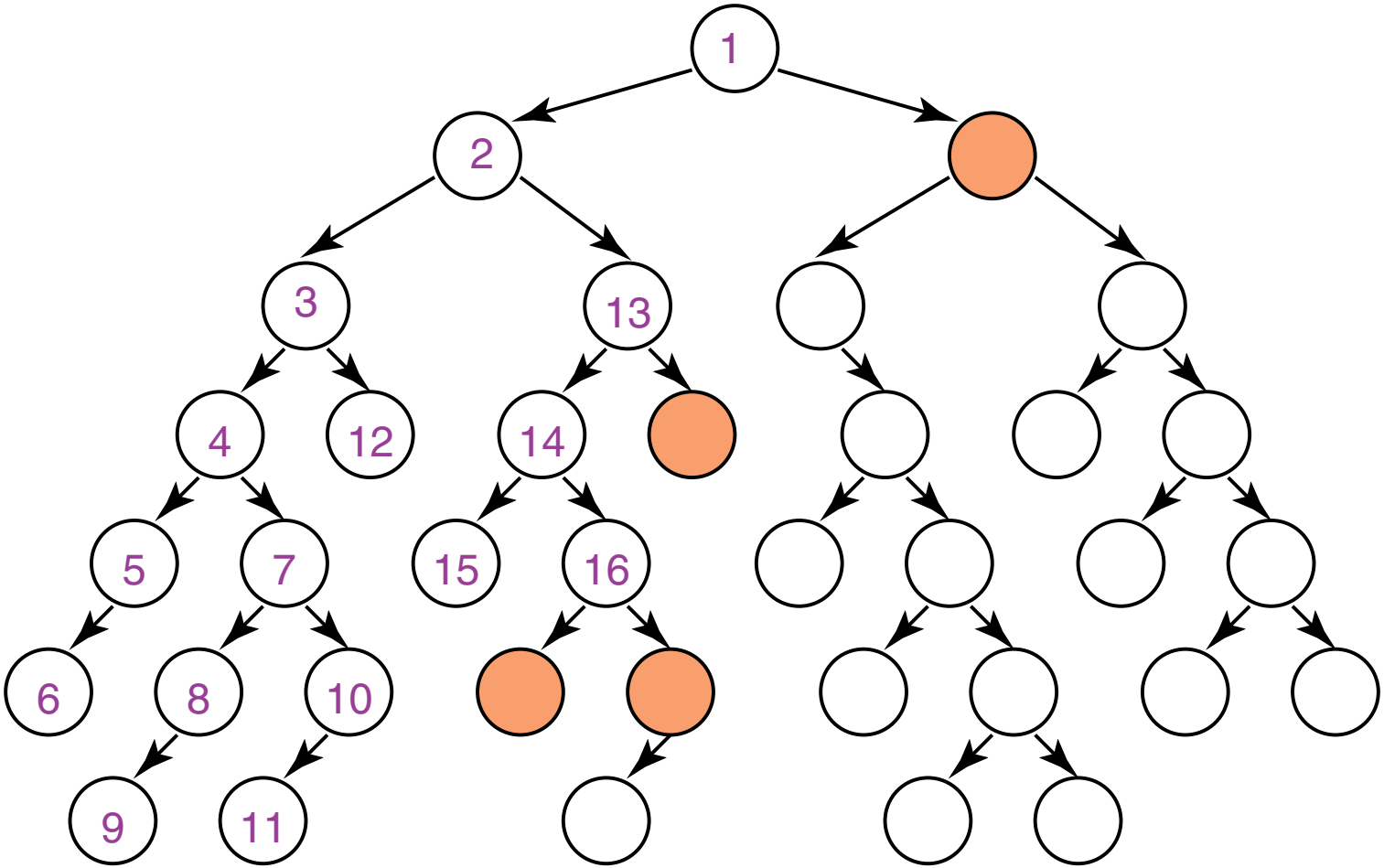
- A problem determines the graph and the goal but not which path to select from the frontier. This is the job of a search strategy.
- A search strategy specifies which paths are selected from the frontier.
- Different strategies are obtained by modifying how the selection of paths in the frontier is implemented.

- Three uninformed search strategies that do not take into account the location of the goal.
- Intuitively, these algorithms ignore where they are going until they find a goal and report success.
  - Depth-First Search
  - Breadth-First Search
  - Lowest-Cost-First Search

Depth-first Search

- Depth-first search treats the frontier as a stack.
- It always selects one of the last elements added to the frontier.
- If the list of paths on the frontier is $[p_1, p_2, \ldots]$
  - p_1 is selected. Paths that extend p_1 are added to the front of the stack (in front of p_2).
  - p_2 is only selected when all paths from p_1 have been explored.

Illustrative Graph — Depth-first Search



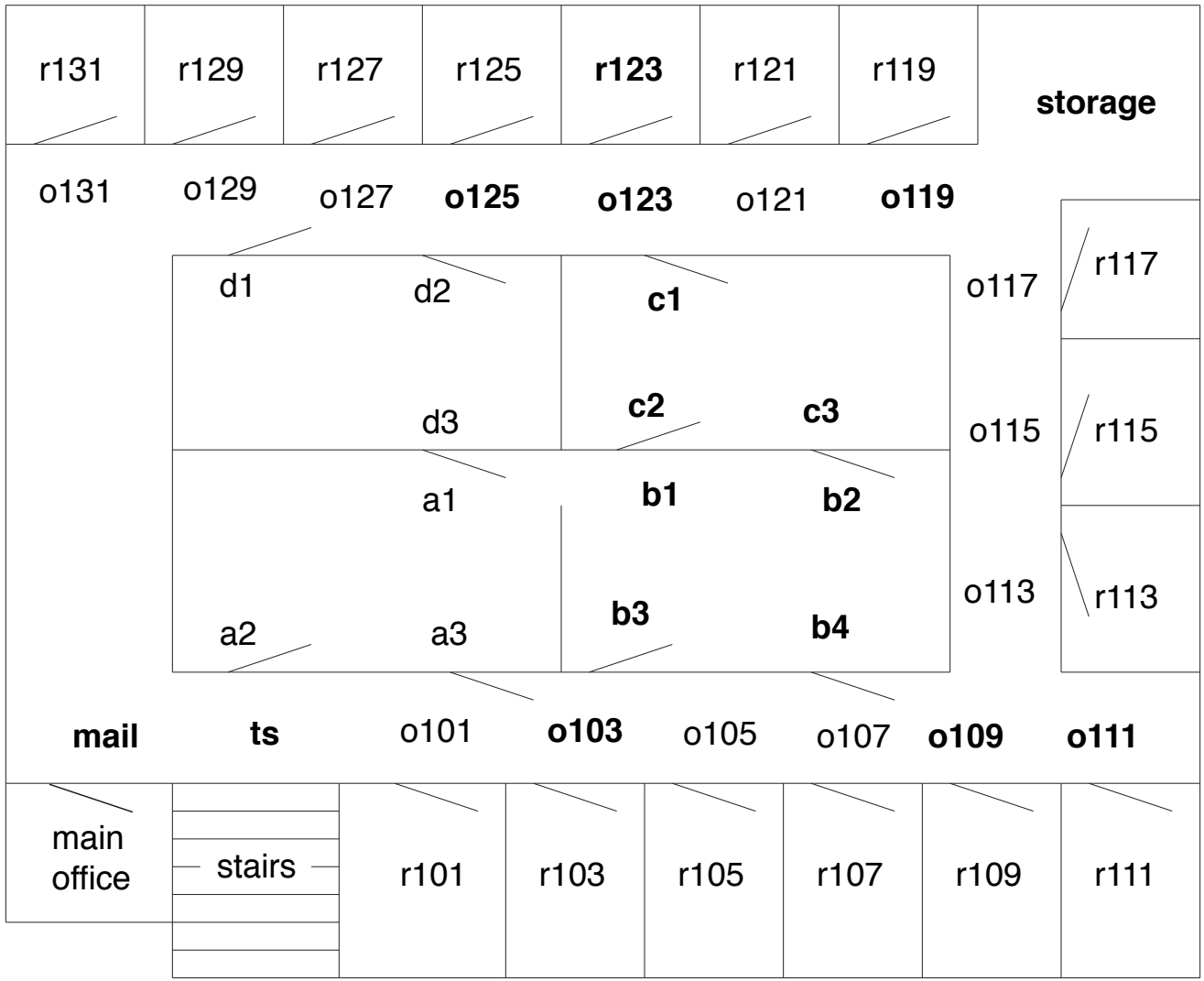
The shaded nodes are the nodes at the ends of the paths on the frontier after the first sixteen steps.

Backtracking

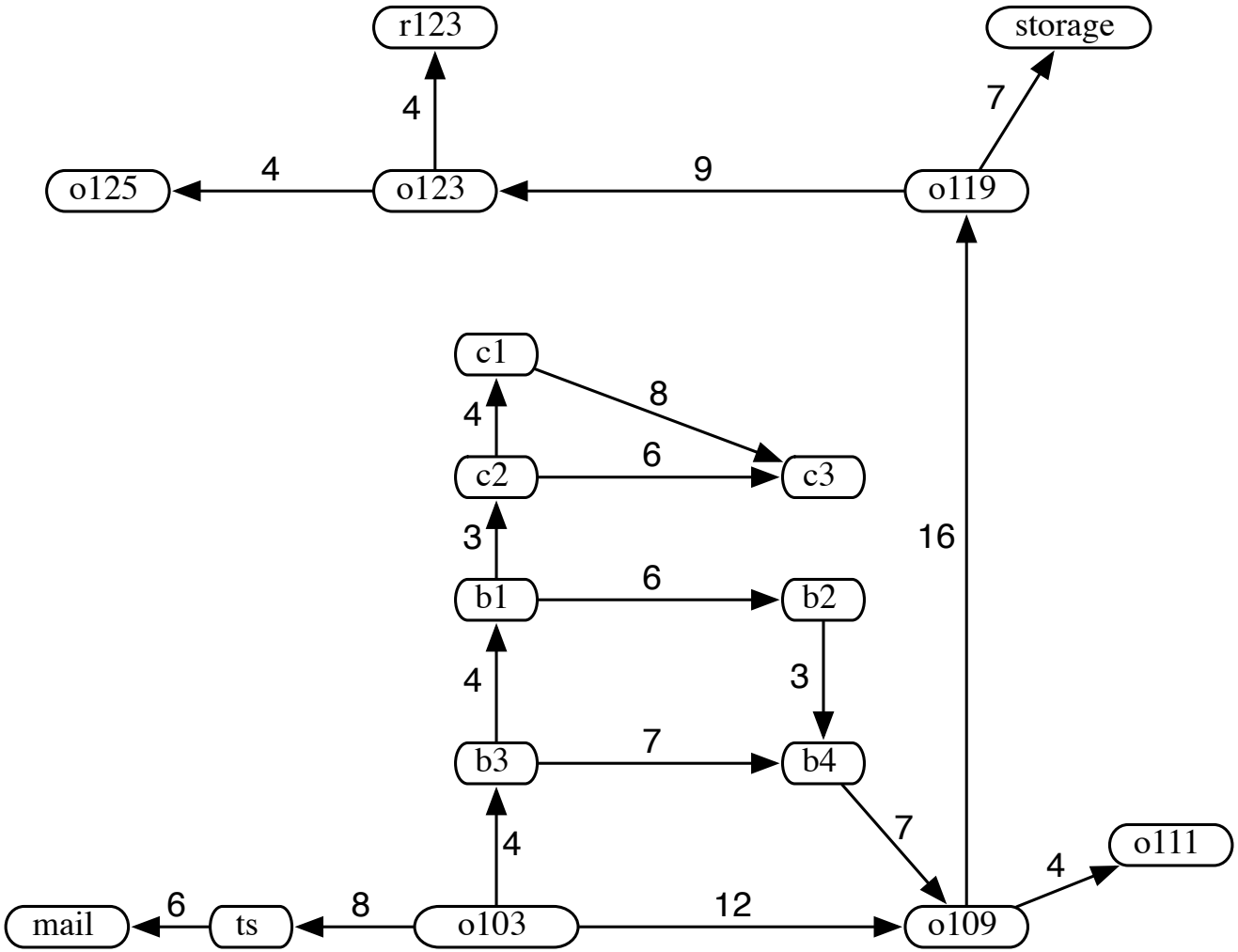
- Searching one path to its completion before trying an alternative path: backtracking.
- The algorithm selects a first alternative at each node, and it backtracks to the next alternative when it has pursued all of the paths from the first selection.
- Some paths may be infinite when the graph has cycles or infinitely many nodes, in which case a depth-first search may never stop.
- This algorithm does not specify the order in which the neighbors are added to the stack that represents the frontier. The efficiency of the algorithm is sensitive to this ordering.

Depth-first for the Delivery Robot

The robot wants to get from outside room 103 to the inside of room 123.



Depth-first for the Delivery Robot



There are three paths from o103 to r123.

Complexity of Depth-first Search

- If there is a solution on the first branch searched, then the time complexity is linear in the length of the path; it considers only those elements on the path, along with their siblings.
- If the graph is a finite tree, with the forward branching factor bounded by b and depth n, the worst-case complexity is $O(b^n)$.
- The worst-case complexity is infinite. Depth-first search can get trapped on infinite branches and never find a solution, even if one exists, for infinite graphs or for graphs with loops.

- Imagine the example of the robot in the case the robot can go back.
- An infinite path leads from *ts* to *mail*, back to *ts*, back to *mail*, and so forth. As presented, depth-first search follows this path forever, never considering alternative paths from *b*3 or *o*109.

Plus and Minus

Depth-first search is appropriate when either

- space is restricted;
- many solutions exist, perhaps with long path lengths, particularly for the case where nearly all paths lead to a solution;
- or the order of the neighbors of a node are added to the stack can be tuned so that solutions are found on the first try.

It is a poor method when

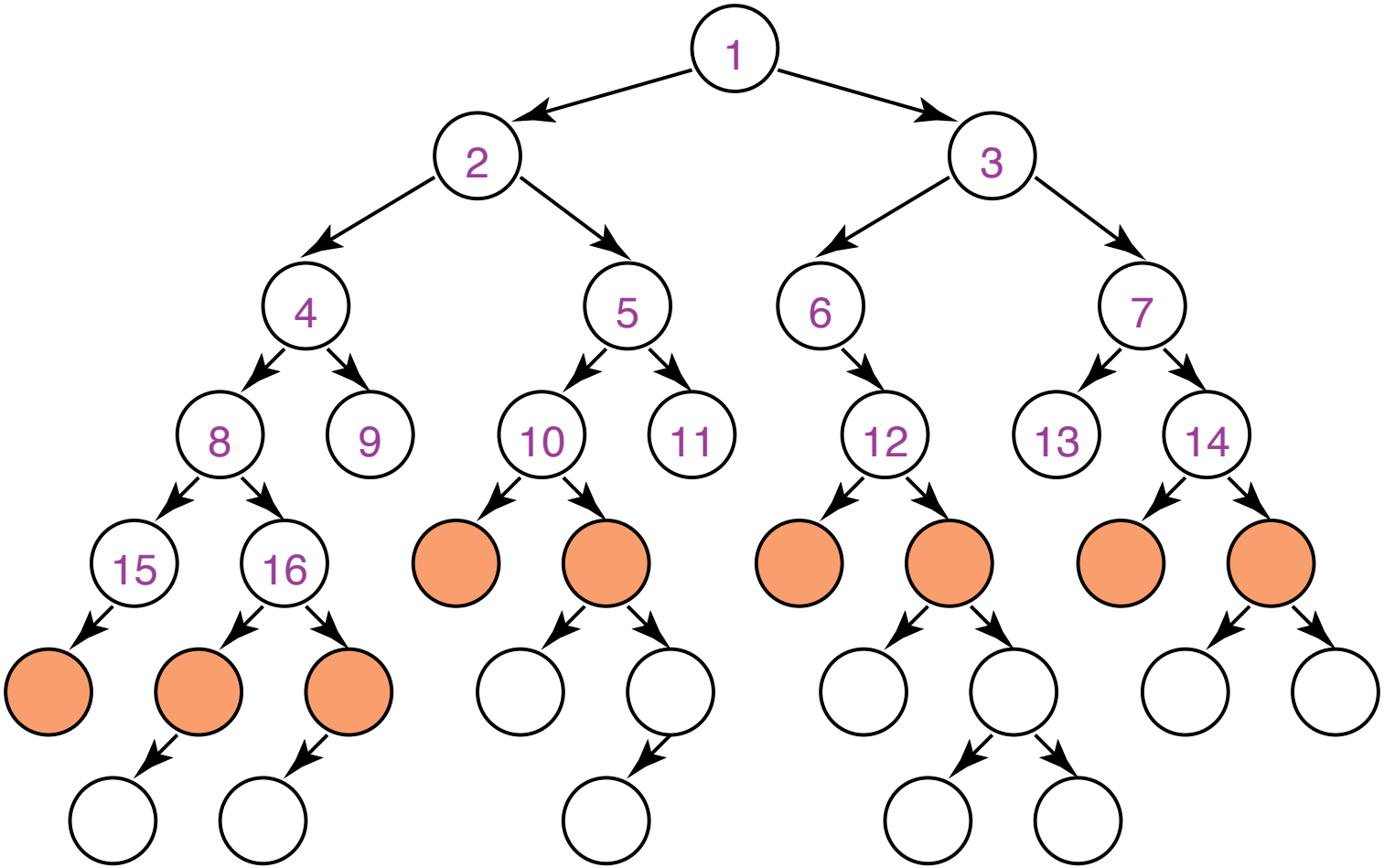
- it is possible to get caught in infinite paths; this occurs when the graph is infinite or when there are cycles in the graph; or
- solutions exist at shallow depth, because in this case the search may look at many long paths before finding the short solutions.

Breadth-first Search

- Breadth-first search treats the frontier as a FIFO (first-in, first-out) queue.
- It always selects one of the earliest elements added to the frontier.
- If the list of paths on the frontier is $[p_1, p_2, \ldots, p_r]$:
  - p_1 is selected. Its neighbors are added to the end of the queue, after p_r.
  - p_2 is selected next.

- This approach implies that the paths from the start node are generated in order of the number of arcs in the path.
- One of the paths with the fewest arcs is selected at each stage.

Illustrative Graph — Breadth-first Search



The shaded nodes are the nodes at the ends of the paths of the frontier after the first sixteen steps.

Complexity of Breadth-first Search

- If the branching factor for all nodes is finite, breadth-first search is guaranteed to find a solution if one exists.
  It is guaranteed to find the path with fewest arcs.
- Time complexity is exponential in the path length:
  b^n, where b is branching factor, n is path length.
- The space complexity is exponential in path length: b^n.

- Breadth-first search finds a solution with the fewest arcs first.

Plus and Minus

Breadth-first search is useful when

- space is not a problem;
- you want to find the solution containing the fewest arcs;
- few solutions may exist, and at least one has a short path length; and
- infinite paths may exist, because it explores all of the search space, even with infinite paths.

It is a poor method when

- all solutions have a long path length or
- there is some heuristic knowledge available.

It is not used very often because of its space complexity.

Lowest-cost-first Search

- Sometimes there are costs associated with arcs. The cost of a path is the sum of the costs of its arcs.

$$cost(\langle n_0, \ldots, n_k \rangle) = \sum_{i=1}^{k} |\langle n_{i-1}, n_i \rangle|$$

- For example, for a delivery robot, costs may be distances and we may want a solution that gives the minimum total distance.
- At each stage, lowest-cost-first search selects a path on the frontier with lowest cost.
- The frontier is a priority queue ordered by path cost.
- It finds a least-cost path to a goal node.
- When arc costs are equal $\Longrightarrow$ breadth-first search.

Features of Lowest-cost-first Search

- If the costs of the arcs are bounded below by a positive constant and the branching factor is finite, the lowest-cost-first search is guaranteed to find an optimal solution - a solution with lowest path cost - if a solution exists.
- Moreover, the first path to a goal that is found is a path with least cost.
- Such a solution is optimal, because the algorithm generates paths from the start in order of path cost.
- If a better path existed than the first solution found, it would have been selected from the frontier earlier.

Complexity of Lowest-cost-first Search

- Like breadth-first search, lowest-cost-first search is typically exponential in both space and time.
- It generates all paths from the start that have a cost less than the cost of the solution.

The bounded arc cost assumption

- The bounded arc cost is used to guarantee the lowest-cost search will find an optimal solution.
- Without such a bound there can be infinite paths with a finite cost.
- For example, there could be nodes $n_0, n_1, \ldots$ with an arc $\langle n_{i-1}, n_i \rangle$ for each $i > 0$ with cost $(1/2)^i$. Infinitely many paths of the form $\langle n_0, n_1, \cdots, n_k \rangle$ exist, all of which have a cost of less than 1. If there is an arc from n_0 to a goal node with a cost greater than or equal to 1, it will never be selected. This is the basis of Zeno's paradoxes that Aristotle wrote about more than 2,300 years ago.