On the Difference between Updating a Knowledge Base and Revising it:

Survey Talk on the KR '1991 Paper by H. Katsuno and A. Mendelzon

Mathew Joseph[1,2], Vadim Savenkov[3]

[1]Data and Knowledge Management unit, FBK-IRST, Trento, Italy

[2]DISI, University of Trento, Trento, Italy

[3]Vienna University of Technology, Vienna, Austria

Research School FCCOD '2014, Bolzano, Italy

Outline

KB Evolution: Revision vs. Update

A Knowledge Base (KB) can eventually become inadequate and require change.

Notation

- ψ is a KB. Models, *Mod*(ψ), of ψ describe *possible worlds*.
- μ specifies the change to be incorporated into ϕ.

- The authors argue that change caused by adding μ to ψ are mainly of two different kinds.

Possible Causes for KB Evolution

- The world described by the KB ψ changes.
  μ is called update in this case. Notation: $\psi \diamond \mu$.
- New knowledge about the world becomes available.
  ψ requires revision μ, denoted as $\psi \circ \mu$.

Update	Revision
Make each possible world a model of μ by some minimal change.	Invalidate possible worlds which are far enough from μ.

Possible Causes for KB Evolution

- The world described by the KB ψ changes.
  μ is called update in this case. Notation: $\psi \diamond \mu$.
- New knowledge about the world becomes available.
  ψ requires revision μ, denoted as $\psi \circ \mu$.

Example

$\psi =$ “Joe’s GF often cancels their dates lately”
$\wedge$ “She is 30 minutes late now”
$\wedge$ ($\heartsuit$: “She is serious about Joe” $\vee$ $\clubsuit$: “...far less than about her cat”)
$\mu =$ “Came late because she was at a movie with another guy.”
$\mu \Rightarrow \neg\heartsuit$:-(

Update: doesn’t allow you to infer $\clubsuit$

Revision: allows you to also infer $\clubsuit$

Outline

KB Revision

Suppose old KB is given by ψ and new knowledge μ, the knowledge revision operator $\circ$ is defined as:

Definition (KB Revision)

$\psi \circ \mu$ is the propositional theory s.t. $Mod(\psi \circ \mu)$ are the set of models of μ that are closest to the set of models of ψ

Closeness could be defined using Dalal's [Dalal, 1988] notion of distance, hence,

$$Mod(\psi \circ \mu) = \{I \in Mod(\mu) |\ \nexists I' \in Mod(\mu) \text{ s.t. } distance(Mod(\psi), I') < distance(Mod(\psi), I)\}$$

Distance [Dalal, 1988]

$$diff(I_1, I_2) = \{p \in \mathbb{P} \mid I_1(p) \neq I_2(p)\},$$
$$distance(I_1, I_2) = |diff(I_1, I_2)|.$$

For a set of models M,

$$distance(M, I_1) = min\{distance(I_2, I_1) | I_2 \in M\}.$$

Example

5 objects A, B, C, D, E and a *table* are in a room. The 5 Objects may be on or off the table. The sentence a intuitively means "Object A is on the table". Similarly b, c, d, e are interpreted. Suppose old KB ψ is the sentence

$$\psi = (a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e) \vee (\neg a \wedge \neg b \wedge c \wedge d \wedge e)$$

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4$, $\quad distance(I_2, I_3) = 2$,

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4$, $distance(I_2, I_3) = 2$,

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4, \qquad distance(I_2, I_3) = 2,$

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4$, $distance(I_2, I_3) = 2$,

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4, \qquad distance(I_2, I_3) = 2,$

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4$, $\quad distance(I_2, I_3) = 2$,

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4$, $distance(I_2, I_3) = 2$,

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$\mu = (a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e)$

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{c, d, e\}$

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{a, b, c, d, e\}$, $I_4 = \{\}$.

$diff(I_1, I_3) = \{b, c, d, e\}$, $diff(I_2, I_3) = \{a, b\}$,

$distance(I_1, I_3) = 4$, $\quad distance(I_2, I_3) = 2$,

hence, $distance(Mod(\psi), I_3) = min\{4, 2\} = 2$.

$diff(I_1, I_4) = \{a\}$, $diff(I_2, I_4) = \{c, d, e\}$,

Example (Contd.)

$distance(I_1, I_4) = 1$, $distance(I_2, I_4) = 3$,

hence, $distance(Mod(\psi), I_4) = min\{1, 3\} = 1$.

Recall, $distance(Mod(\psi), I_3) = 2$

which means $Mod(\psi \circ \mu) = I_4$.

Hence, $\psi \circ \mu \equiv \neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e$.

Example (Contd.)

$distance(I_1, I_4) = 1$, $distance(I_2, I_4) = 3$,

hence, $distance(Mod(\psi), I_4) = min\{1, 3\} = 1$.

Recall, $distance(Mod(\psi), I_3) = 2$

which means $Mod(\psi \circ \mu) = I_4$.

Hence, $\psi \circ \mu \equiv \neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e$.

Example (Contd.)

$distance(I_1, I_4) = 1$, $distance(I_2, I_4) = 3$,

hence, $distance(Mod(\psi), I_4) = min\{1, 3\} = 1$.

Recall, $distance(Mod(\psi), I_3) = 2$

which means $Mod(\psi \circ \mu) = I_4$.

Hence, $\psi \circ \mu \equiv \neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e$.

Example (Contd.)

$distance(I_1, I_4) = 1$, $distance(I_2, I_4) = 3$,

hence, $distance(Mod(\psi), I_4) = min\{1, 3\} = 1$.

Recall, $distance(Mod(\psi), I_3) = 2$

which means $Mod(\psi \circ \mu) = I_4$.

Hence, $\psi \circ \mu \equiv \neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e$.

Example (Contd.)

$distance(I_1, I_4) = 1$, $distance(I_2, I_4) = 3$,

hence, $distance(Mod(\psi), I_4) = min\{1, 3\} = 1$.

Recall, $distance(Mod(\psi), I_3) = 2$

which means $Mod(\psi \circ \mu) = I_4$.

Hence, $\psi \circ \mu \equiv \neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e$.

Example (Contd.)

$distance(I_1, I_4) = 1$, $distance(I_2, I_4) = 3$,

hence, $distance(Mod(\psi), I_4) = min\{1, 3\} = 1$.

Recall, $distance(Mod(\psi), I_3) = 2$

which means $Mod(\psi \circ \mu) = I_4$.

Hence, $\psi \circ \mu \equiv \neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e$.

Revision Postulates [Alchourrón et al. 1985]

R1 $\psi \circ \mu$ implies μ.

R2 If $\psi \wedge \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \wedge \mu$.

R3 If μ is satisfiable then $\psi \circ \mu$ is satisfiable.

R4 If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$.

R5 If $(\psi \circ \mu) \wedge \phi$ implies $\psi \circ (\mu \wedge \phi)$.

R6 If $(\psi \circ \mu) \wedge \phi$ is satisfiable then $\psi \circ (\mu \wedge \phi)$ implies $(\psi \circ \mu) \wedge \phi$.

Orders between interpretations

Let $\mathcal{I}$ be the set of all interpretations over a language $\mathcal{L}$.
A *preorder* $\leq$ over $\mathcal{I}$ is a *reflexive* and *transitive* relation on $\mathcal{I}$.
Define $<$ as $I < I'$ iff $I \leq I'$ and $I' \not\leq I$.

Suppose we assign every formula ψ, a preorder $\leq_\psi$ over $\mathcal{I}$.
This assignment is faithful iff:

1. If $I, I' \in Mod(\psi)$ then $I <_\psi I'$ does not hold.
2. If $I \in Mod(\psi)$ and $I' \notin Mod(\psi)$ then $I <_\psi I'$ holds.
3. If $\psi \equiv \phi$ then $\leq_\psi = \leq_\phi$.

For any $M \subseteq \mathcal{I}$, $Min(M, \leq_\psi)$ be the set of all interpretations I s.t. I is minimal in M w.r.t. $\leq_\psi$.

Soundness and Completeness

Theorem (Soundness and Completeness)

Revision operator $\circ$ satisfies postulates (R1)-(R6) iff there exists a faithful assignment that maps each KB ψ to a total preorder $\leq_\psi$ s.t. Mod$(\psi \circ \mu) =$ Min(Mod$(\mu), \leq_\psi)$.

KB Update

Suppose old KB is given by ψ and new knowledge μ, the knowledge update operator $\diamond$ is defined as:

Definition (KB Update)

$\psi \diamond \mu$ is the propositional theory s.t.

$$Mod(\psi \diamond \mu) = \bigcup_{I \in Mod(\psi)} closest(Mod(\mu), I)$$

Closeness could be the following notion: for any interpretations I, J_1, J_2, $J_1 \leq_I J_2$ iff $diff(J_1, I) \subseteq diff(J_2, I)$.

$closest(Mod(\mu), I) = Min(Mod(\mu), \leq_I)$, i.e the set of all minimal elements in $Mod(\mu)$ w.r.t. $\leq_I$ relation.

Example

Suppose now there are only two objects A, B, and the table. Proposition a means “object A is on the table”, similarly for b. Now our KB ψ is s.t.
$\psi \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$,

and the new knowledge μ is s.t. $\mu \equiv b$.

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$diff(I_1, I_3) = \{a, b\}$, $diff(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$
$diff(I_2, I_3) = \emptyset$, $diff(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$

Example

Suppose now there are only two objects A, B, and the table. Proposition a means “object A is on the table”, similarly for b. Now our KB ψ is s.t.
$\psi \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$,

and the new knowledge μ is s.t. $\mu \equiv b$.

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$diff(I_1, I_3) = \{a, b\}$, $diff(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$
$diff(I_2, I_3) = \emptyset$, $diff(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$

Example

Suppose now there are only two objects A, B, and the table. Proposition a means "object A is on the table", similarly for b.
Now our KB ψ is s.t.
$\psi \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$,

and the new knowledge μ is s.t. $\mu \equiv b$.

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$diff(I_1, I_3) = \{a, b\}$, $diff(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$
$diff(I_2, I_3) = \emptyset$, $diff(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$

Example

Suppose now there are only two objects *A*, *B*, and the table. Proposition *a* means "object *A* is on the table", similarly for *b*. Now our KB ψ is s.t.
$\psi \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$,

and the new knowledge μ is s.t. $\mu \equiv b$.

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$diff(I_1, I_3) = \{a, b\}$, $diff(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$
$diff(I_2, I_3) = \emptyset$, $diff(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$

Example

Suppose now there are only two objects A, B, and the table. Proposition a means "object A is on the table", similarly for b. Now our KB ψ is s.t.
$\psi \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$,

and the new knowledge μ is s.t. $\mu \equiv b$.

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$diff(I_1, I_3) = \{a, b\}$, $diff(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$
$diff(I_2, I_3) = \emptyset$, $diff(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$

Example

Suppose now there are only two objects A, B, and the table. Proposition a means "object A is on the table", similarly for b. Now our KB ψ is s.t.
$\psi \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$,

and the new knowledge μ is s.t. $\mu \equiv b$.

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$diff(I_1, I_3) = \{a, b\}$, $diff(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$
$diff(I_2, I_3) = \emptyset$, $diff(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$

Example

Suppose now there are only two objects A, B, and the table. Proposition a means "object A is on the table", similarly for b. Now our KB ψ is s.t.

$\psi \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$,

and the new knowledge μ is s.t. $\mu \equiv b$.

$Mod(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$Mod(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$diff(I_1, I_3) = \{a, b\}$, $diff(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$
$diff(I_2, I_3) = \emptyset$, $diff(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$

Example (Contd.)

Hence, we have

$closest(Mod(\mu), I_1) = I_4$, and $closest(Mod(\mu), I_2) = I_3$.

Hence, $Mod(\psi \diamond \mu) = \bigcup_{I \in Mod(\psi)} closest(Mod(\mu), I) = \{I_3, I_4\}$,

and hence, updated KB $\psi \diamond \mu \equiv b$

Whereas $Mod(\psi \circ \mu) = I_3$, and

hence, revised KB $\psi \circ \mu \equiv \neg a \wedge b$.

Update Postulates

- **U1** $\psi \diamond \mu$ implies μ
- **U2** If ψ implies μ then $\psi \diamond \mu$ is equivalent to ψ
- **U3** If both ψ and μ are satisfiable then $\psi \diamond \mu$ is also satisfiable.
- **U4** If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2$.
- **U5** $(\psi \diamond \mu) \wedge \phi$ implies $\psi \diamond (\mu \wedge \phi)$.
- **U6** If $\psi \diamond \mu_1$ implies μ_2 and $\psi \diamond \mu_2$ implies μ_1 then $\psi \diamond \mu_1 \equiv \psi \diamond \mu_2$.
- **U7** If ψ is complete then $(\psi \diamond \mu_1) \wedge (\psi \diamond \mu_2)$ implies $\psi \diamond (\mu_1 \vee \mu_2)$.
- **U8** $(\psi_1 \vee \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \vee (\psi_2 \diamond \mu)$.

Lemma

If ψ is inconsistent, then $\psi \diamond \mu$ is inconsistent for any μ.

Orders between interpretations

Let $\mathcal{I}$ be the set of all interpretations over a language $\mathcal{L}$. Suppose we assign, to each interpretation I, a partial preorder $\leq_I$ over $\mathcal{I}$. This assignment is said to be faithful iff:

- For any $J \in \mathcal{I}$, If $J \neq I$ then $I <_I J$.

Theorem (Soundness and Completeness)

The update operator $\diamond$ satisfies postulates U1-U8 iff there exists a faithful assignment that maps each interpretation I to a partial pre-order $\leq_I$ s.t.

$$Mod(\psi \diamond \mu) = \bigcup_{I \in Mod(\psi)} Min(Mod(\mu), \leq_I).$$

Outline

Example (Now Joe is certain.)

$\psi =$ "Joe's GF often cancels their dates lately"
$\wedge$ "She is 30 minutes late now"
$\wedge$ $\heartsuit$:"She is serious about Joe"

$\mu =$ "Late because she was at a movie with another guy."

- $\psi \circ \mu = \psi \wedge \mu \wedge \neg\heartsuit$ makes $\psi \circ \mu$ inconsistent.
- Contraction operator: give up compromised beliefs ($\heartsuit$ in our case).

Contraction

Eliminating sentences from the KB which are no longer trusted.

Postulates [Alchourrón et al. 1985]

C1 $\psi \Longrightarrow \psi \bullet \mu$

C2 $\psi \not\to \mu \Longrightarrow \psi \bullet \mu \equiv \psi$

C3 $\mu \not\equiv \top \Longrightarrow \psi \bullet \mu \not\to \mu$

C4 $\psi_1 \equiv \psi_2 \wedge \mu_1 \equiv \mu_2 \Longrightarrow \psi_1 \bullet \mu_1 \equiv \psi_2 \bullet \mu_2$

C5 $(\psi \bullet \mu) \wedge \mu \Longrightarrow \psi$

Contraction vs. Revision [Alchourrón et al. 1985]

R1 $\psi \circ \mu \Longrightarrow \mu$.

R2 $\psi \wedge \mu \not\equiv \bot \Longrightarrow \psi \circ \mu \equiv \psi \wedge \mu$.

R3 $\mu \not\equiv \bot \Longrightarrow \psi \circ \mu \not\equiv \bot$.

R4 $\psi_1 \equiv \psi_2 \wedge \mu_1 \equiv \mu_2$
$\Longrightarrow \psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$.

...

C1 $\psi \Longrightarrow \psi \bullet \mu$

C2 $\psi \not\Rightarrow \mu \Longrightarrow \psi \bullet \mu \equiv \psi$

C3 $\mu \not\equiv \top \Longrightarrow \psi \bullet \mu \not\Rightarrow \mu$

C4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2)$
$\Longrightarrow \psi_1 \bullet \mu_1 \equiv \psi_2 \bullet \mu_2$

C5 $(\psi \bullet \mu) \wedge \mu \Longrightarrow \psi$

Revision ⇒ Contraction

- If $\circ$ is a revision operator satisfying properties (R1)–(R4), then $\bullet$ defined as $\psi \bullet \mu \equiv \psi \vee (\psi \circ \neg\mu)$ satisfies (C1)–(C5)

Contraction ⇒ Revision

- If $\bullet$ is a contraction operator satisfying (C1)–(C5), then $\circ$ defined as $\psi \circ \mu \equiv (\psi \bullet \neg\mu) \wedge \mu$ satisfies (P1)–(P4)

Erasure: Contracting All Possible Worlds

Contraction only works for facts known for sure:

- Recall the postulate (C2) $\psi \not\to \mu \Longrightarrow \psi \bullet \mu \equiv \psi$

Example (Original version)

$\psi =$ "Joe's GF often cancels their dates lately"
$\wedge$ "She is 30 minutes late now"
$\wedge$ ($\heartsuit$:"She is serious about Joe" $\vee$ $\clubsuit$: "...far less than about her cat")

Contraction of $\heartsuit$ does nothing here: $\psi \bullet \heartsuit = \psi$, since $\psi \not\to \heartsuit$. That is, $\heartsuit$ is not part of all possible worlds.
Suppose Joe is fed up and decides to break up. He is determined and therefore sure that $\heartsuit$ should not be implied by any possible world.

The version of contraction that works on all possible worlds is called erasure. It is a form of *update*.

Erasure: Contracting All Possible Worlds

Contraction only works for facts known for sure:

- Recall the postulate (C2) $\psi \not\to \mu \Longrightarrow \psi \bullet \mu \equiv \psi$

Example (Original version)

$\psi =$ "Joe's GF often cancels their dates lately"
$\wedge$ "She is 30 minutes late now"
$\wedge$ ($\heartsuit$:"She is serious about Joe" $\vee$ $\clubsuit$: "…far less than about her cat")

Contraction of $\heartsuit$ does nothing here: $\psi \bullet \heartsuit = \psi$, since $\psi \not\to \heartsuit$. That is, $\heartsuit$ is not part of all possible worlds.
Suppose Joe is fed up and decides to break up. He is determined and therefore sure that $\heartsuit$ should not be implied by any possible world.

The version of contraction that works on all possible worlds is called erasure. It is a form of *update*.

Erasure: Contraction-like Counterpart to Update

Postulates of the Erasure operator

E1 $\psi \Longrightarrow \psi \blacklozenge \mu$

E2 $\psi \rightarrow \neg\mu \Longrightarrow \psi \blacklozenge \mu \equiv \psi$ (C2) $\psi \not\rightarrow \mu \Longrightarrow \psi \bullet \mu \equiv \psi$

E3 $\psi \not\equiv \bot \wedge \mu \not\equiv \top \Longrightarrow \psi \blacklozenge \mu \not\rightarrow \mu$ (C3) $\mu \not\equiv \top \Longrightarrow \psi \bullet \mu \not\rightarrow \mu$

E4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2$

E5 $(\psi \blacklozenge \mu) \wedge \mu \Longrightarrow \psi$

E8 $(\psi_1 \vee \psi_2) \blacklozenge \mu \Longleftrightarrow (\psi_1 \blacklozenge \mu) \vee (\psi_2 \blacklozenge \mu)$

Example (Erasure works on every possible world = disjunct)

Let $\psi = \theta \wedge (\heartsuit \vee \clubsuit)$.

$\psi \blacklozenge \heartsuit \overset{(E8)}{=} ((\theta \wedge \heartsuit) \blacklozenge \heartsuit) \vee ((\theta \wedge \clubsuit) \blacklozenge \heartsuit) \overset{(E3)}{=} \theta \vee (\theta \wedge \clubsuit) \blacklozenge \heartsuit$

Erasure: Contraction-like Counterpart to Update

Postulates of the Erasure operator

E1 $\psi \Longrightarrow \psi \blacklozenge \mu$

E2 $\psi \rightarrow \neg\mu \Longrightarrow \psi \blacklozenge \mu \equiv \psi$ (C2) $\psi \not\rightarrow \mu \Longrightarrow \psi \bullet \mu \equiv \psi$

E3 $\psi \not\equiv \bot \wedge \mu \not\equiv \top \Longrightarrow \psi \blacklozenge \mu \not\rightarrow \mu$ (C3) $\mu \not\equiv \top \Longrightarrow \psi \bullet \mu \not\rightarrow \mu$

E4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2$

E5 $(\psi \blacklozenge \mu) \wedge \mu \Longrightarrow \psi$

E8 $(\psi_1 \vee \psi_2) \blacklozenge \mu \Longleftrightarrow (\psi_1 \blacklozenge \mu) \vee (\psi_2 \blacklozenge \mu)$

Example (Erasure works on every possible world = disjunct)

Let $\psi = \theta \wedge (\heartsuit \vee \clubsuit)$.

$\psi \blacklozenge \heartsuit \overset{(E8)}{=} ((\theta \wedge \heartsuit) \blacklozenge \heartsuit) \vee ((\theta \wedge \clubsuit) \blacklozenge \heartsuit) \overset{(E3)}{=} \theta \vee (\theta \wedge \clubsuit) \blacklozenge \heartsuit$

Erasure: Contraction-like Counterpart to Update

Postulates of the Erasure operator

- E1 $\psi \implies \psi \blacklozenge \mu$
- E2 $\psi \rightarrow \neg\mu \implies \psi \blacklozenge \mu \equiv \psi$ (C2) $\psi \not\rightarrow \mu \implies \psi \bullet \mu \equiv \psi$
- E3 $\psi \not\equiv \bot \wedge \mu \not\equiv \top \implies \psi \blacklozenge \mu \not\rightarrow \mu$ (C3) $\mu \not\equiv \top \implies \psi \bullet \mu \not\rightarrow \mu$
- E4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \implies \psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2$
- E5 $(\psi \blacklozenge \mu) \wedge \mu \implies \psi$
- E8 $(\psi_1 \vee \psi_2) \blacklozenge \mu \iff (\psi_1 \blacklozenge \mu) \vee (\psi_2 \blacklozenge \mu)$

Example (Erasure works on every possible world = disjunct)

Let $\psi = \theta \wedge (\heartsuit \vee \clubsuit)$.

$\psi \blacklozenge \heartsuit \overset{(E8)}{=} ((\theta \wedge \heartsuit) \blacklozenge \heartsuit) \vee ((\theta \wedge \clubsuit) \blacklozenge \heartsuit) \overset{(E3)}{=} \theta \vee (\theta \wedge \clubsuit) \blacklozenge \heartsuit$

Erasure vs. Update

U1 $\psi \diamond \mu \Longrightarrow \mu$

U2 $\psi \rightarrow \mu \Longrightarrow \psi \diamond \mu \equiv \psi$

U3 $\psi \wedge \mu \not\equiv \bot \Longrightarrow \psi \diamond \mu \not\equiv \bot$

U4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow$
$\psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2$.

...

U8 $(\psi_1 \vee \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \vee (\psi_2 \diamond \mu)$.

E1 $\psi \Longrightarrow \psi \blacklozenge \mu$

E2 $\psi \rightarrow \neg\mu \Longrightarrow \psi \blacklozenge \mu \equiv \psi$

E3 $\psi \not\equiv \bot \wedge \mu \not\equiv \top \Longrightarrow \psi \blacklozenge \mu \not\rightarrow \mu$

E4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow$
$\psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2$

E5 $(\psi \blacklozenge \mu) \wedge \mu \Longrightarrow \psi$

E8 $(\psi_1 \vee \psi_2) \blacklozenge \mu \equiv (\psi_1 \blacklozenge \mu) \vee (\psi_2 \blacklozenge \mu)$

Theorem

1. *If an update operator $\diamond$ satisfies (U1)–(U4) and (U8), then the erasure operator $\blacklozenge$ defined by $\psi \blacklozenge \mu \equiv \psi \vee (\psi \diamond \neg\mu)$ satisfies (E1)–(E5) and (E8).*

Erasure vs. Update

U1 $\psi \diamond \mu \Longrightarrow \mu$

U2 $\psi \rightarrow \mu \Longrightarrow \psi \diamond \mu \equiv \psi$

U3 $\psi \wedge \mu \not\equiv \bot \Longrightarrow \psi \diamond \mu \not\equiv \bot$

U4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2.$

...

U8 $(\psi_1 \vee \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \vee (\psi_2 \diamond \mu).$

E1 $\psi \Longrightarrow \psi \blacklozenge \mu$

E2 $\psi \rightarrow \neg\mu \Longrightarrow \psi \blacklozenge \mu \equiv \psi$

E3 $\psi \not\equiv \bot \wedge \mu \not\equiv \top \Longrightarrow \psi \blacklozenge \mu \not\rightarrow \mu$

E4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2$

E5 $(\psi \blacklozenge \mu) \wedge \mu \Longrightarrow \psi$

E8 $(\psi_1 \vee \psi_2) \blacklozenge \mu \equiv (\psi_1 \blacklozenge \mu) \vee (\psi_2 \blacklozenge \mu)$

Theorem

2. *If an erasure operator* $\blacklozenge$ *satisfies (E1)–(E4) and (E8), then the update operator* $\diamond$ *defined by* $\psi \diamond \mu \equiv (\psi \blacklozenge \neg\mu) \wedge \mu$ *satisfies (U1)–(U4) and (U8).*

Erasure vs. Update

U1 $\psi \diamond \mu \Longrightarrow \mu$

U2 $\psi \rightarrow \mu \Longrightarrow \psi \diamond \mu \equiv \psi$

U3 $\psi \wedge \mu \not\equiv \bot \Longrightarrow \psi \diamond \mu \not\equiv \bot$

U4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2.$

...

U8 $(\psi_1 \vee \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \vee (\psi_2 \diamond \mu).$

E1 $\psi \Longrightarrow \psi \blacklozenge \mu$

E2 $\psi \rightarrow \neg\mu \Longrightarrow \psi \blacklozenge \mu \equiv \psi$

E3 $\psi \not\equiv \bot \wedge \mu \not\equiv \top \Longrightarrow \psi \blacklozenge \mu \not\rightarrow \mu$

E4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2$

E5 $(\psi \blacklozenge \mu) \wedge \mu \Longrightarrow \psi$

E8 $(\psi_1 \vee \psi_2) \blacklozenge \mu \equiv (\psi_1 \blacklozenge \mu) \vee (\psi_2 \blacklozenge \mu)$

Theorem

3. *Suppose that an update operator* $\diamond$ *satisfies (U1)–(U4) and (U8). Then, we can define an erasure operator by* $\psi \blacklozenge \mu \equiv \psi \vee (\psi \diamond \neg\mu)$. *The update operator obtained from the erasure operator by* $\psi \diamond \mu \equiv (\psi \blacklozenge \neg\mu) \wedge \mu$ *is equal to the original update operator.*

Erasure vs. Update

U1 $\psi \diamond \mu \Longrightarrow \mu$

U2 $\psi \rightarrow \mu \Longrightarrow \psi \diamond \mu \equiv \psi$

U3 $\psi \wedge \mu \not\equiv \bot \Longrightarrow \psi \diamond \mu \not\equiv \bot$

U4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2$.

...

U8 $(\psi_1 \vee \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \vee (\psi_2 \diamond \mu)$.

E1 $\psi \Longrightarrow \psi \blacklozenge \mu$

E2 $\psi \rightarrow \neg\mu \Longrightarrow \psi \blacklozenge \mu \equiv \psi$

E3 $\psi \not\equiv \bot \wedge \mu \not\equiv \top \Longrightarrow \psi \blacklozenge \mu \not\rightarrow \mu$

E4 $(\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \Longrightarrow \psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2$

E5 $(\psi \blacklozenge \mu) \wedge \mu \Longrightarrow \psi$

E8 $(\psi_1 \vee \psi_2) \blacklozenge \mu \equiv (\psi_1 \blacklozenge \mu) \vee (\psi_2 \blacklozenge \mu)$

Theorem

4. *Suppose that an erasure operator* $\blacklozenge$ *satisfies (E1)–(E5) and (E8). Then, we can define an update operator by* $\psi \diamond \mu \equiv (\psi \blacklozenge \neg\mu) \wedge \mu$. *The erasure operator obtained from the update operator by* $\psi \blacklozenge \mu \equiv \psi \vee (\psi \diamond \neg\mu)$ *is equal to the original erasure operator.*

Outline

How to tell if μ is a revision or an update?

- Time parameter: t.
- Parameterized KB has the form $\langle \psi, t \rangle$.
- New operator: $Tell(\mu, t')\langle \psi, t \rangle = \begin{cases} \langle \psi \circ \mu, t \rangle & \text{if } t' = t \\ \langle \psi \diamond \mu, t' \rangle & \text{if } t' > t \end{cases}$

In this framework, the type of the change is done automatically based on the relationship between the time instant of the KB and that of the change:

- Change now ($t' = t$) $\Longrightarrow$ That's about the knowledge.
- Change in the future ($t' > t$) $\Longrightarrow$ That's about the world.

Example

Recall the example with two objects A, B on the table.

- $\langle \psi = (a \wedge \neg b) \vee (\neg a \wedge b), \texttt{10:00} \rangle$.
- New knowledge: it's surely the object B on the table.
  $Tell(b, \texttt{10:00})\langle \psi, \texttt{10:00} \rangle$
  $\Longrightarrow \langle \psi \circ b, \texttt{10:00} \rangle = \langle (b \wedge \neg a), \texttt{10:00} \rangle$
- Sent robot to put the object B on the table.
  $Tell(b, \texttt{10:05})\langle \psi, \texttt{10:00} \rangle$
  $\Longrightarrow \langle \psi \diamond b, \texttt{10:05} \rangle = \langle b, \texttt{10:05} \rangle$

Example

Recall the example with two objects A, B on the table.

- $\langle \psi = (a \wedge \neg b) \vee (\neg a \wedge b), \texttt{10:00} \rangle$.
- New knowledge: it's surely the object B on the table.
  $Tell(b, \texttt{10:00})\langle \psi, \texttt{10:00} \rangle$
  $\implies \langle \psi \circ b, \texttt{10:00} \rangle = \langle (b \wedge \neg a), \texttt{10:00} \rangle$
- Sent robot to put the object B on the table.
  $Tell(b, \texttt{10:05})\langle \psi, \texttt{10:00} \rangle$
  $\implies \langle \psi \diamond b, \texttt{10:05} \rangle = \langle b, \texttt{10:05} \rangle$

THANKS

Thanks for your attention

Questions?

[Katsuno & Mendelzon. 1991]
On the logic of theory change: partial meets contraction and revision functions
Katsuno H. and Mendelzon A.O., 1991
In Proceedings of KR '1991 (387-394).

[Alchourrón et al. 1985]
On the logic of theory change: partial meets contraction and revision functions
Alchourrón C.E. and Gardenfors P.E. and Makinson D., 1985
Journal of Symbolic Logic 50 (510-530).

[Dalal, 1988]
Investigations into a theory of knowledge base revision: Preliminary Report
Dalal, M., 1988, AAAI 17 (475-479).