Multifarious Uncertainty in Ontologies

Where we are and where we might go

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**Dictionary**

**multifarious** | \(\text{ˌmælt(ə)ˈfə(ə)rēəs}\)  
**adjective**  
many and of various types: **multifarious activities**.  
• having many varied parts or aspects: a vast multifarious organization.

**DERIVATIVES**  
**multifariously** adverb.  
**multifariousness** noun

**ORIGIN** late 16th cent.: from Latin **multifarius** + -ous.

**Thesaurus**

**multifarious**  
adjective  
*our multifarious ethnic traditions: DIVERSE, many, numerous, various, varied, diversified, multiple, multitudinous, multiplex, manifold, multifaceted, different, heterogeneous, miscellaneous, assorted; literary myriad, divers.* ANTONYMS homogeneous.
Motivation

Viral meningitis is a type of meningitis
Bacterial meningitis is a type of meningitis
Meningitis is either viral or bacterial
Motivation

Medical terminology

- Viral meningitis is a type of meningitis
- Bacterial meningitis is a type of meningitis
- Meningitis is either viral or bacterial

- Meningitis is usually not fatal
- Meningitis and caused by bacteria is usually fatal
- Meningitis affects most of skull, Dura mater is mostly made of fibre
- B. menin. is similar to v. menin., Pia mater is analogous to dura mater
- Cases of B. meningitis can be treated with antibiotics
- But Mary has meningitis and is pregnant
Uncertainty in Ontologies

Several nuances

- **Exceptions**: special cases, overriding of properties
  
  meningitis, *bact.* meningitis

- **Similarity** or **analogy**: focus on relevant aspects, tolerance
  
  *pia mater*, *dura mater*

- **Vagueness**: notions of ‘generally’, ‘rarely’, ‘most’
  
  meningitis rarely kills

- **Incomplete information**: take chances, be venturous
  
  give antibiotics to cases of meningitis

- **Dynamicty**: incorporate new information, backtracking
  
  not during pregnancy

- **Others**
Uncertainty in Ontologies

Various takes

- **Quantitative**: probabilistic, statistical
- **Qualitative**: logical
- **Combinations thereof**
Uncertainty in Ontologies

Various takes

- **Quantitative:** probabilistic, statistical
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- Combinations thereof

Logical approaches

- Qualitative analysis of uncertainty in reasoning
- a.k.a. nonmonotonic reasoning
- Broader than the usual understanding of NMR
Outline

Overview

Where Are We?

Where To?

Conclusion
Outline
Monotonicity

\[ Cn(\alpha) \subseteq Cn(\alpha \land \beta) \]

\[ Mod(\alpha \land \beta) \subseteq Mod(\alpha) \]

In reasoning

- It means knowledge is always \textit{incremental}
- \textbf{Not suitable} when facing uncertainty
Reasoning under Uncertainty

In the logic landscape

- Shares aims of non-classical logics
- But does not reject classical reasoning
- Builds on classical logic, extending it
Reasoning under Uncertainty

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Two fundamental aspects

- Ampliativeness and defeasibility
Reasoning under Uncertainty

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Two fundamental aspects

- Ampliativeness and defeasibility

Happens at three levels (at least)

- Object, entailment and meta-reasoning levels
Ampliative Aspect of Uncertainty

Allowing more conclusions by venturing beyond what is known

Default reasoning

- Jumping to conclusions: $\mathcal{T} \not\models \neg \alpha$, $\therefore \mathcal{T} \cup \{\alpha\}$ OK

- E.g.: negation as failure, closed-world assumption
Ampliative Aspect of Uncertainty

Allowing more conclusions by venturing beyond what is known

Default reasoning

▶ Jumping to conclusions: $T \not\models \neg \alpha$, $\therefore T \cup \{\alpha\}$ OK
▶ E.g.: negation as failure, closed-world assumption

Abductive reasoning

▶ Finding tentative explanations: $T \not\models \alpha$, $T \cup ? \models \alpha$
▶ E.g.: diagnosis, forensics
Ampliative Aspect of Uncertainty

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Abductive reasoning

- Finding tentative explanations: $\mathcal{T} \not\models \alpha$, $\mathcal{T} \cup ? \models \alpha$
- E.g.: diagnosis, forensics

Inductive reasoning

- Making generalizations: $P(a), P(b), P(c), \ldots$, $\therefore \forall x. P(x)$ OK
- E.g.: physical laws, stereotypes
Defeasible Aspect of Uncertainty

Allowing less conclusions by disregarding or blocking some of them

Retractive reasoning

▶ Withdrawing conclusions already derived

\[ \alpha \in Cn(T) \quad \rightarrow \quad \alpha \notin Cn(T) \]

▶ Ex.: ontology change, dialectics
Defeasible Aspect of Uncertainty

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Retractive reasoning

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- Ex.: ontology change, dialectics

Preemptive reasoning

- Preventing the derivation of some conclusion

\[ \gamma \rightarrow \alpha, \; \alpha \rightarrow \beta, \; \text{not} \; (\gamma \rightarrow \beta) \]

- Ex.: special cases in taxonomies, exceptions in regulations
Central Research Question in Uncertainty

How to sanction more conclusions and how to sanction fewer of them
Central Research Question in Uncertainty

How to sanction more conclusions and how to sanction fewer of them
Uncertainty at the Object Level

Logical symbols of the language

- Connectives can behave nonmonotonically

Nonmonotonic version of material implication '$\alpha \rightarrow \beta$';

Ampliative aspect: $\alpha; \beta$ holds even if $\alpha \rightarrow \beta$ doesn't;

Defeasible (preemptive) aspect: $\alpha; \beta$ is the case but $\alpha \land \gamma; \beta$ not

Negation in logic programming

- Ampliative and retractive
Uncertainty at the Object Level

Logical symbols of the language

- Connectives can behave nonmonotonically

Nonmonotonic version of material implication ‘$\rightsquigarrow$’

- Ampliative aspect: $\alpha \rightsquigarrow \beta$ holds even if $\alpha \rightarrow \beta$ doesn’t

\[ \textit{meningitis} \rightsquigarrow \neg \textit{fatal} \]

- Defeasible (preemptive) aspect: $\alpha \rightsquigarrow \beta$ is the case but $\alpha \land \gamma \rightsquigarrow \beta$ not

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Nonmonotonic version of material implication ‘\(\sim\)’

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Negation in logic programming

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Uncertainty at the Entailment Level

Sanctioned inferences or reasoning

- Entailment $\models$ behaves nonmonotonically
Uncertainty at the Entailment Level

Sanctioned inferences or reasoning

- Entailment $\models$ behaves nonmonotonically

Nonmonotonic version of $\models$

- Historically the most extensively studied
- Ampliative aspect: we may have $\{\alpha\} \models \beta$ even if $\{\alpha\} \not\models \beta$

$$\{\text{hasMeningitis}(\text{mary})\} \models \text{AntiBioOK}$$

- Defeasible (retractive) aspect: $\{\alpha\} \models \beta$ is the case but $\{\alpha, \gamma\} \not\models \beta$

$$\{\text{hasMeningitis}(\text{mary}), \text{pregnant}(\text{mary})\} \not\models \text{AntiBioOK}$$
Uncertainty at the Meta-reasoning Level

Reasoning about sanctioned inferences

- Nonmonotonicity happens ‘outside’ the logic

Theory change

- Theory expansion: make sure $\alpha \in \text{Cn}(\mathcal{T})$
- Theory contraction: make sure $\alpha \notin \text{Cn}(\mathcal{T})$
Uncertainty at the Meta-reasoning Level

Reasoning about sanctioned inferences

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Theory change

- Theory expansion: make sure $\alpha \in Cn(T)$
- Theory contraction: make sure $\alpha \not\in Cn(T)$

Ampliative aspect

- Usually more ‘conservative’: primacy of new information
- Even when not conservative, not venturous enough: minimal change
- New information must follow classically from the new theory
Outline

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Where Are We?

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Conclusion
Standard Logical Frameworks for Uncertainty

- Conditional logics
- Default logic
- Circumscription
- Autoepistemic logic
- AGM belief revision
- Ontology evolution
- Abstract argumentation frameworks
- Dynamic epistemic logic
- Adaptive logics
- Preferential logics
- ...
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Ivan Varzinczak (CAIR) Multifarious Uncertainty in Ontologies
Default Logic

Motivation

- Account of conclusions by default (based on absence of knowledge)
- Default rules of the form

$$\alpha : \beta, \neg \gamma \quad hasMeningitis(mary) : AntiBioOK, \neg pregnant(mary)$$

$$\beta \quad AntiBioOK$$
Default Logic

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▶ Account of conclusions by default (based on absence of knowledge)

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Operational semantics

▶ Notion of extension: ‘closure’ of default rules

▶ Related to negation as failure
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- Account of conclusions by default (based on absence of knowledge)
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\alpha : \beta, \neg \gamma \quad \text{hasMeningitis(mary)} : \text{AntiBioOK}, \neg \text{pregnant(mary)}
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Operational semantics

- Notion of *extension*: ‘closure’ of default rules
- Related to negation as failure

Aspects and levels

- Both ampliative and defeasible (retractive)
- Only at the entailment level
Circumscription

Motivation

▶ Assumption that everything is normal by default
▶ Exceptional cases should be minimized
Circumscription

Motivation

- Assumption that everything is normal by default
- Exceptional cases should be minimized

Semantic intuition

- Minimize the extension of predicates (different policies)
- Look at some models of the premises
- \( \alpha \models_{\text{Circ}(\gamma)} \beta \) if the \( \gamma \)-minimized \( \alpha \)-models are \( \beta \)-models
- E.g. minimize extension of *pregnant* to infer *AntiBioOK*
Circumscription

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AGM Belief Revision

Motivation

- Account of theory change
- Additions and removals of theorems
- Several guiding principles (postulates), e.g. minimal change
AGM Belief Revision

Motivation

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- Additions and removals of theorems
- Several guiding principles (postulates), e.g. minimal change

Approaches and construction methods

- Belief bases and belief sets
- Partial-meet, kernels, system of spheres, etc.
AGM Belief Revision

Motivation

- Account of theory change
- Additions and removals of theorems
- Several guiding principles (postulates), e.g. minimal change

Example

\[ \mathcal{K} = \{ \ldots, \exists \text{hasDisease.} \text{Menin(mary)}, \ldots \} \models \exists \text{hasDisease.} \neg \text{Fatal(mary)} \]

Revise \( \mathcal{K} \) with \( \exists \text{hasDisease.} \text{BacMenin(mary)} \)
AGM Belief Revision

Motivation

- Account of *theory change*
- Additions and removals of *theorems*
- Several guiding principles (*postulates*), e.g. *minimal change*

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\[ K = \{ \ldots, \exists \text{hasDisease. Menin}(\text{mary}), \ldots \} \models \exists \text{hasDisease. } \neg \text{Fatal}(\text{mary}) \]

Revise \( K \) with \( \exists \text{hasDisease. BacMenin}(\text{mary}) \)

Aspects and levels

- Both *ampliative* and *defeasible* (retractive)
- Only at the *metareasoning level*
Ontology Evolution

Motivation

- Ontology revision and repair
- Essentially the same as AGM …
- …but from a different angle (more ‘operational’)
Ontology Evolution

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Approaches

- Justifications, MUPs, etc.
- Repairs
Ontology Evolution

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Approaches

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Aspects and levels

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Dynamic Epistemic Logic(s)

Motivation

- Logic of (group) knowledge change
- Information flows via informative events:

\[ \text{hasDisease.BacMenin(mary)!} \]
Dynamic Epistemic Logic(s)

Motivation

▶ Logic of (group) knowledge change
▶ Information flows via informative events:

\[
\text{[hasDisease}.\text{BacMenin(mary)}!\text{]}
\]

Semantic intuition

▶ Epistemic possibilities held by multiple agents
▶ Model transformations: \( \mathcal{M}_E \otimes \mathcal{M}_A \Rightarrow \mathcal{M}'_E \)

\[
\mathcal{M}_E \text{hasDisease}.\text{Menin(mary)} \otimes \mathcal{M}_A \text{hasDisease}.\text{BacMenin(mary)} \Rightarrow \mathcal{M}_E \text{hasDisease}.\text{Fatal(mary)}
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Dynamic Epistemic Logic(s)

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- Logic of (group) knowledge change
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\[\mathcal{M}_E \text{hasDisease.Menin(mary)} \otimes \mathcal{M}_A \text{hasDisease.BacMenin(mary)} \Rightarrow \mathcal{M}_E \text{hasDisease.Fatal(mary)}\]

Aspects and levels

- Only defeasible (retractive): \(\neg K\alpha \rightarrow [\alpha!]\neg K\alpha\) not valid
- Only at the object level
Preferential Logics and Rational Closure (KLM)

Motivation

- Nonmonotonic conditional $\leadsto$ satisfying rationality properties:

  $\begin{align*}
  \text{(Ref)} & \quad \alpha \leadsto \alpha \\
  \text{(LLE)} & \quad \frac{\alpha \equiv \beta, \alpha \leadsto \gamma}{\beta \leadsto \gamma} \\
  \text{(And)} & \quad \frac{\alpha \leadsto \beta, \alpha \leadsto \gamma}{\alpha \leadsto \beta \land \gamma} \\
  \text{(Or)} & \quad \frac{\alpha \leadsto \gamma, \beta \leadsto \gamma}{\alpha \lor \beta \leadsto \gamma} \\
  \text{(RW)} & \quad \frac{\alpha \leadsto \beta, \models \beta \rightarrow \gamma}{\alpha \leadsto \gamma} \\
  \text{(CM)} & \quad \frac{\alpha \leadsto \beta, \alpha \leadsto \gamma}{\alpha \land \gamma \leadsto \beta} \\
  \text{(RM)} & \quad \frac{\alpha \leadsto \beta, \alpha \not\leadsto \gamma}{\alpha \land \gamma \leadsto \beta}
  \end{align*}$
Preferential Logics and Rational Closure (KLM)

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- Nonmonotonic conditional $\sim$ satisfying rationality properties:

- Semantic intuition

  - Extra structure: preference relation on worlds
  - Notion of minimal entailment à la circumscription
  - Different strategies: prototypical and presumptive reasoning
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(\text{RM}) & \quad \frac{\alpha \sim \beta, \alpha \land \neg \gamma}{\alpha \land \gamma \sim \beta}
\end{align*}
$$

Semantic intuition

- Extra structure: *preference relation* on worlds
- Notion of *minimal entailment à la circumscription*
- Different strategies: *prototypical* and *presumptive* reasoning

Aspects and levels

- Both *ampliative* and defeasible (preemptive and retractive)
- Only at the *object* and *entailment* levels
Preferential DLs

Defeasible subsumption

- E.g. Menin $\sqsubseteq \neg$ Fatal, BacMenin $\sqsubseteq$ Fatal

- Properties

\[
\begin{align*}
\text{(Cons)} & \quad \top \not\sqsubseteq \bot \\
\text{(Ref)} & \quad C \sqsubseteq C \\
\text{(LLE)} & \quad \frac{C \equiv D, \ C \sqsubseteq E}{D \sqsubseteq E} \\
\text{(And)} & \quad \frac{C \sqsubseteq D, \ C \sqsubseteq E}{C \sqsubseteq D \cap E} \\
\text{(Or)} & \quad \frac{C \sqsubseteq E, \ D \sqsubseteq E}{C \sqcup D \sqsubseteq E} \\
\text{(RW)} & \quad \frac{C \sqsubseteq D, \ D \sqsubseteq E}{C \sqsubseteq E} \\
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\]

Typicality operator

- E.g. $\mathbf{T}(\text{Menin}) \sqsubseteq \neg \text{Fatal}$, $\mathbf{T}(\text{BacMenin}) \sqsubseteq \text{Fatal}$
Preferential DLs

Semantics

- Enriched DL Interpretations $\mathcal{P} := \langle \Delta^\mathcal{I}, \cdot^\mathcal{I}, \preceq \rangle$
- $\Delta^\mathcal{I}$ and $\cdot^\mathcal{I}$ as before
- $\preceq$ is a preference (or normality) relation

$\mathcal{I}$:  
\[
\begin{align*}
    x_1 \{A\} & \quad x_2 \{A, B\} & \quad x_3 \{A, B\} & \quad x_4 \{B\} & \quad x_5 \{\} \\
    \end{align*}
\]

$\mathcal{P}$:  
\[
\begin{align*}
    (T(C))^\mathcal{P} & = \min_{\preceq} C^\mathcal{P} \\
    \mathcal{P} \models C \subseteq D \iff \min_{\preceq} C^\mathcal{P} \subseteq D^\mathcal{P} \\
\end{align*}
\]

Representation results: soundness and completeness of postulates
# Existing Frameworks: Summary

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Existing Frameworks: Summary

Not all *levels* and *aspects* have been dealt with!
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The Need for Multifarious Uncertainty

Other nonmonotonic logical symbols

- Nonmonotonic connectives
- Nonmonotonic modalities and quantifiers
The Need for Multifarious Uncertainty

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New nonmonotonic consequence relations

- Levels of venturousness
- Links with abduction, induction and other forms of reasoning
The Need for Multifarious Uncertainty

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More powerful accounts of theory change

- Languages that are more expressive
- With nonmonotonic connectives
The Need for Multifarious Uncertainty

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New and more general theories of uncertainty are called for
Uncertainty in Concept Constructors

Defeasible disjointness

- Normally mutually exclusive

\[ \text{BacMenin} \sqsubseteq \neg \text{ViralMen} \text{ 'OR' } \text{ViralMen} \sqsubseteq \neg \text{BacMenin} \]
Uncertainty in Concept Constructors

Defeasible disjointness

- Normally mutually exclusive

  \[ \text{BacMenin} \nsubseteq \neg \text{ViralMen} \text{ ‘OR’ ViralMen} \nsubseteq \neg \text{BacMenin} \]

Defeasible equivalence

- Normally equivalent (similar, analogous?)

  \[ \text{Cortisol} \cong \text{Dexamethasone} \]
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Other layers of typicality

- Talk about other levels

\[ T_1(\text{Menin}), \ T_2(\text{Menin}), \ldots, \ T_n(\text{Menin}) \]
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Typicality for roles

- Some relations are more normal than others

\[ T(\text{infectedBy}), \ T(\text{marriedTo}) \]
Uncertainty in Concept Constructors

Nonmonotonic role restrictions

- Normal role fillers

\[ \text{Skull} \equiv \forall \text{madeOf}.\text{Bone} \]
Uncertainty in Concept Constructors

Nonmonotonic role restrictions

» Normal role fillers

\[ \text{Skull} \equiv \forall \text{madeOf}.\text{Bone} \]

Defeasible role subsumption

» Normal relationship between roles

\[ \text{parentOf} \sqsubseteq \text{progenitorOf} \]
Uncertainty in Concept Constructors

Nonmonotonic role restrictions

- Normal role fillers

\[ \text{Skull} \equiv \forall \text{madeOf}. \text{Bone} \]

Defeasible role subsumption

- Normal relationship between roles

\[ \text{parentOf} \sqsubseteq \text{progenitorOf} \]

Defeasible role properties

- Holding in the most normal cases

\[ \text{marriedTo} : \text{usually functional}, \quad \text{partOf} : \text{usually transitive} \]
Uncertainty in Entailment

Allow for extra expressivity

- Transfer propositional constructions to DLs, modal logics, etc
Uncertainty in Entailment

Allow for extra expressivity

- Transfer propositional constructions to DLs, modal logics, etc

But also make use of it

- Extra postulates beyond the Boolean ones
- Further semantic constraints

- (Naïve) Existential Restriction Introduction

\[
C \sqsubseteq D \\
\exists r. C \sqsubseteq \exists r. D
\]
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\[
C \sqsubseteq D \\
\exists r. C \sqsubseteq \exists r. D
\]

\[\checkmark\]
BacMenin \sqsubseteq \text{Fatal}

\[\exists \text{hasDisease}.\text{BacMenin} \sqsubseteq \exists \text{hasDisease}.\text{Fatal}\]

\[\times\]
Menin \sqsubseteq \neg \text{Fatal}

\[\exists \text{dieOf}.\text{Menin} \sqsubseteq \exists \text{dieOf}.\neg \text{Fatal}\]
Uncertainty in Entailment

Make use of extra expressivity

- **Existential Restriction Introduction**

  \[
  C \sqsubseteq D \\
  \exists r. \mathbf{T}(C) \sqsubseteq \exists r. D
  \]

  \[
  \checkmark \\
  \text{Menin} \sqsubseteq \neg \text{Fatal} \\
  \exists \text{hasDisease. } \mathbf{T}(\text{Menin}) \sqsubseteq \exists \text{hasDisease. } \neg \text{Fatal}
  \]

- **Value Restriction Introduction**

  \[
  C \sqsubseteq D \\
  \forall r. \mathbf{T}(C) \sqsubseteq \forall r. D
  \]
Uncertainty in Entailment

Make use of extra expressivity

- Rational Existential Monotonicity

\[
\begin{align*}
\text{(REM)} & \quad \exists r. C \subseteq \exists r. D, \quad \exists r. C \not\subseteq \neg T(\exists r. E) \\
& \quad \exists r. (C \cap E) \subseteq \exists r. D
\end{align*}
\]

- Rational Value Monotonicity

\[
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\end{align*}
\]
Uncertainty in Entailment

Instance-level ampliative and defeasible reasoning

Let the TBox

\[ \mathcal{T} = \begin{cases} 
    \text{BacMenin} \sqsubseteq \text{Menin}, \\
    \text{Menin} \sqsubseteq \neg \text{Fatal}, \\
    \text{BacMenin} \sqsubseteq \text{Fatal} 
\end{cases} \]
Uncertainty in Entailment

Instance-level ampliative and defeasible reasoning

Let the TBox

\[ \mathcal{T} = \left\{ \begin{array}{l} \text{BacMenin} \sqsubseteq \text{Menin}, \\ \text{Menin} \sqsubseteq \neg \text{Fatal}, \\ \text{BacMenin} \sqsubseteq \text{Fatal} \end{array} \right\} \]

If we learn hasDisease.\text{Menin(mary)} \ldots
Uncertainty in Entailment

Instance-level ampliative and defeasible reasoning

- Let the TBox

\[ \mathcal{T} = \{ \begin{array}{l}
BacMenin \sqsubseteq Menin, \\
Menin \sqsubseteq \neg Fatal, \\
BacMenin \sqsubseteq Fatal
\end{array} \} \]

- If we learn `hasDisease.Menin(mary)` ...

- ...it is plausible to (defeasibly) conclude `hasDisease.\neg Fatal(mary)` ...
Uncertainty in Entailment

Instance-level ampliative and defeasible reasoning

▶ Let the TBox

\[ T = \begin{cases} 
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▶ If we learn hasDisease.Menin(mary) …

▶ …it is plausible to (defeasibly) conclude hasDisease.\neg\text{Fatal}(mary) …

▶ …allowing for retracting it if we learn hasDisease.BacMenin(mary)
Uncertainty in Entailment

Instance-level ampliative and defeasible reasoning

▶ Let the TBox

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\end{cases} \]

▶ If we learn \text{hasDisease.Menin}(\text{mary}) \ldots

▶ ...it is plausible to (defeasibly) conclude \text{hasDisease.\neg Fatal}(\text{mary}) \ldots

▶ ...allowing for retracting it if we learn \text{hasDisease.BacMenin}(\text{mary})

Different levels of ‘venturousness’

▶ Skeptical, credulous, and in between
Uncertainty in Entailment

Standard constructions

- **Usually:** Strengthening the premises or relaxing the conclusions

\[ \mathcal{K} \models \alpha \text{ iff } \downarrow \text{Mod}(\mathcal{K}) \subseteq \text{Mod}(\alpha) \]

\[ \mathcal{K} \models \alpha \text{ iff } \text{Mod}(\mathcal{K}) \subseteq \uparrow \text{Mod}(\alpha) \]

- **Usually:** \( \downarrow \text{Mod}(\mathcal{K}) = \) the most preferred \( \mathcal{K} \)-worlds
Uncertainty in Entailment

Standard constructions

- Usually: **Strengthening** the premises or **relaxing** the conclusions

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- Usually: \( \downarrow \text{Mod}(\mathcal{K}) \) = the most preferred \( \mathcal{K} \)-worlds

Beyond standard constructions

- Go beyond the **dichotomy** “preferred v. non-preferred”
- Look for other notions of **preferences** and **minimality**
- New forms of reasoning beyond induction and abduction?
Enhanced Theory Change

Beyond propositional languages

- Modal logics, description logics, ...

\[ \mathcal{T} \star \Box \alpha \?, \quad \mathcal{T} \star (C \subseteq D) \?, \quad \mathcal{T} \rightarrow r(a, b) \?

- Also fragments thereof (Horn, $\mathcal{EL}$, etc.)
Enhanced Theory Change

Beyond propositional languages

- Modal logics, description logics, …

\[ T \star \Box \alpha ? , \quad T \star (C \sqsubseteq D) ? , \quad T - r(a, b) ? \]

- Also fragments thereof (Horn, \( \mathcal{EL} \), etc.)

Beyond classical constructors

- Makes sense for languages with nonmonotonic connectives
- A whole family of AGM-like new postulates
- Links with various \( \bowtie \)
Desiderata for a General Framework

Remember

- The two aspects and the three levels
Desiderata for a General Framework

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- The two aspects and the three levels

But we also want a framework that

- accounts for languages of various expressive power
- has good balance between expressivity and computational complexity
- is general yet elegant
- can serve as a core formalism for further extensions
- abides by principles of software ergonomics (usability)
Desiderata for a General Framework

Remember

- The two *aspects* and the three *levels*

But we also want a framework that

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- has *good balance* between expressivity and computational complexity
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- can serve as a *core formalism* for further extensions
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Principle

- Not to diverge from existing approaches, rather *build on* them
What to Use as a Springboard?

Several frameworks available

- In principle, anyone can serve as the **basis** for such constructions
What to Use as a Springboard?

Several frameworks available

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Promising starting point

- Formalisms general enough in the propositional case: Preferential
What to Use as a Springboard?

Several frameworks available

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Promising starting point

- Formalisms general enough in the propositional case: Preferential

Why?

- Provides a general proof-theoretic characterization of $\sim$ (and $\not\sim$)
- Basis for nonmonotonic entailment $\approxeq$, e.g. rational closure
- Links with AGM belief revision (inter-definability)
- Simple and elegant (cf. our desiderata)
- Recently extended to modal and description logics
Outline

Overview

Where Are We?

Where To?

Conclusion
Conclusion

What we have seen

- Uncertainty has **two aspects**: ampliative and defeasible
- It happens at **three levels**: object, entailment, and meta-reasoning
- There are still many **open issues** not fully addressed
- There is a need for **more general theories**
- We saw some **possible directions to pursue**
Conclusion

What we have seen

- Uncertainty has two aspects: ampliative and defeasible
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Immediate next steps

- A thorough investigation of uncertainty in the object language
- Study of corresponding appropriate entailment relations
- Assessment of what theory change would mean in these contexts
Conclusion

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Thank you!