A non-monotonic DL for reasoning about typicality

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Description Logics (DLs)

- One of the most important formalism for knowledge representation
- First Order Logic (FOL) based formal semantics
- Good trade-off between expressivity and reasoning complexity
- Underpinning many real systems and languages (*e.g.*, OWL)

Description Logics And Typicality

- DLs encode taxonomy using TBox axioms, and properties either hold or do not hold for a class as a whole
- Real world scenarios requires to express typical/default (but not necessary) properties for a given class (not possible in DLs without extensions)
- Default properties may lead to overgeneralization, addressed using inheritance exceptions mechanisms (for subclasses)

Prototypical Property Example

- "Normally, a department member has lunch at the restaurant"
- We need a typicality operator **T** for expressing it:
  $\mathbf{T}(DepartmentMember) \sqsubseteq LunchAtRestaurant$
- DLs are monotone, while **T** is inherently non-monotone:
  $\mathbf{T}(DepartmentMember) \sqsubseteq LunchAtRestaurant$
  $\mathbf{T}(DepartmentMember \sqcap TemporaryWorker) \sqsubseteq \neg LunchAtRestaurant$
  $\mathbf{T}(DepartmentMember \sqcap TemporaryWorker \sqcap \exists Owns.RestaurantTicket) \sqsubseteq LunchAtRestaurant$
- We need $C \sqsubseteq D \not\Rightarrow \mathbf{T}(C) \sqsubseteq \mathbf{T}(D)$

Monotonic vs non-monotonic reasoning

- Monotonicity: adding new knowledge does not reduce the entailment set
- Monotonic reasoning is computationally and conceptually simpler
- Non-monotonic aspects arise when dealing with advanced aspects, such as updates (*e.g.*, belief revision), defaults etc.

Research Question:

How can prototypical properties be formally represented in order to reason about them using a Description Logic?

Literature Overview

Dealing with defeasible inheritance and non-monotonic inference requires the integration of DLs with non-monotonic reasoning formalisms:

- DLs + default [BH95]
- DLs + epistemic operators [DNR02, MR10, KS08]
- DLs + ASP [ELST04]
- DLs + circumscription [BLW09, BFS11]
- DLs + rational closure [CS10]
- DLs + preferential subsumption (rational logic R) [BHM08]

This work applies a model-theoretic approach (minimal models on the basis of a preferential logic).

Logic $\mathcal{ALC} + \mathbf{T}$

- $\Sigma = \mathcal{C} \cup \mathcal{R} \cup \mathcal{O}$ (concepts, roles, individuals)
- $\mathcal{L}^{\Sigma}$ (language over Σ):
    - $\top, \bot, A \in \mathcal{C}$, and if $C, D \in \mathcal{L}^{\Sigma}, R \in \mathcal{R}$, then $C \sqcap D$, $C \sqcup D$, $\neg C$, $\forall R.C$, $\exists R.C \in \mathcal{L}^{\Sigma}$ (standard $\mathcal{ALC}$ concept expressions)
    - if $C \in \mathcal{L}^{\Sigma}$, then C and $\mathbf{T}(C)$ are extended concepts, as well as boolean combinations of extended concepts
- $KB = \langle \mathcal{T}, \mathcal{A} \rangle$ (TBox, ABox, resp.)
    - TBox: $C \sqsubseteq D$, C extended concept, D concept
    - ABox: $C(a)$, $R(a, b)$, C extended concept, $R \in \mathcal{R}$, and $a, b \in \mathcal{O}$

Intuitively, $\mathbf{T}$ selects the "most typical" element(s) of a class.

$\mathcal{ALC} + \mathbf{T}$ Semantics

- Extended concept aside, it coincides with classic FOL semantics for $\mathcal{ALC}$
- *Unique Name Assumption* (different individual constants interpreted with different domain elements)
- $\mathbf{T}$ semantics based on *preference relation* $<$ over domain Δ, that is partial and global (typicality is class unaware)
- $<$ is irreflexive, transitive and well-founded (no infinite descending chains):
  1. for every non empty set $S \subseteq \Delta$, a minimum always exists (possibly not unique): $Min_<(S) = \{x \in S \mid \nexists y \in S \ . \ y < x\}$
  2. if $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ s.t. $y < x$
- $\mathbf{T}(C)^\mathcal{I} = Min_<(C^\mathcal{I})$

Model and Satisfiability

Model $\mathcal{M} = \langle \Delta, \mathcal{I}, < \rangle$ *satisfies*:

- a TBox $\mathcal{T}$, if for any $C \sqsubseteq D \in \mathcal{T}$, $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$,
- an ABox $\mathcal{A}$, if for any $C(a)$ (resp. $R(a, b)$) $\in \mathcal{A}$, if $a^{\mathcal{I}} \in A^{\mathcal{I}}$ (resp. $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$).
- a $KB = \langle \mathcal{T}, \mathcal{A} \rangle$, if it satisfies both $\mathcal{T}$ and $\mathcal{A}$

A query F of the form $C(a)$, C an extended concept, is entailed by an $\mathcal{ALC} + \mathbf{T}$ KB, $KB \models_{\mathcal{ALC}+\mathbf{T}} F$, iff F holds in any model satisfying KB.

Modal Formulation of Typicality

- $x \in \mathbf{T}(C)^{\mathcal{I}}$ iff (1) $x \in C^{\mathcal{I}}$ and (2) $\nexists y \in C^{\mathcal{I}} \ . \ y < x$
- $(\Box C)^{\mathcal{I}} = \{x \in \Delta \mid \forall y \in \Delta \ . \ y < x \implies y \in C^{\mathcal{I}}\}$
- $(\Box \neg C)^{\mathcal{I}} = \{x \in \Delta \mid \forall y \in \Delta \ . \ y < x \implies y \in \neg C^{\mathcal{I}}\}$, this implies that each x is "a most typical" element of C, given that preferable elements (w.r.t. $<$) are not in $C^{\mathcal{I}}$
- Condition (2) is then equivalent to $x \in (\Box \neg C)^{\mathcal{I}}$
- Therefore, $x \in \mathbf{T}(C)^{\mathcal{I}}$ iff $x \in (C \sqcap \Box \neg C)^{\mathcal{I}}$

Inheritance exception, non-monotonic features (Example)

1. $\mathcal{T} = \{\mathbf{T}(DepartmentMember) \sqsubseteq LunchAtRestaurant$
2. $\mathbf{T}(DepartmentMember \sqcap TempResearcher) \sqsubseteq \neg LunchAtRestaurant$
3. $\mathbf{T}(DepartmentMember \sqcap TempResearcher \sqcap \exists Owns.TicketRestaurant) \sqsubseteq LunchAtRestaurant\}$

$\mathcal{A} = \{\mathbf{T}(DepartmentMember \sqcap TempResearcher \sqcap \exists Owns.TicketRestaurant)(greg)\}$
$\mathcal{A}' = \{(DepartmentMember \sqcap TempResearcher \sqcap \exists Owns.TicketRestaurant)(greg)\}$

- $\langle \mathcal{T}, \mathcal{A} \rangle \models_{\mathcal{ALC}+\mathbf{T}} LunchAtRestaurant(greg)$
- $\langle \mathcal{T}, \mathcal{A}' \rangle \not\models_{\mathcal{ALC}+\mathbf{T}} LunchAtRestaurant(greg)$

Non-monotonic extension: Logic $\mathcal{ALC} + \mathbf{T}_{min}$

- $\mathcal{ALC} + \mathbf{T}_{min}$ considers only *minimal models* for (non-monotonic) inference
- minimized quantity is "concept atypicality", that is, the number of atypical instances of a given set of concepts $\mathcal{L}_{\mathbf{T}}$
- Atypical instances:
  $(\neg\Box\neg C)^{\mathcal{I}} = \{x \in \Delta \mid \exists y \in \Delta \;.\; y < x \wedge y \in C^{\mathcal{I}}\})$
- more formally, we aim at minimizing, for a given model $\mathcal{M} = \langle \Delta, \mathcal{I}, < \rangle$, the cardinality of $\mathcal{M}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}} = \{x \mid x \in \neg\Box\neg C^{\mathcal{I}} \wedge x \in \Delta \wedge C \in \mathcal{L}_{\mathbf{T}}\}$

Minimal and preferred models

Given two models $\mathcal{M} = \langle \Delta_{\mathcal{M}}, \mathcal{I}_{\mathcal{M}}, <_{\mathcal{M}} \rangle$ and $\mathcal{N} = \langle \Delta_{\mathcal{N}}, \mathcal{I}_{\mathcal{N}}, <_{\mathcal{N}} \rangle$, $\mathcal{M}$ is preferred to $\mathcal{N}$ w.r.t. $\mathcal{L}_{\mathbf{T}}$, denoted as $\mathcal{M} <_{\mathcal{L}_{\mathbf{T}}} \mathcal{N}$, if:

1. $\Delta_{\mathcal{M}} = \Delta_{\mathcal{N}}$,
2. $\forall a \in \mathcal{O} \; . \; a^{\mathcal{I}_{\mathcal{M}}} = a^{\mathcal{I}_{\mathcal{N}}}$,
3. $\mathcal{M}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}} \subset \mathcal{N}^{\Box^-}_{\mathcal{L}_{\mathbf{T}}}$.

A model $\mathcal{M}$ is a *minimal model* for a KB (w.r.t. to $\mathcal{L}_{\mathbf{T}}$), if it is a model for *KB* and no other model $\mathcal{M}'$ exists s.t. $\mathcal{M}' <_{\mathcal{L}_{\mathbf{T}}} \mathcal{M}$.

Minimal entailment in $\mathcal{ALC} + \mathbf{T}_{min}$

- Queries are of the form $C(a)$, with C an extended concept and $a \in \mathcal{O}$
- Given an $\mathcal{ALC} + \mathbf{T}_{min}$ KB with model $\mathcal{M}$, query $F = C(a)$ holds in $\mathcal{M}$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$.
- F is *minimally entailed* from KB w.r.t. $\mathcal{L}_{\mathbf{T}}$, denoted as $KB \models_{min}^{\mathcal{L}_{\mathbf{T}}} F$, if it holds in any minimal model of KB
- In case of conflict, typicality in the more specific concept is preferred

Specificity Example (1/3)

TBox $\mathcal{T}$ composed by:

1. $\mathbf{T}(DepartmentMember) \sqsubseteq LunchAtRestaurant$
2. $\mathbf{T}(DepartmentMember \sqcap TemporaryResearcher) \sqsubseteq \neg LunchAtRestaurant$

- For ABox $\mathcal{A} = \{DepartmentMember(greg), TemporaryResearcher(greg)\}$ we have that $\langle \mathcal{T}, \mathcal{A} \rangle \models^{\mathcal{L}_{\mathbf{T}}}_{min} \neg LunchAtRestaurant(greg)$ holds.
- In all the minimal models, $greg^{\mathcal{I}} \in \mathbf{T}(DepartmentMember \sqcap TemporaryResearcher)^{\mathcal{I}}$, and $greg^{\mathcal{I}} \not\in \mathbf{T}(DepartmentMember)^{\mathcal{I}}$, because they are in contrast and the former ensures minimality.
- Intuitively, minimality has the side-effect of preferring typicality in the more specific concept.

Specificity Example (2/3)

Model $\mathcal{M}_1$ (minimal, one negated box):

1. $DepartmentMember(greg) \sqcap TemporaryResearcher(greg)$
2. $\mathbf{T}(DepartmentMember \sqcap TemporaryResearcher)(greg)$
3. $\neg LunchAtRestaurant(greg)$
4. $\neg\mathbf{T}(DepartmentMember)(greg)$
5. $(\neg\Box\neg DepartmentMember)(greg)$

1. Given $DepartmentMember(greg)$, 5 requires x s.t. $x < greg$ and $DepartmentMember(x)$, so $\mathbf{T}(DepartmentMember)(x)$

Specificity Example (3/3)

Model $\mathcal{M}_2$ (two negated boxes):

1. $DepartmentMember(greg) \sqcap TemporaryResearcher(greg)$
2. $\neg \mathbf{T}(DepartmentMember \sqcap TemporaryResearcher)(greg)$
3. $(\neg \Box \neg (DepartmentMember \sqcap TemporaryResearcher))(greg)$

- 3 requires x s.t. $x < greg$ and $\mathbf{T}(DepartmentMember \sqcap TemporaryResearcher)(x)$, for consistency $\neg \mathbf{T}(DepartmentMember)(x)$
- $(\neg \Box \neg DepartmentMember)(x)$ requires y s.t. $y < x$ and $\mathbf{T}(DepartmentMember)(y)$
- $y < x \wedge x < greg \implies y \neq greg$, and therefore also $\neg \mathbf{T}(DepartmentMember)(greg)$

Tableaux calculus for $\mathcal{ALC} + \mathbf{T}_{min}$

- $\mathcal{TAB}_{min}^{\mathcal{ALC}+\mathbf{T}}$ two-phase, sound and complete tableau calculus for deciding query (F) minimal entailment, given KB
- $\mathcal{TAB}_{min}^{\mathcal{ALC}+\mathbf{T}} = \mathcal{TAB}_{PH1}^{\mathcal{ALC}+\mathbf{T}} + \mathcal{TAB}_{PH2}^{\mathcal{ALC}+\mathbf{T}}$
- $\mathcal{TAB}_{PH1}^{\mathcal{ALC}+\mathbf{T}}$ tries to build models (open branches) for $KB \cup \{\neg F\}$
- $\mathcal{TAB}_{PH2}^{\mathcal{ALC}+\mathbf{T}}$ chases the models of $\mathcal{TAB}_{PH1}^{\mathcal{ALC}+\mathbf{T}}$, trying to build a "smaller" one

Tableau phase 1

- Tableau is a tree having nodes of the form $\langle S, U\rangle$, where S is a set of constraints, and U a set of labeled concept inclusions (subsumption relations in the TBox, labelled using variables in $\mathcal{V}$)
- Each branch is a sequence of nodes $\langle S_1, U_1\rangle, \ldots, \langle S_n, U_n\rangle$, with $n \geq 0$, where $\langle S_i, U_i\rangle$ is obtained by $\langle S_{i-1}, U_{i-1}\rangle$ through rule application
- A branch is either open or closed (due to a clash)
- A tableau is closed (*i.e.*, no possible models) iff all the branches are closed
- Open branches are either saturated (no rules are applicable, it corresponds to a model) or not (model computation to be completed)

Tableau phase 1: Constraints and Formulas

- Constraint: $x \xrightarrow{R} y$, $x < y$, $x : C$, where x, y are labels, R is a role, C is either an extended concept or has the form $\Box\neg D$ or $\neg\Box\neg D$, where D is a concept
- Formula: $C \sqsubseteq D^L$, where L is a list of labels (to ensure termination)
- Initialization (tableau root node):
  - ABox $\mathcal{A}$: $S = \{a : C \mid C(a) \in \mathcal{A}\} \cup \{a \xrightarrow{R} b \mid R(a, b) \in \mathcal{A}\}$
  - TBox $\mathcal{T}$: $U = \{C \sqsubseteq D^{\emptyset} \mid C \sqsubseteq D \in \mathcal{T}\}$

$$\frac{\langle S, x : C, x : \neg C \mid U \rangle}{} \ (\text{Clash}) \qquad \frac{\langle S, x : \neg \top \mid U \rangle}{} \ (\text{Clash})_{\top} \qquad \frac{\langle S, x : \bot \mid U \rangle}{} \ (\text{Clash})_{\bot}$$

$$\frac{\langle S, x : C \sqcap D \mid U \rangle}{\langle S, x : C \sqcap D, x : C, x : D \mid U \rangle} \ (\sqcap^{+}) \quad \text{if } \{x : C, x : D\} \not\subseteq S$$

$$\frac{\langle S, x : \forall R.C, x \xrightarrow{R} y \mid U \rangle}{\langle S, x : \forall R.C, x \xrightarrow{R} y, y : C \mid U \rangle} \ (\forall^{+}) \quad \text{if } y : C \notin S$$

$$\frac{\langle S, x : \neg(C \sqcap D) \mid U \rangle}{\langle S, x : \neg(C \sqcap D), x : \neg C \mid U \rangle \qquad \langle S, x : \neg(C \sqcap D), x : \neg D \mid U \rangle} \ (\sqcap^{-}) \quad \text{if } x : \neg C \notin S \text{ and } x : \neg D \notin S$$

$$\frac{\langle S, x : C \sqcup D \mid U \rangle}{\langle S, x : C \sqcup D, x : C \mid U \rangle \qquad \langle S, x : C \sqcup D, x : D \mid U \rangle} \ (\sqcup^{+}) \quad \text{if } x : C \notin S \text{ and } x : D \notin S$$

$$\frac{\langle S, x : \neg(C \sqcup D) \mid U \rangle}{\langle S, x : \neg(C \sqcup D), x : \neg C, x : \neg D \mid U \rangle} \ (\sqcup^{-}) \quad \text{if } \{x : \neg C, x : \neg D\} \not\subseteq S$$

$$\frac{\langle S, x : \neg\neg C \mid U \rangle}{\langle S, x : \neg\neg C, x : C \mid U \rangle} \ (\neg) \quad \text{if } x : C \notin S$$

Classic $\mathcal{ALC}$ tableau rules.

$$\frac{\langle S, x : \mathbf{T}(C) \mid U \rangle}{\langle S, x : \mathbf{T}(C), x : C, x : \Box\neg C \mid U \rangle} (\mathbf{T}^+)$$
$$\text{if } \{x : C, x : \Box\neg C\} \not\subseteq S$$

$$\frac{\langle S, x : \neg\mathbf{T}(C) \mid U \rangle}{\langle S, x : \neg\mathbf{T}(C), x : \neg C \mid U \rangle \quad \langle S, x : \neg\mathbf{T}(C), x : \neg\Box\neg C \mid U \rangle} (\mathbf{T}^-)$$
$$\text{if } x : \neg C \notin S \text{ and } x : \neg\Box\neg C \notin S$$

$$\frac{\langle S \mid U \rangle}{\langle S, x : \Box\neg C \mid U \rangle \qquad \langle S, x : \neg\Box\neg C \mid U \rangle} (cut)$$
$$\text{if } x : \neg\Box\neg C \notin S \text{ and } x : \Box\neg C \notin S$$
$$C \in \mathcal{L}_{\mathbf{T}}$$
$$x \text{ occurs in } S$$

$$\frac{\langle S \mid U, C \sqsubseteq D^L \rangle}{\langle S, x : \neg C \sqcup D \mid U, C \sqsubseteq D^{L,x} \rangle} (\sqsubseteq)$$
$$\text{if } x \text{ occurs in } S \text{ and } x \notin L$$

Typicality rules.

$$\frac{\langle S, x : \exists R.C \mid U\rangle}{\langle S, x : \exists R.C, x \xrightarrow{R} y, y : C \mid U\rangle \quad \langle S, x : \exists R.C, x \xrightarrow{R} v_1, v_1 : C \mid U\rangle \quad \langle S, x : \exists R.C, x \xrightarrow{R} v_2, v_2 : C \mid U\rangle \cdots \langle S, x : \exists R.C, x \xrightarrow{R} v_n, v_n : C \mid U\rangle} \ (\exists^+)$$

y new
if $\nexists z \prec x$ s.t. $z \equiv_{S,x:\exists R.C} x$ and $\nexists u$ s.t. $x \xrightarrow{R} u \in S$ and $u : C \in S$
$\forall v_i$ occurring in S

$$\frac{\langle S, x : \neg\Box\neg C \mid U\rangle}{\langle S, x : \neg\Box\neg C, y < x, y : C, y : \Box\neg C, S^M_{x\rightarrow y} \mid U\rangle \ \langle S, x : \neg\Box\neg C, v_1 < x, v_1 : C, v_1 : \Box\neg C, S^M_{x\rightarrow v_1} \mid U\rangle \cdots \langle S, x : \neg\Box\neg C, v_n < x, v_n : C, v_n : \Box\neg C, S^M_{x\rightarrow v_n} \mid U\rangle} \ (\Box^-)$$

y new
if $\nexists z \prec x$ s.t. $z \equiv_{S,x:\neg\Box\neg C} x$ and $\nexists u$ s.t. $\{u < x, u : C, u : \Box\neg C, S^M_{x\rightarrow u}\} \subseteq S$
$\forall v_i$ occurring in $S, x \neq v_i$

Dynamic rules.

Tableau phase 1: Termination

Non-termination may be caused by:

1. rule re-application on the same premises (they are always copied in the conditions)
2. dynamic rules generate infinite many labels (infinite branches)
3. rule re-applictaion on the same formula, for the same variable

Termination is guaranteed:

1. is prevented by the side conditions of the rules
2. is prevented by the blocking technique
3. is prevented by testing the set of variables used for each formula

Solution Advantages

- All individuals are treated uniformly (minimization is also applied implicit individuals not occurring in the Abox)
- Typicality naturally addresses specificity and irrelevance. It supports defeasible reasoning in the context of inheritance with exceptions
- Instance checking, subsumption and concept satisfiability can be reduced to minimal entailment

Solution Disadvantages

- Typical birds have wings and typical birds fly: if a given bird is typical, it has both, otherwise none (a specific bird, tweety, cannot inherit only some of the typical properties of birds)
- The preference relation is "global": we cannot model the fact that y is more typical than x with respect to concept C, whereas x is more typical than y with respect to another concept D.
- Complexity of this approach is co-NExp$^{\text{NP}}$, higher than that of $\mathcal{ALC}$, and other approaches ([CS10], based on *Rational closure*, and of [MR10])

Open problems and Future Work

- One issue is the extension of the approach to more expressive DLs (up to $\mathcal{SROIQ}$/OWL2)
- Another issue is exploring alternative semantics:
  - several preference relations/tipicality operators $<_{C_i}$ associated with different concepts C_i
  - changing the relation $<$ or the preference among models, gives different semantics (*e.g.*, [BHM08] is based on rational logic R).

Thanks for your attention!

References I

- Piero A. Bonatti, Marco Faella, and Luigi Sauro, *On the complexity of $\mathcal{EL}$ with defeasible inclusions*, Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI 2011) (Barcelona, Spain) (Toby Walsh, ed.), Morgan Kaufmann, July 2011, pp. 762–767.
- F. Baader and B. Hollunder, *Embedding defaults into terminological knowledge representation formalisms*, Journal of Automated Reasoning (JAR) **14** (1995), no. 1, 149–180.

References II

- Katarina Britz, Johannes Heidema, and Thomas Meyer, *Semantic preferential subsumption*, Principles of Knowledge Representation and Reasoning: Proceedings of the Eleventh International Conference (KR 2008) (Sidney, Australia) (G. Brewka and J. Lang, eds.), AAAI Press, September 2008, pp. 476–484.
- Piero A. Bonatti, Carsten Lutz, and Frank Wolter, *The Complexity of Circumscription in DLs*, Journal of Artificial Intelligence Research (JAIR) **35** (2009), 717–773.

References III

G. Casini and U. Straccia, *Rational Closure for Defeasible Description Logics*, Proceedings of the 12th European Conference on Logics in Artificial Intelligence (JELIA 2010) (Helsinki, Finland) (T. Janhunen and I. Niemelä, eds.), Lecture Notes in Artificial Intelligence (LNAI), vol. 6341, Springer, September 2010, pp. 77–90.

F. M. Donini, D. Nardi, and R. Rosati, *Description logics of minimal knowledge and negation as failure*, ACM Transactions on Computational Logic (ToCL) **3** (2002), no. 2, 177–225.

References IV

T. Eiter, T. Lukasiewicz, R. Schindlauer, and H. Tompits, *Combining Answer Set Programming with Description Logics for the Semantic Web*, Principles of Knowledge Representation and Reasoning: Proceedings of the 9th International Conference (KR 2004) (Whistler, Canada) (D. Dubois, C.A. Welty, and M. Williams, eds.), AAAI Press, June 2004, pp. 141–151.

Laura Giordano, Valentina Gliozzi, Nicola Olivetti, and Gian Luca Pozzato, *A non-monotonic description logic for reasoning about typicality*, Artif. Intell. **195** (2013), 165–202.

References V

P. Ke and U. Sattler, *Next Steps for Description Logics of Minimal Knowledge and Negation as Failure*, Proceedings of the 21st International Workshop on Description Logics (DL2008) (Dresden, Germany) (F. Baader, C. Lutz, and B. Motik, eds.), CEUR Workshop Proceedings, vol. 353, CEUR-WS.org, May 2008.

Boris Motik and Riccardo Rosati, *Reconciling Description Logics and rules*, Journal of the ACM **57** (2010), no. 5.