Exercises on Neural Networks

1. What are the values of weights \( w_0, w_1, \) and \( w_2 \) for the perceptron whose decision surface is illustrated in the figure? Assume the surface crosses the \( x_1 \) axis at -1 and the \( x_2 \) axis at 2.

2. Design a two-input perceptron that implements the Boolean function \( A \land \neg B \). Design the two layer network of perceptrons that implements \( A \ XOR B \).

3. Consider two perceptrons \( A \) and \( B \) defined by the threshold expression \( w_0 + w_1 x_1 + w_2 x_2 > 0 \). Perceptron \( A \) has weight values \( w_0=1, w_1=2, w_2=1 \) and perceptron \( B \) has weight values \( w_0=0, w_1=2, w_2=1 \). Is perceptron \( A \) more general than perceptron \( B \)? \( A \) is more general than \( B \) if and only if \( \forall \) instance \( <x_1, x_2>, B(<x_1, x_2>)=1 \rightarrow A(<x_1, x_2>)=1 \).

4. Derive a gradient descent training rule for a single unit with output \( o \), where
   \[ o = w_0 + w_1 x_1 + w_2 x_1^2 + \ldots + w_n x_n + w_n x_n^2 \]

5. Consider a two-layer feed-forward neural network that has the topology shown in the figure.
   - \( X_1 \) and \( X_2 \) are the two inputs.
   - \( Z_1 \) and \( Z_2 \) are the two hidden neurons.
   - \( Y \) is the (single) output neuron.
   - \( w, i=1..4, \) are the weights of the connections from the inputs to the hidden neurons.
   - \( w, j=5..6, \) are the weights of the connections from the hidden neurons to the output neuron.

   Explain the three phases (i.e., input signal forward, error signal backward, and weight update) of the first training iteration of the Back-Propagation algorithm for the current network, given the training example: \( (X_1=x_1, X_2=x_2, Y=y) \). Please use the following notations for the explanation.
   - \( Net_i, Net_2, \) and \( Net_3 \) are the net inputs to the \( Z_1, Z_2, \) and \( Y \) neurons, respectively.
   - \( o_1, o_2, \) and \( o_3 \) are the output values for the \( Z_1, Z_2, \) and \( Y \) neurons, respectively.
   - \( f \) is the activation function used for every neuron in the network, i.e., \( o_k=f(Net_k), k=1..3 \).
   - \( E(w) = (y - o_3)^2 / 2 \) is the Euclidean error function, where \( y \) is the desired network output.
• $\eta$ is the learning rate
• $\delta_1$, $\delta_2$, and $\delta_3$ are the error signals for the $Z_1$, $Z_2$, and $Y$ neurons, respectively.

6. In the Back-Propagation learning algorithm, what is the object of the learning? Does the Back-Propagation learning algorithm guarantee to find the global optimum solution?

7. Other exercises: Mitchell pp. 124-126