Advanced Algorithms

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Lecture 9 – Network Optimization Algorithms (cont.)

Edmonds-Karp algorithm

- **The Ford-Fulkerson algorithm does not specify which alternating augmenting path to use if there is more than one**
- In 1972, Edmonds and Karp proposed a **heuristic** for **choosing** the **augmenting path**
  - choose the augmenting path with fewest edges
- Using the **heuristic**, the path can be found in ***O(|E|)* time**
  - running a **breadth-first search in the residual network**
  - *E* is the set of edges
- The algorithm ends after a **polynomial number of iterations**, independent of the edge capacities

Edmonds-Karp algorithm – Pseudo code

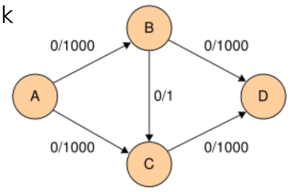
1. create the residual network *Gr*
2. **while** there is some augmented path from *s* to *t* in in *Gr* **do**
   1. let *P* be the path from *s* to *t* in *Gr* with the fewest number of edges
   2. $c^* = min_{(ij) \in P}\, r_{ij}$
   3. send c^* unit of flow along *P*
   4. update r_{ij} for each $(i,j) \in P$

NB: r_{ij} is a pair, including the residual capacities in both directions

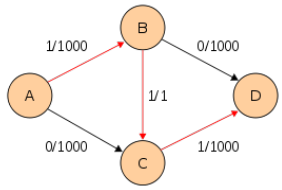
Complexity: Ford-Fulkerson vs Edmonds-Karp

Example of worst-case behavior of the Ford–Fulkerson algorithm
In each iteration, only a flow of 1 is sent across the network

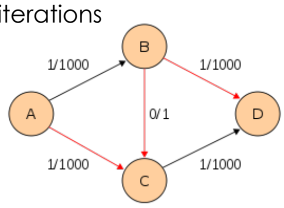
A: source
D: sink



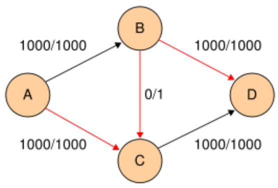
1 iteration



2 iterations



…. 2000 iterations

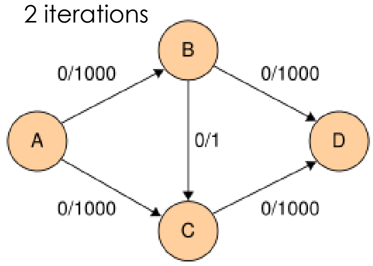


Complexity: Ford-Fulkerson vs Edmonds-Karp

Let's apply Edmonds-Karp's on the same example
The algorithm ends in 2 iterations

A: source
D: sink

2 iterations

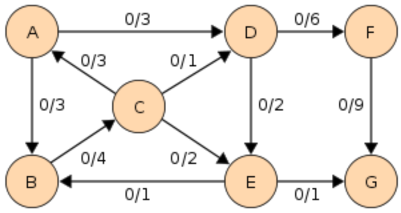


The first iteration selects the path A-B-D (or A-C-D)
The flow is increased by 1000 units

The second iteration selects path A-C-D (or A-B-D)
The flow is increased by 1000 units

Edmonds-Karp algorithm – Example

Initial residual network G



A: source
G: sink

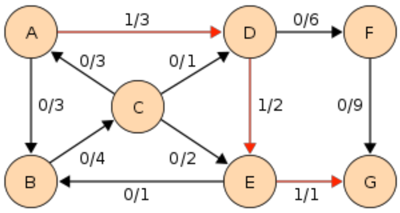
In the pairs ***f/c*** written on the edges, ***f*** is the current flow, and is ***c*** the capacity

- The residual capacity from u to v is $c_f(u,v)=c(u,v)-f(u,v)$, the total capacity, minus the flow that is already used
- If the net flow from u to v is negative, it contributes to the residual capacity

Edmonds-Karp algorithm – Example (cont.)

Iteration 1

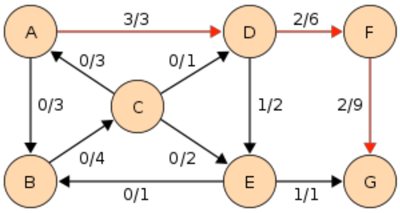
- the shortest augmenting path $A \rightarrow D \rightarrow E \rightarrow G$ is selected
- the residual capacity $c*$ of the augmenting path is
  - $c* = min\{3\text{-}0,\ 2\text{-}0,\ 1\text{-}0\} = 1$
- The total flow is increased by 1
- The new residual network is



Edmonds-Karp algorithm – Example (cont.)

Iteration 2

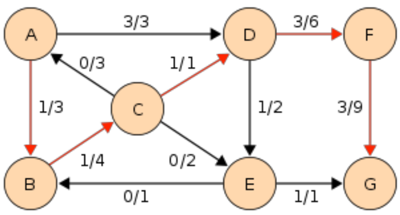
- the shortest augmenting path $A \rightarrow D \rightarrow F \rightarrow G$ is selected
- the residual capacity $c*$ of the augmenting path is
  - $c* = min\{3\text{-}1, 6\text{-}0, 9\text{-}0\} = 2$
- The total flow is increased by 2
- The new residual network is



Edmonds-Karp algorithm – Example (cont.)

Iteration 3

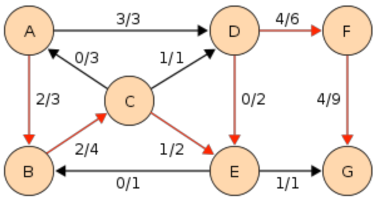
- the shortest augmenting path $A \rightarrow B \rightarrow C \rightarrow D \rightarrow F \rightarrow G$ is selected
- the residual capacity $c*$ of the augmenting path is
  - $c* = min\{3\text{-}0, 4\text{-}0, 1\text{-}0, 6\text{-}2, 9\text{-}2\} = 1$
- The total flow is increased by 1
- The new residual network is



Edmonds-Karp algorithm – Example (cont.)

Iteration 4

- the shortest augmenting path $A \rightarrow B \rightarrow C \rightarrow E \rightarrow D \rightarrow F \rightarrow G$ is selected
- the residual capacity $c*$ of the augmenting path is
  - $c* = min\{3\text{-}1,\ 4\text{-}1,\ 2\text{-}0,\ 0\text{-}(\text{-}1),\ 6\text{-}3,\ 9\text{-}3\} = 1$
- The total flow is increased by 1
- The new (and final) residual network is



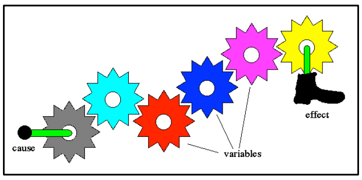
- The length of the augmenting path found by the algorithm (in red) never decreases
- The maximum flow is 5

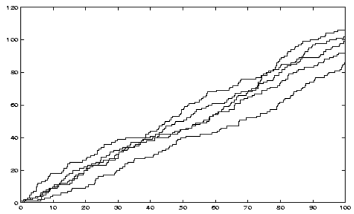
Lecture 9 – Evolutionary algorithms

These slides are mainly taken from A.E. Eiben and J.E. Smith, Introduction to Evolutionary Computing

Randomized algorithms

- A **deterministic** algorithm behaves "**predictably**"
  - **Given** a particular **input**, it will always produce the **same output** through the **same execution**
- A **randomized** algorithm employs a degree of **randomness** as part of its logic
  - **Given** a particular **input**, its **output** is a **random variable** and also its **execution cannot be predicted deterministically**





5 executions of a random algorithm on the same input

Randomized algorithms (cont.)

- Randomized algorithms are often very **simple** and **elegant**, and their **output** is **correct** with **high probability**
- This **success probability** does **not depend** on the **randomness** of the **input**; it only **depends** on the **random choices** made by the algorithm itself
- There are **two varieties** of random algorithms
  - **Monte Carlo** algorithms **always run fast** but their **output has a small chance of being incorrect**
  - **Las Vegas** algorithms **always output** the **right answer**, but there is a **small chance their running time is high**

Consider the primality test algorithm presented in Lecture 3

```
function primality(N)
Input:  Positive integer N
Output:  yes/no

Pick a positive integer a < N at random
if a^(N-1) ≡ 1 (mod N):
   return yes
else:
   return no
```

Is it a Monte Carlo or a Las Vegas random algorithm? Why?

It is a Monte Carlo Algorithm because it runs fast - $O(n^2)$ - but it has a not null probability of error

Evolutionary Computing metaphor

- A **population** of individuals exists in an **environment** with **limited resources**
- ***Competition*** for those resources causes selection of those **fitter individuals** that are **better adapted** to the environment
- These individuals act as **seeds for the generation of new individuals** through recombination and mutation
- The **new individuals** have their fitness evaluated and **compete** (possibly also with parents) **for survival**
- Over time **natural selection** causes a **rise in the fitness of the population**

EC Metaphor

EVOLUTION		PROBLEM SOLVING
Environment	$\longleftrightarrow$	Problem
Individual	$\longleftrightarrow$	Candidate Solution
Fitness	$\longleftrightarrow$	Quality

Fitness → chance for survival and reproduction

Quality → chance for seeding new solutions

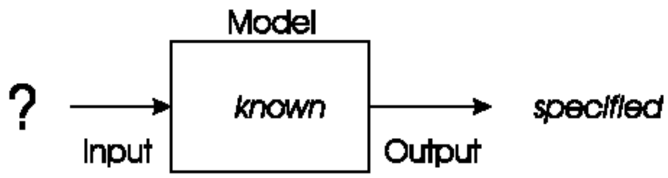
Fitness in nature: **observed**, **2ndary**
Fitness in EC: **primary**

Motivations for EC

- Developing, analyzing, applying problem solving methods a.k.a. algorithms is a central theme in mathematics and computer science
- Time for thorough problem analysis decreases
- Complexity of problems to be solved increases
- Consequence: ROBUST PROBLEM SOLVING technology needed

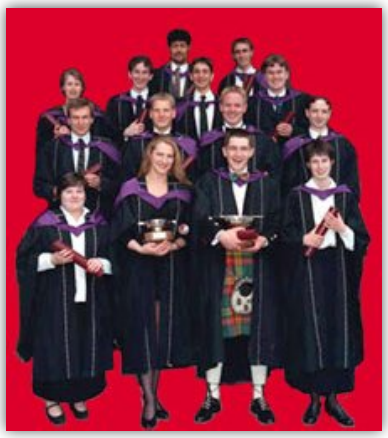
Problem type 1: Optimization

- We have a **model** of our system and **seek inputs** that give us a **specified goal**



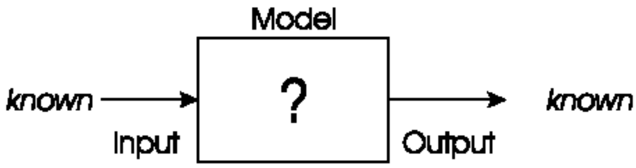
Optimization example 1: university timetabling

- Enormously big search space
- Timetables must be **feasible**
  - Vast majority of search space is infeasible
- Timetables must be **good**
  - Good is defined by a number of competing criteria



Problem types 2: Modelling

- We have corresponding sets of inputs & outputs and seek model that delivers correct output for every known input



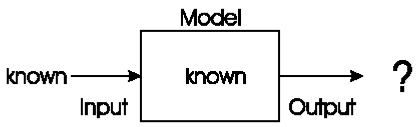
Modelling example: mushroom edibility

- **Classify** mushrooms as edible or not edible
- Evolving: **classifications models**
- **Fitness**: classification **accuracy** on training set of edible and not edible mushrooms

This is an example of **evolutionary machine learning**

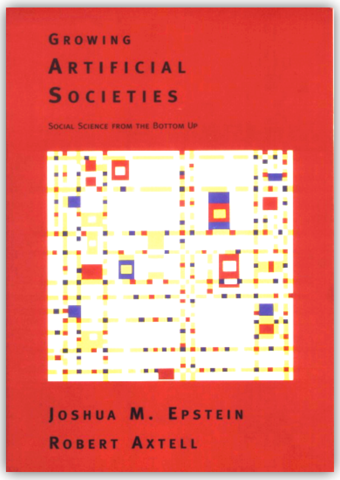
Problem type 3: Simulation

- We have a given model and wish to know the outputs that arise under different input conditions



- Often used to answer "what-if" questions in evolving dynamic environments

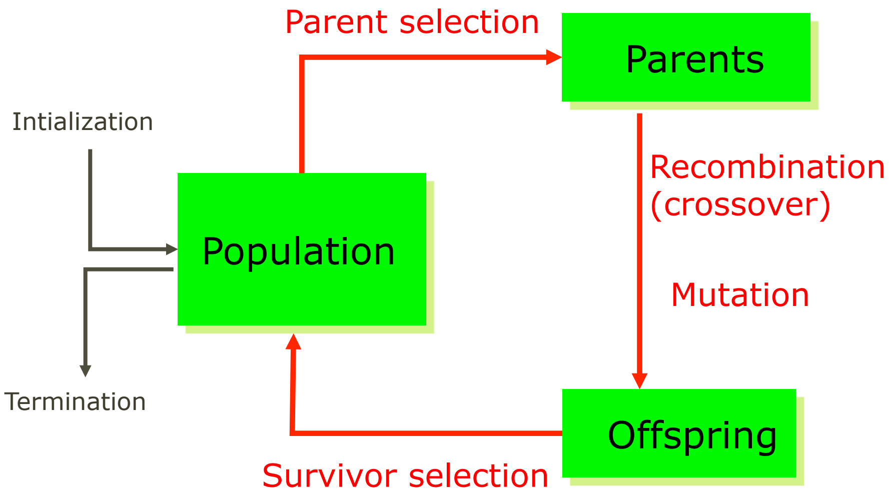
Simulation example: evolving artificial societies



Simulating trade, economic competition, etc. to calibrate models

Use models to optimize strategies and policies

General schema of EAs



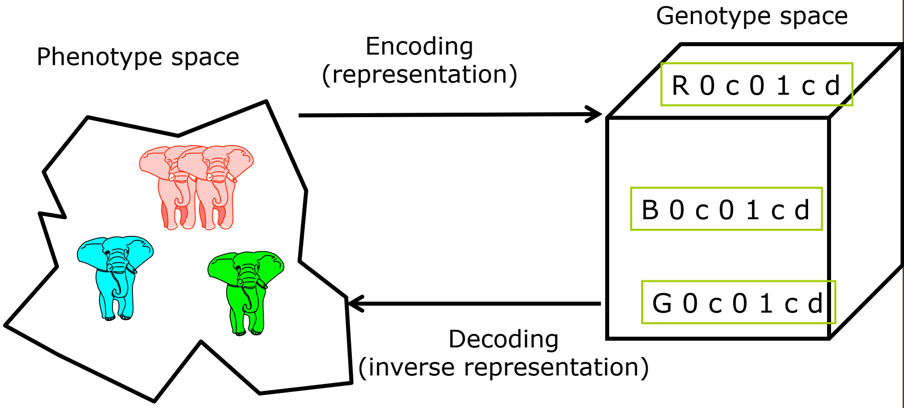
EA schema in pseudo-code

```
BEGIN
  INITIALISE population with random candidate solutions;
  EVALUATE each candidate;
  REPEAT UNTIL ( TERMINATION CONDITION is satisfied ) DO
    1 SELECT parents;
    2 RECOMBINE pairs of parents;
    3 MUTATE the resulting offspring;
    4 EVALUATE new candidates;
    5 SELECT individuals for the next generation;
  OD
END
```

Main EA components

- Representation (encoding) of solutions into individuals
- Population
  - How big? How is it initialized?
- Evaluation
  - How to measure the goodness of individuals?
- Selection (parent selection, survivor selection)
  - How to pick up?
- Variation (mutation, recombination)
  - How to create new solutions?
- Termination condition
  - When the algorithm has found the best solution?

Representation



In order to find the global optimum, every feasible solution must be represented in genotype space

Population

- **Role**: holds the candidate solutions of the problem as individuals (genotypes)
- Formally, a population is a **multiset** of **individuals**, i.e. repetitions are possible
- Population is the **basic unit of evolution**, i.e., the population is evolving, not the individuals
- **Selection** operators act **on population level**
- **Variation** operators act on **individual level**

Evaluation (fitness) function

- A.k.a. **quality** function or **objective** function
- **Role**:
  - Represents the **task** to solve, the **requirements** to adapt to (can be seen as "the **environment**")
  - **enables selection** (provides basis for comparison)
    - e.g., some phenotypic traits are advantageous, desirable, e.g. big ears cool better, these traits are rewarded by more offspring that will expectedly carry the same trait
- Assigns a single **real-valued fitness** to each **phenotype** which forms the basis for selection
  - So the more discrimination (different values) the better
- Typically we talk about **fitness being maximised**
  - Some problems may be best posed as minimisation problems, but conversion is trivial

Selection

Role:

- Identifies **individuals**
  - to become **parents**
  - to **survive**
- **Pushes** population towards **higher fitness**
- Usually **probabilistic**
  - high quality solutions more likely to be selected than low quality
  - but not guaranteed
  - even worst in current population usually has non-zero probability of being selected
- This **stochastic** nature can aid **escape from local optima**

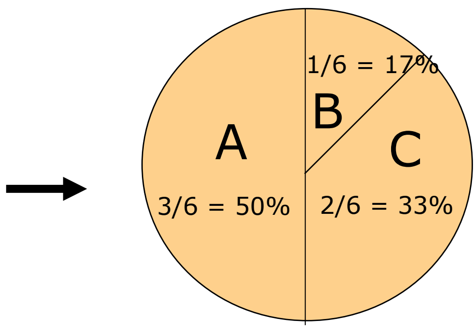
Selection mechanism example

Example: **roulette wheel selection**

fitness(A) = 3

fitness(B) = 1

fitness(C) = 2



In principle, any selection mechanism can be used for parent selection as well as for survivor selection

Survivor selection

- A.k.a. ***replacement***
- Most EAs use **fixed population size** so need a way of going from (parents + offspring) to next generation
- Often **deterministic** (while parent selection is usually stochastic)
  - **Fitness based** : e.g., rank parents+offspring and take best
  - **Age based**: make as many offspring as parents and delete all parents
- Sometimes a combination of stochastic and deterministic (elitism)

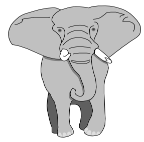
Variation operators

- **Role**: to generate **new** candidate **solutions**
- Usually divided into **two types** according to their **arity** (number of inputs):
  - **Arity 1** : **mutation** operators
  - **Arity >1** : **recombination** operators
    - **Arity = 2** typically called **crossover**
    - Arity > 2 is formally possible, seldomly used in EC
- There has been much **debate about relative importance of recombination and mutation**
  - Nowadays **most EAs use both**
  - Variation operators must **match** the given **representation**

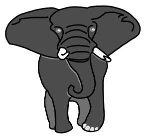
Mutation

- **Role**: causes small, **random variance**
- Acts on one genotype and delivers another
- Element of **randomness** is essential and differentiates it from other unary heuristic operators

before 1 1 1 1 1 1 1



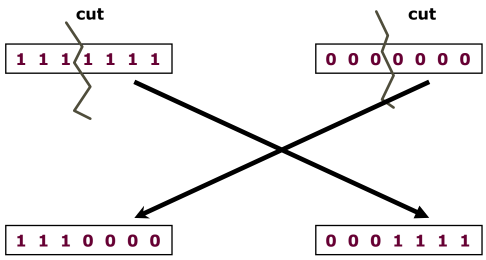
after 1 1 1 0 1 1 1



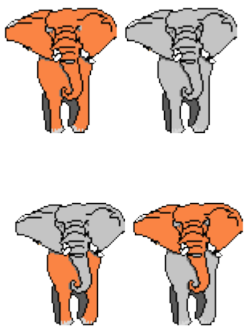
Recombination

- **Role**: **merges** information from **parents** into **offspring**
- **Choice** of what **information** to merge is **stochastic**
- Most **offspring** may be **worse**, or the same as the parents
- Hope is that **some** are **better** by **combining** elements of **genotypes** that lead to **good traits**

Parents



Offspring



Initialisation / Termination

- **Initialisation** usually done at **random**
  - Need to ensure even spread and mixture of possible allele values
  - Can include existing solutions, or use problem-specific heuristics, to "**seed**" the population

- **Termination** condition **checked every generation**
  - Reaching some (known/hoped for) **fitness**
  - Reaching some **maximum** allowed **number** of **generations**
  - Reaching some **minimum level** of **diversity**
  - Reaching some specified **number** of **generations without** fitness **improvement**