Advanced Algorithms

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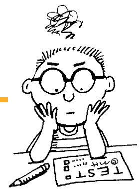
Lab 9 – Solution of assignments

Assignment 08



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Solution

A. Suppose we have a flow network G that has multiple sources S_1, S_2, …, S_n and multiple sinks T_1, T_2, …, T_m. Each source S_i has a set of outgoing edges E_i (i=1,…n), and each sink T_j has a set of incoming edges Fj, (j=1,…,m). The max-flow problem on G can be reduced to the original max-flow problem by constructing a network G' from G as follows:

- We introduce two additional vertices S and T
- We construct n edges e_1, e_2, …, e_n, each of them going from S to S_1, …, S_n.
- We construct m edges f_1, f_2, …, f_m each of them going from T_1, …, T_m to T.
- For each e_i from S to S_i, e_i has capacity equal to the sum of the capacities of all edges in E_i.
- For each f_j from T_j to T, f_j has capacity equal to the sum of the capacities of all edges in F_j.
- S is the single source of G' and T is the single sink of G'.
- The original S_1, .. S_n and T_1, …, T_m are treated as transshipment nodes.

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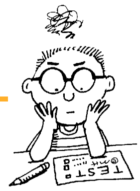

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Solution (cont.)

B. Suppose we have a flow network G with a single source S, a single sink T and n transshipment nodes N_1, …, N_n. For each vertex but the source, G has also a maximum flow that can enter the vertex. Let f_i be the maximum flow that can enter N_i, i=1,..n, and let f_t the maximum flow that can enter the sink node T. The max-flow problem on G can be reduced to the original max-flow problem by constructing a network G' from G as follows:

- G' has S as source and T as sink
- For each N_i , i=1,..n, G' has two nodes M_i and $M_{i'}$ such that for each incoming edge of N_i there is an equivalent incoming edge of M_i, and for each outgoing edge of N_i there is an equivalent outgoing edge of $M_{i'}$; in addition, there is an edge e_i from M_i to $M_{i'}$, whose capacity is f_i.
- For the sink T, we introduce in G' a new sink node U' and an additional transshipment node U such that for each incoming edge of T there is an equivalent incoming edge of U, and there is an edge e_t from U to U', whose capacity is f_t

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Solution (cont.)

C. Suppose we have a flow network G and each edge of G has not only a capacity, but also a lower bound on the flow it can carry. The max-flow problem on G can be modeled in LP similarly to reducing a normal max-flow problem. The only difference is that for every edge e in G, instead of the capacity and non negativity constraints 0 <= e <= c_e (where c_e denotes the capacity of edge e), we have c_{e_min} <= e <= c_{e_max} (where c_{e_min} and c_{e_max} denote the lower and the upper bounds of the edge capacity, respectively). The flow conservation constraints are the same and also the objective function to be maximized does not change.

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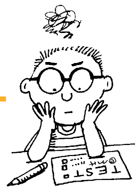

Exercise

Solution

A. Some greedy strategies are:

1. Repeatedly pick the item with the maximum value
   Counter example:
   n (number of items) = 3, m (max weight in knapsack) = 3 kg
   i_1: v_1 = 7 \$; w_1 = 0.5 kg
   i_2: v_2 = 9 \$; w_2 = 1 kg
   i_3: v_3 = 14 \$; w_3 = 2.8 kg
   Greedy solution: {i_3}
   Optimal solution: {i_1, i_2}
2. Repeatedly pick the item with the minimum weight
   Counter example:
   n = 3, m = 3 kg
   i_1: v_1 = 15 \$; w_1 = 0.5 kg
   i_2: v_2 = 6 \$; w_2 = 1 kg
   i_3: v_3 =14 \$; w_3 = 2 kg
   Greedy solution: {i_1, i_2}
   Optimal solution: {i_1, i_3}
3. Repeatedly pick the item with the maximum weight
   Counter example
   n = 3, m = 3 kg
   i_1: v_1 = 15 \$; w_1 = 0.5 kg
   i_2: v_2 = 6 \$; w_2 = 1 kg
   i_3: v_3 =14 \$; w_3 = 2 kg
   Greedy solution: {i_2, i_3}
   Optimal solution: {i_1, i_3}

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Exercise

Solution (cont.)

B. Repeatedly pick the object with the maximum value per unit of weight

Counter example:
n (number of items) = 3, m (max weight in knapsack) = 3 kg
i_1: v_1 = 5 $; w_1 = 0.5 kg; v_1/w_1 = 10
i_2: v_2 = 6 $; w_2 = 1 kg; v_2/w_2 = 6
i_3: v_3 = 14 $; w_3 = 2 kg; v_3/w_3 = 7
Greedy solution: {i_1, i_3}
Optimal solution: {i_2, i_3}