Advanced Algorithms

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Lab 8 – Exercises on network optimization
Exercise 1

Solution

1. Find value of the maximum flow from A to F using the Ford-Fulkerson algorithm

2. Find the final flow on each edge

Iteration 1

We choose the path \( P_1 = (1, 4, 8) \), identified by the list of edges in the path, and the flow can be increased by 5 units
Exercise 1

Solution (cont.)

Iteration 2

- Another flow augmenting path is $P_2 = (2, 5, 6, 9)$. We can augment the flow on this path by 5 units.

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\text{Residual network}
\]

Iteration 3

- Now we select the augmenting path $P_3 = (2, 7, 4, 3, 6, 9)$. Notice that edge 4 is traversed in the opposite direction and therefore the flow allocated to this edge decreases. We can augment the flow on this path by 5 units.
Exercise 1

Solution (cont.)

- Iteration 4
  - There isn’t an augmenting path: STOP!!
  - The final flow from A to F is of 15 units

Exercise 2

Network Flow

- The figure below shows a flow network on which an s-t flow is shown
- The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge
- Edges without boxed numbers have no flow being sent on them

1. What is the value of this flow?
2. Is this a maximum s-t flow in this graph? If not, find a maximum s-t flow
Exercise 2

Solution

1. The value of the flow is 4+7+6=17
2. The value of the flow it is not maximum. We can send 2 units of flow along the path: s – z – y – x – w - t obtaining a flow of value 17+2=19 (figure on the left). The residual capacities are reported in the figure on the right.

Assignment 07

Exercise 7.10 page 224 DPV

For the following network, with edge capacities as shown, find the maximum flow from S to T. Use both Linear programming and the Ford-Fulkerson algorithm.
Assignment 07

Exercise 7.17 page 226 DPV
Consider the following network (the numbers are edge capacities)

1. Find the maximum flow $f$
2. Draw the residual graph $G_f$ (along with its edge capacities). In this residual network, mark the vertices reachable from $S$ and the vertices from which $T$ is reachable.
3. An edge of a network is called a bottleneck edge if increasing its capacity results in an increase in the maximum flow. List all bottleneck edges in the above network.
4. Give a very simple example (containing at most four nodes) of a network which has no bottleneck edges.
5. (Optional) Give an efficient algorithm to identify all bottleneck edges in a network. (Hint: Start by running the usual network flow algorithm, and then examine the residual graph.)