Advanced Algorithms

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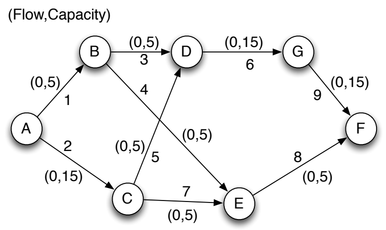
Lab 8 – Exercises on network optimization

Exercise 1

Network Flow

- The figure below shows a network
- For each edge the information about the current flow (first number in parenthesis) and the capacity (second number in parenthesis) is provided

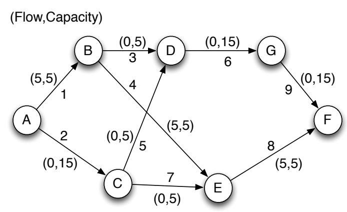


1. Find value of the maximum flow from A to F using the Ford-Fulkerson algorithm
2. Find the final flow on each edge

Exercise 1

Solution

- Iteration 1
  - We choose the path P1 = (1, 4, 8), identified by the list of edges in the path, and the flow can be increased by 5 units

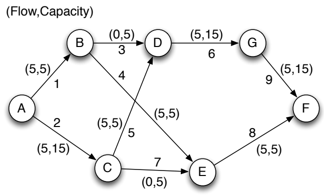


Residual network

Exercise 1

Solution (cont.)

- Iteration 2
  - Another flow augmenting path is P2 = (2, 5, 6, 9). We can augment the flow on this path by 5 units

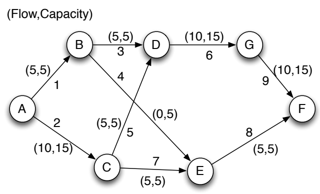


Residual network

Exercise 1

Solution (cont.)

- Iteration 3
  - Now we select the augmenting path P3 = (2, 7, 4, 3, 6, 9). Notice that edge 4 is traversed in the opposite direction and therefore the flow allocated to this edge decreases. We can augment the flow on this path by 5 units



Residual network

Exercise 1

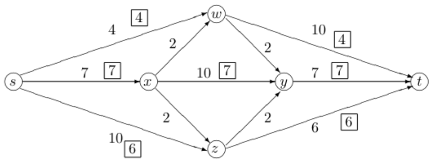
Solution (cont.)

- Iteration 4
  - There isn't an augmenting path: STOP!!
  - The final flow from A to F is of 15 units

Exercise 2

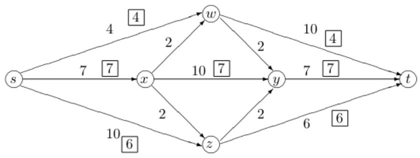
Network Flow

- The figure below shows a flow network on which an *s*-*t* flow is shown
- The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge
  - Edges without boxed numbers have no flow being sent on them



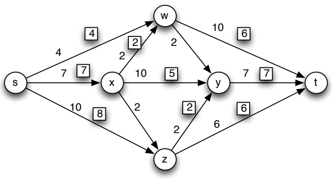
1. What is the value of this flow?
2. Is this a maximum *s*-*t* flow in this graph? If not, find a maximum *s*-*t* flow

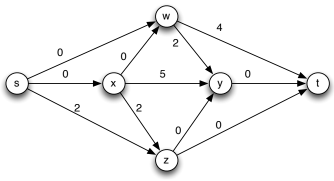
Exercise 2



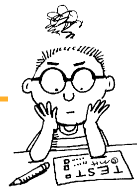
Solution

1. The value of the flow is 4+7+6=17
2. The value of the flow it is not maximum. We can send 2 units of flow along the path: s – z – y – x – w - t obtaining a flow of value 17+2=19 (figure on the left). The residual capacities are reported in the figure on the right



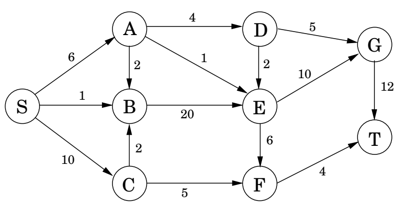


Assignment 07

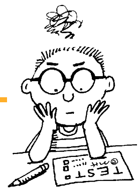


Exercise 7.10 page 224 DPV

- For the following network, with edge capacities as shown, find the maximum flow from *S* to *T*. Use both Linear programming and the Ford-Fulkerson algorithm

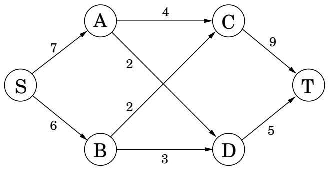


Assignment 07



Exercise 7.17 page 226 DPV

Consider the following network (the numbers are edge capacities)



1. Find the maximum flow *f*
2. Draw the residual graph G_f (along with its edge capacities). In this residual network, mark the vertices reachable from *S* and the vertices from which *T* is reachable
3. An edge of a network is called a *bottleneck edge* if increasing its capacity results in an increase in the maximum flow. List all bottleneck edges in the above network.
4. Give a very simple example (containing at most four nodes) of a network which has no bottleneck edges
5. (Optional) Give an efficient algorithm to identify all bottleneck edges in a network. (*Hint:* Start by running the usual network flow algorithm, and then examine the residual graph.)