Advanced Algorithms

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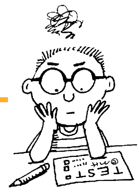
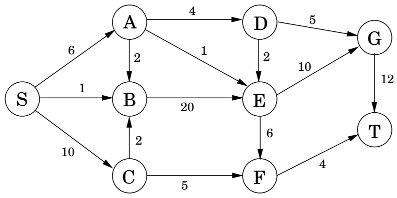
Lab 8 – Solution of assignments

Assignment 07

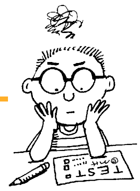


Exercise 7.10 page 224 DPV

- For the following network, with edge capacities as shown, find the maximum flow from *S* to *T*. Use both Linear programming and the Ford-Fulkerson algorithm



Assignment 07



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Solution with LP

- Variables:
  a=SA, b=SB, c=SC, d=AB, e=CB, f=AD, g=AE, h=BE, i=CF,
  j=DE, k=EF, l=DG, m=EG, n=FT, o=GT
- Objective function
  maximize a + b + c + d + e + f + g + h + i + j + k + l + m + n + o
- Constraints
  - Capacity and non negativity
    0 <= a <= 6
    0 <= b <= 1
    0 <= c <= 10
    0 <= d <= 2
    0 <= e <= 2
    0 <= f <= 4
    0 <= g <= 1
    0 <= h <= 20
    0 <= i <= 5
    0 <= j <= 2
    0 <= k <= 6
    0 <= l <= 5
    0 <= m <= 10
    0 <= n <= 4
    0 <= o <= 12
  - Flow conservation
    a = d + f + g
    b + d + e = h
    c = e + i
    f = j + l
    g + h + j = k + m
    i + k = n
    l + m = o

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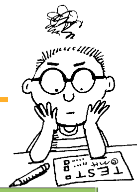

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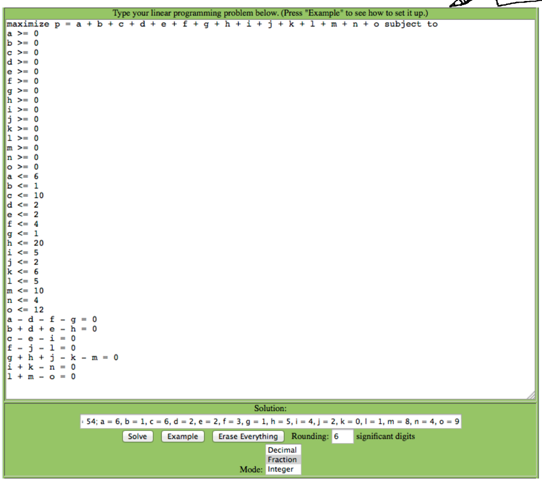
Solution with LP (cont.)

The solution is on the right and is:
a = 6, b = 1, c = 6, d = 2, e = 2, f = 3, g = 1, h = 5, i = 4, j = 2, k = 0, l = 1, m = 8, n = 4, o = 9

To obtain the maximum flow we consider either the variables corresponding to the out-links of the source node or those corresponding the in-links of the sink node

Max flow = a + b + c = n + o = 13



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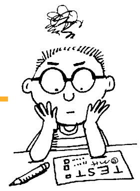

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Solution with Ford-Fulkerson

We use the following notation for the edges in G: a=SA, b=SB, c=SC, d=AB, e=CB, f=AD, g=AE, h=BE, i=CF, j=DE, k=EF, l=DG, m=EG, n=FT, o=GT

We apply the FF algorithm to the graph G.

1. Take augmenting path ***c→i→n***. The residual capacity ***c* = min{c,i,n} = 4***. We allocate **4** flow units from this path. Update ***e := e − 4*** for every ***e*** along the path ***c→i→n***
2. Take augmenting path ***a→f→l→o***. The residual capacity ***c* = min{a,f,l,o} = 4***. We allocate **4** flow units from this path. Update ***e := e - 4*** every ***e*** along the path ***a→f→l→o***
3. Take augmenting path ***c→e→h→m→o***. The residual capacity ***c* = min{c,e,h,m,o} = 2***. We allocate ***2*** flow units from this path. Update ***e := e - 2*** every ***e*** along the path ***c→e→h→m→o.***
4. Take augmenting path ***a→d→h→m→o***. The residual capacity ***c* = min{a,d,h,m,o} = 2***. We allocate ***2*** flow units from this path. Update ***e := e - 2*** every ***e*** along the path ***a→d→h→m→o.***
5. Take augmenting path ***b→h→m→o***. The residual capacity ***c* = min{b,h,m,o} = 1***. We allocate ***1*** flow units from this path. Update ***e := e - 1*** every ***e*** along the path ***b→h→m→o.***
6. Now we cannot find any other valid path from source to sink → STOP!
7. Total flow = 4 + 4 + 2 + 2 + 1 = 13.

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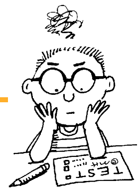

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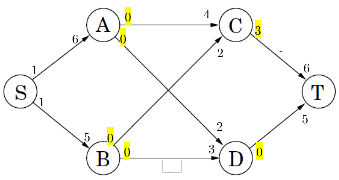
1. The maximum flow *f=11* can be found solving the LP model corresponding to graph *G* or running the Ford-Fulkerson algorithm on *G*.
2. The residual graph G_f is



In this residual network, vertices A and B reachable from *S*, and C is the vertex from which *T* is reachable (by reachable we mean that there is still some flow that can be further allocated for S to A and B or from C to T).

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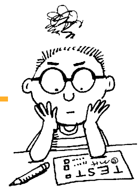

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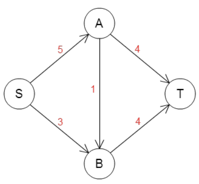
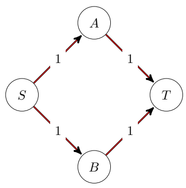
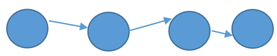
Solution (cont.)

3. The bottlenecks are AC and BC
4. Examples of networks without bottlenecks are:







All the edges have the same capacity

5. On the residual graph, check every edge e that has remaining flow = 0. Check whether there is a path in the graph from source to sink such that the path contains e and e is the only edge in the path that has flow = 0.