Advanced Algorithms

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Lab 7 – Exercises on linear programming

Exercise 7.7 page 224 DPV

Find necessary and sufficient conditions on the real numbers *a* and *b* under which the linear program

$max\ x + y$
$a*x + b*y \leq 1$
$x, y \geq 0$

a. Is infeasible
b. Is unbounded

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Solution

a) This LP is never infeasible as the origin will satisfy $a*x + b*y \leq 1$ for any choice of *a* and *b*

b) It is sufficient that $a \leq 0$ or $b \leq 0$
   - If $a \leq 0$, then we can increase *x* (and the objective function) arbitrarily without violating any constraint
   - The same argument works for *b* and *y*

   Conversely, suppose that the linear program is unbounded and both *a* and *b* are positive
   - Let $m = min\{a,b\}$ and notice $m > 0$
   - $m*x + m*y \leq a*x+b*y \leq 1 \rightarrow x+y \leq 1/m$
   - Hence, the LP cannot be unbounded

Exercise

Product Mixture Problem

- The nutritionist at a food research lab is trying to develop a new type of multigrain flour
- The grains that can be included have the following composition and price

	% of Nutrient in Grain			
	1	2	3	4
Starch	30	20	40	25
Fiber	40	65	35	40
Protein	20	15	5	30
Gluten	10	0	20	5
Cost (cents/kg.)	70	40	60	80

- Because of taste considerations, the amount of grain 2 in the mix cannot exceed 20%, the amount of grain 3 in the mix has to be at least 30%, and the amount of grain 1 in the mix has to be between 10% to 25%
- The protein content in the flour must be at least 18%, the gluten content has to be between 8% and 13%, and the fiber content should be at most 50%

Find the LP modeling this problem. Find (if it exists) the least costly way of blending the grains to make the flour, subject to the constraints given (you can use an online solver as http://www.zweigmedia.com/RealWorld/simplex.html)

Exercise

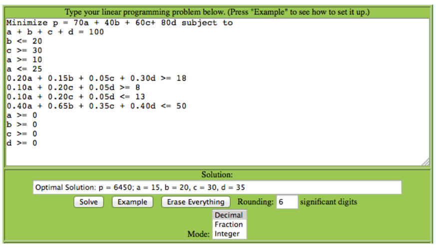
Solution

- The model is:

  x_i = amount of grain i

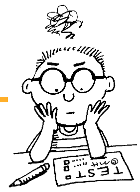
  $\min 70 x1 + 40 x2 + 60 x3 + 80 x4$
  $x1 + x2 + x3 + x4 = 100$
  $x2 \leq 20$
  $x3 \geq 30$
  $x1 \geq 10$
  $x1 \leq 25$
  $0.20 x1 + 0.15 x2 + 0.05 x3 + 0.30 x4 \geq 18$
  $0.10 x1 + 0.20 x3 + 0.05 x4 \geq 8$
  $0.10 x1 + 0.20 x3 + 0.05 x4 \leq 13$
  $0.40 x1 + 0.65 x2 + 0.35 x3 + 0.40 x4 \leq 50$
  $x1 \geq 0, x2 \geq 0, x3 \geq 0, x4 \geq 0$

- The optimal solution calculated with the solver http://www.zweigmedia.com/RealWorld/simplex.html is



In the picture, *a, b, c*, and *d* are used instead of *x1, x2, x3*, and *x4*

Assignment 06



Solve the following LP graphically. For each model state clearly whether it is infeasible, it is unbounded, or it has multiple solutions

$a)$	$\text{Max } 3A + 5B$ $A \geq 5$ $B \leq 10$ $A + 2B \geq 10$ $B \geq 0$
$b)$	$\text{Max } P + Q$ $P + 2Q \leq 6$ $2P + Q \leq 8$ $P \geq 7$ $Q \geq 0$
$c)$	$\text{Max } M + 2N$ $M + N \leq 25$ $2M + N \leq 30$ $N \leq 35$ $M, N \geq 0$
$d)$	$\text{Max } 5X + 2Y$ $7{,}5X + Y \leq 15$ $5X + 2Y \leq 20$ $X, Y \geq 0$