# Advanced Algorithms

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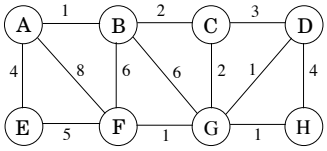
# Lab 5 – Exercises on greedy algorithms and linear programming

# Exercise 5.2a page 148 DPV

Suppose we want to find the minimum spanning tree of the following graph




- Run Prim's algorithm; whenever there is a choice of nodes, always use alphabetic ordering (e.g., start from node A)
- Draw a table with 3 column (node included, edge included, intermediate total cost) showing the evolution of the algorithm

# Exercise 5.2a page 148 DPV

**Solution**

| Vertex included | Edge included | Cost |
|---|---|---|
| A | | 0 |
| B | AB | 1 |
| C | BC | 3 |
| G | CG | 5 |
| D | GD | 6 |
| F | GF | 7 |
| H | GH | 8 |
| E | AE | 12 |

## Exercise 5.13 page 150 DPV

- A DNA sequence can be represented by a string over the 4 characters A, C, G, T, which represent the 4 nucleic acids
- A long string consists of the four characters A,C,G,T
- They appear with frequency 31%, 20%, 9%, and 40%, respectively
- What is the Huffman encoding of these four characters?

## Exercise 5.13 page 150 DPV

**Solution**

- Huffman's algorithm assigns codewords of length 1 to T, length 2 to A and length 3 to G and C
- So, one possible encoding is 0 for T, 10 for A, 110 for C and 111 for G

## Exercise 5.14 page 150 DPV

- Suppose the symbols a,b,c,d,e occur with frequencies 1/2, 1/4, 1/8, 1/16, 1/16, respectively
  a. What is the Huffman encoding of the alphabet?
  b. If this encoding is applied to a file consisting of 1,000,000 characters with the given frequencies, what is the length of the encoded file in bits?

## Exercise 5.14 page 150 DPV

**Solution**

a. a → 0, b → 10, c → 110, d → 1110, e → 1111
b. length = $10^6/2 * 1 + 10^6/4 * 2 + 10^6/8 * 3 +$
   $+ 10^6/16 * 4 + 10^6/16 * 4 =$
   $= 1875000$

## LP Exercise - Clothing Factory

- A **factory** produces **cloths for men**: trousers, jackets, shirts
- Each item **costs** to the factory a **different amount** and it is sold for a **different price**:

| | Cost | Price | Profit |
|---|---|---|---|
| Trousers | 25 | 45 | 20 |
| Jackets | 35 | 70 | 35 |
| Shirts | 20 | 35.50 | 15.50 |

- The factory has **three divisions**: cutting, sewing, ironing

| | Cutting | Sewing | Ironing |
|---|---|---|---|
| Trousers | 10 *min* | 15 *min* | 20 *min* |
| Jackets | 20 *min* | 20 *min* | 20 *min* |
| Shirts | 15 *min* | 20 *min* | 20 *min* |
| Daily availability | 20 *hours* | 18 *hours* | 16 *hours* |

- **How many items of each kind the factory has to produce each day to?**
  1. **Minimize the daily cost?**
  2. **Maximize the daily revenue?**
  3. **Maximize the daily profit?**
- **Set up a LP problem for each question**

## LP Exercise - Clothing Factory (sol.)

First step – Identification of the problem

- Individuate (or choose) the objective
  1. ***Minimize the daily production costs***
  2. ***Maximize the daily revenue***
  3. ***Maximize the daily profit***
- Individuate the available resources
  - ***Cutting division***
  - ***Sewing division***
  - ***Ironing division***
- Individuate available data
  - ***unitary production costs***
  - ***unitary sell prices***
  - ***daily availability of the three divisions***

## LP Exercise - Clothing Factory (sol.)

Second step – Formulation of the linear programming problem:

- Define the variables
  - $x_t$ : number of **trousers produced per day**
  - $x_j$ : number of **jackets produced per day**
  - $x_s$: number of **shirts produced per day**
- Define the mathematical relations among variables and data
  - Constraints
    - The cutting division is available for 20 hours per day, that is 1200 mins per day
      each pair of trousers needs 10 mins
      each jacket needs 20 mins
      each shirt needs 15 mins
      $10x_t + 20x_j + 15x_s \leq 1200$
    - The sewing division is available for 18 hours per day, that is 1080 mins per day
      each pair of trousers needs 15 mins
      each jacket needs 20 min
      each shirt needs 20 mins
      $15x_t + 20x_j + 20x_s \leq 1080$
    - The ironing division is available for 16 hours per day, that is 960 mins per day
      each pair of trousers require 20 mins
      each jacket needs 20 mins
      each shirt needs 20 mins
      $20x_t + 20x_j + 20x_s \leq 960$

## LP Exercise - Clothing Factory (sol.)

Second step – Formulation of the linear programming problem:

- Define the mathematical relations among variables and data
  - Objective function
    1. **minimize the daily production costs**
       $min\ 25x_t + 35x_j + 20x_s$
    2. **maximize the daily revenue**
       $max\ 45x_t + 70x_j + 35.50x_s$
    3. **maximize the daily profit**
       $max\ 20x_t + 35x_j + 15.50x_s$

## LP Exercise - Clothing Factory (sol.)

To summarize:

**Mathematical models**:

$x_t$:trousers, $x_j$: jackets, $x_s$: shirts.

Minimize the daily production costs

$$min\ 25x_t + 35x_j + 20x_s$$
$$10x_t + 20x_j + 15x_s \leq 1200$$
$$15x_t + 20x_j + 20x_s \leq 1080$$
$$20x_t + 20x_j + 20x_s \leq 960$$
$$x_t, x_j, x_s \geq 0$$

Maximize the daily revenue

$$max\ 45 + 70x_j + 35.50x_s$$
$$10x_t + 20x_j + 15x_s \leq 1200$$
$$15x_t + 20x_j + 20x_s \leq 1080$$
$$20x_t + 20x_j + 20x_s \leq 960$$
$$x_t, x_j, x_s \geq 0$$

Maximize the daily profit

$$max\ 20x_t + 35x_j + 15.50x_s$$
$$10x_t + 20x_j + 15x_s \leq 1200$$
$$15x_t + 20x_j + 20x_s \leq 1080$$
$$20x_t + 20x_j + 20x_s \leq 960$$
$$x_t, x_j, x_s \geq 0$$

## Assignment 04


**Exercise 5.6 page 149 DPV**

- Let *G = (V, E)* be an undirected graph
- Prove that if all its edge weights are distinct, then it has a unique minimum spanning tree

# Assignment 04 (cont.)


**Exercise 5.15 page 150 DPV**

- We use Huffman's algorithm to obtain an encoding of alphabet *{a,b,c}* with frequencies $f_a$, $f_b$, $f_c$
- In each of the following cases, either give an example of frequencies ($f_a$, $f_b$, $f_c$) that would yield the specified code and say if the code is optimal, or explain why the code cannot possibly be obtained (no matter what the frequencies are)
  a. Code: *{0, 10, 11}*
  b. Code: *{0, 1, 00}*
  c. Code: *{10, 01, 00}*

# Assignment 04 (cont.)


**Exercise 7.2 page 223 DPV**

- Duckwheat is produced in Kansas and Mexico and consumed in NewYork and California
- Kansas produces 15 shnupells of duckwheat and Mexico 8.
- Meanwhile, New York consumes 10 shnupells and California 13
- The transportation costs per shnupell are $4 from Mexico to New York, $1 from Mexico to California, $2 from Kansas to New York, and $3 and from Kansas to California.
- Write a linear program that decides the amounts of duckwheat (in shnupells and fractions of a shnupell) to be transported from each producer to each consumer, so as to minimize the overall transportation cost

# Assignment 04 (cont.)


**Exercise 7.3 page 223 DPV**

- A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, up to the maximum available limits given below
  - Material 1 has density 2 tons/cubic meter, maximum available amount 40 cubic meters, and revenue $1,000 per cubic meter
  - Material 2 has density 1 ton/cubic meter, maximum available amount 30 cubic meters, and revenue $1,200 per cubic meter
  - Material 3 has density 3 tons/cubic meter, maximum available amount 20 cubic meters, and revenue $12,000 per cubic meter
- Write a linear program that optimizes revenue within the constraints