Advanced Algorithms

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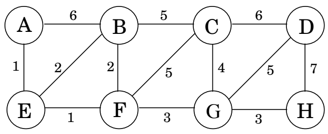
Lab 4 – Exercises on greedy algorithms

Exercise 5.1 page 148 DPV

Consider the following graph



- What is the cost of its minimum spanning tree?
- How many minimum spanning trees does it have?
- Suppose Kruskal's algorithm is run on this graph. In what order are the edges added to the MST? For each edge in this sequence, give a cut that justifies its addition

Exercise 5.1 page 148 DPV

Solution

- What is the cost of its minimum spanning tree?
  - **19**
- How many minimum spanning trees does it have?
  - **2**
- Suppose Kruskal's algorithm is run on this graph. In what order are the edges added to the MST? For each edge in this sequence, give a cut that justifies its addition

Edge included	Cut
AE	$\{A, B, C, D\}$ & $\{E, F, G, H\}$
EF	$\{A, B, C, D, E\}$ & $\{F, G, H\}$
BE	$\{A, E, F, G, H\}$ & $\{B, C, D\}$
FG	$\{A, B, E\}$ & $\{C, D, F, G, H\}$
GH	$\{A, B, E, F, G\}$ & $\{C, D, H\}$
CG	$\{A, B, E, F, G, H\}$ & $\{C, D\}$
GD	$\{A, B, C, E, F, G, H\}$ & $\{D\}$

Exercise 5.5.a page 148 DPV

Consider an undirected graph $G = (V, E)$ with nonnegative edge weights $w_e \geq 0$.
Suppose that you have computed a minimum spanning tree of G.
Now suppose each edge weight is increased by 1: the new weights are $w_e' = w_e + 1$.

- Does the minimum spanning tree change? Give an example where it changes or prove it cannot change

Exercise 5.5.a page 148 DPV

Solution

The MST does not change because 1 is added to all edges, so their ordering per weight is the same as before and Kruskal's algorithms build the MTS exactly as before.

Exercise 5.7 page 149 DPV

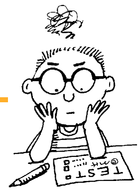
Show how to find the *maximum* spanning tree of a graph, that is, the spanning tree of largest total weight

Exercise 5.7 page 149 DPV

Solution

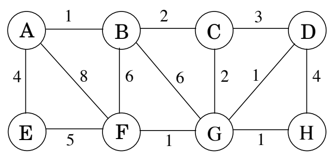
Multiply the weights of all the edges by -1. Since Kruskal's algorithm works for positive as well as negative weights, we can find the minimum spanning tree of the new graph. This is the same as the maximum spanning tree of the original graph.

Assignment 03



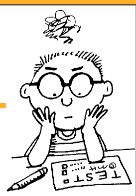
Exercise 5.2.b page 148 DPV

Suppose we want to find the minimum spanning tree of the following graph



Run Kruskal’s algorithm on the graph. Show how the disjoint-sets data structure looks at every intermediate stage (including the structure of the directed trees), assuming path compression is used

Assignment 03 (cont.)



Exercise 5.9 page 149 DPV (partially)

The following statements may or may not be correct. In each case, either prove it (if it is correct) or give a counterexample (if it isn’t correct). Always assume that the graph G = (V, E) is undirected. Do not assume that edge weights are distinct unless this is specifically stated.

1. If graph G has more than $|V|-1$ edges, and there is a unique heaviest edge, then this edge cannot be part of a minimum spanning tree
2. If G has a cycle with a unique heaviest edge e, then e cannot be part of any MST
3. Let e be any edge of minimum weight in G. Then e must be part of some MST
4. If the lightest edge in a graph is unique, then it must be part of every MST.
5. If e is part of some MST of G, then it must be a lightest edge across some cut of G