Exact and Approximate Algorithms for Finding k-Shortest Paths with Limited Overlap

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MOTIVATION

Finding multiple short yet different routes between two locations in a road network is a problem with various real-world applications:

- ✓ Commercial Route Planners
- ✓ Evacuation planning
- ✓ Humanitarian aid

k-SPwLO PROBLEM

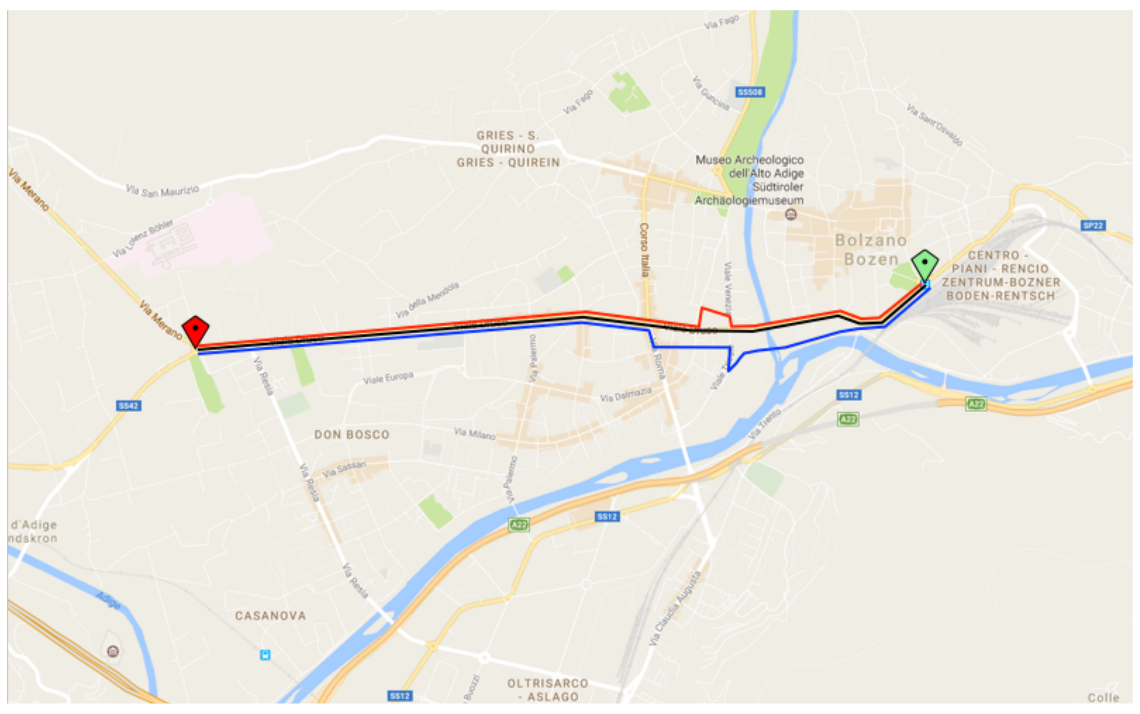
Given a source *s* and a target *t*, the **k-SPwLO** is a set of *k* paths from *s* to *t*, sorted by length in increasing order, such that:

(a) the set includes the shortest path $p_0(s \rightarrow t)$,

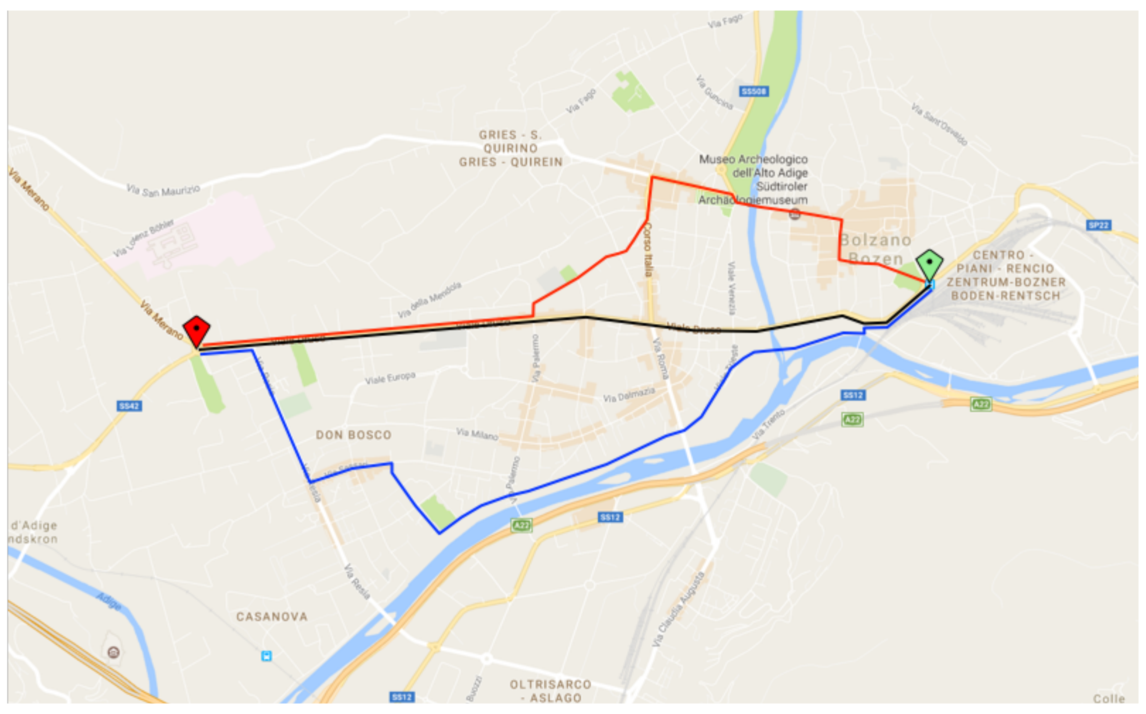
(b) every path is dissimilar to its predecessors w.r.t. a similarity threshold θ,

(c) all *k* paths are as short as possible.

EXAMPLE



K-SP: Short but very similar paths



k-SPwLO: Longer but dissimilar paths

Result Set 1 - K-Shortest Paths

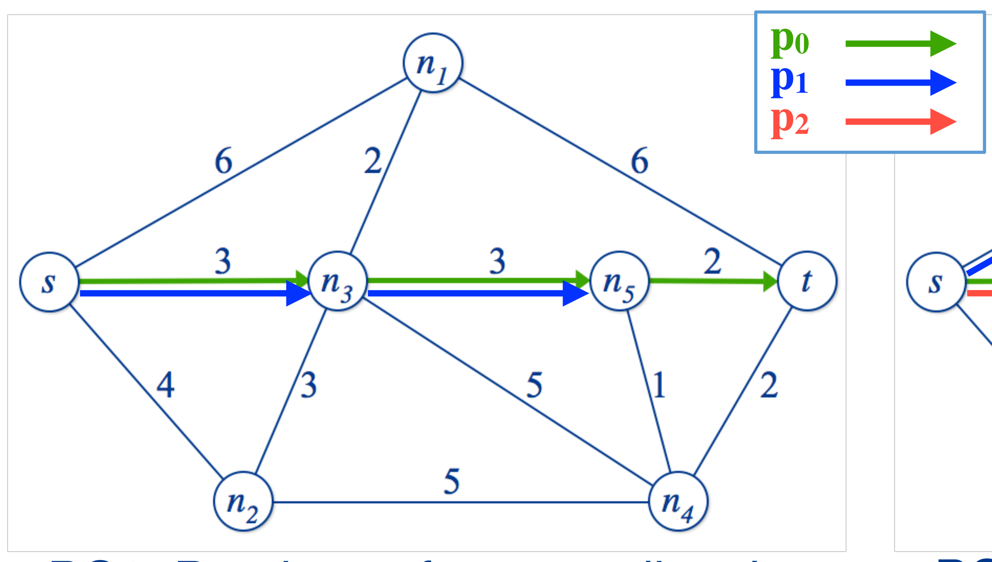
p	*ℓ*	*Sim(p,p₀)*	*Sim(p,p₁)*
Shortest path	3.4 km	-	-
1st alternative	3.6 km	95%	-
2nd alternative	3.7 km	80%	76%%

Result Set 2 - kSPwLO (θ = 50%)

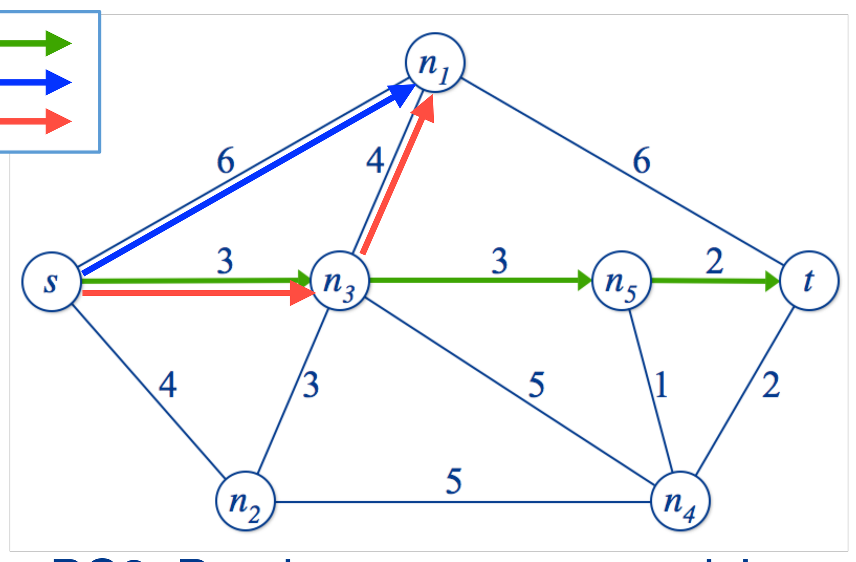
p	*ℓ*	*Sim(p,p₀)*	*Sim(p,p₁)*
Shortest path	3.4 km	-	-
1st alternative	4.1 km	47%	-
2nd alternative	4.5 km	20%	6%

EXACT ALGORITHM

- **MultiPass**
  - ✓ Traverses the road network k-1 times
  - ✓ Restarts the expansion after each alternative path is found
  - ✓ Employs two powerful pruning criteria



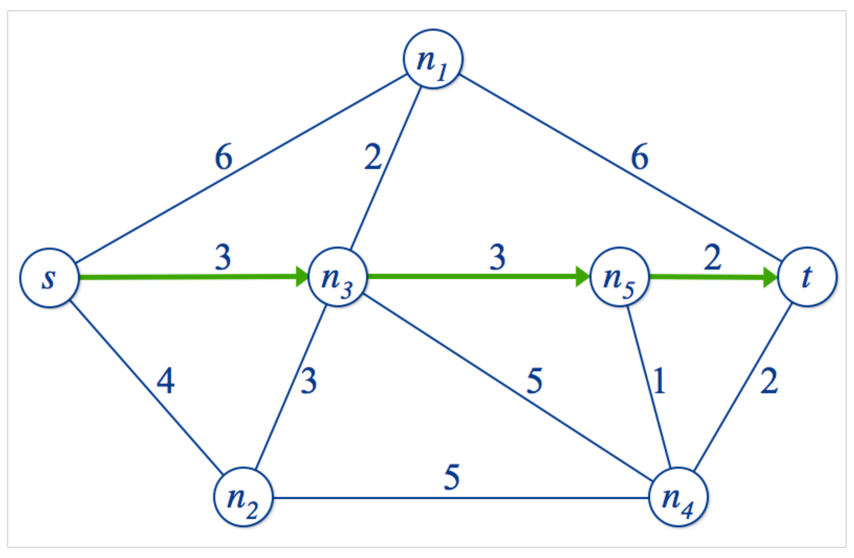
PC1: Pruning p_1 for exceeding the similarity threshold θ=50%



PC2: Pruning p_2 as non-promising

APPROXIMATE ALGORITHMS

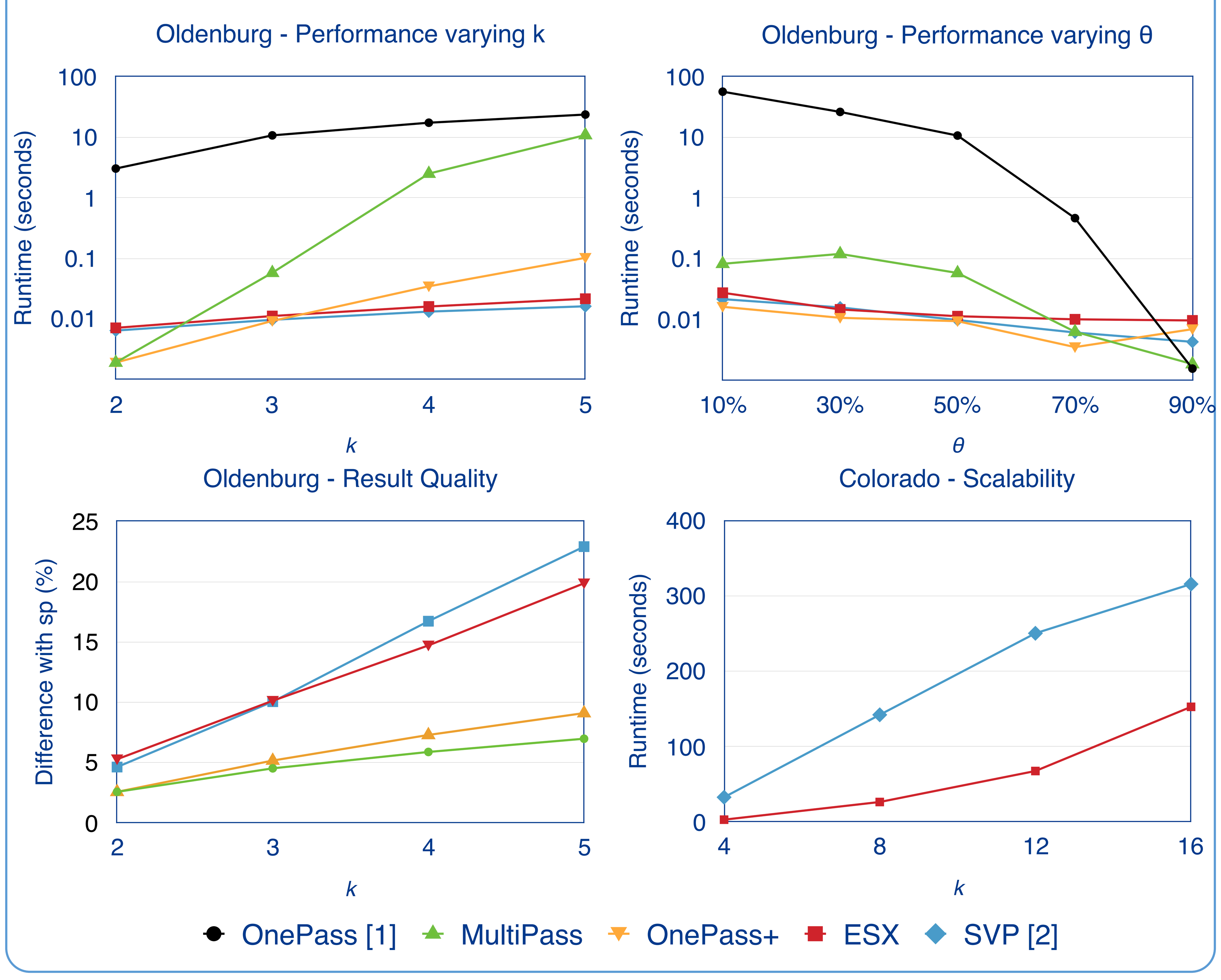
- **OnePass+**
  - ✓ Traverses the road network once
  - ✓ Prunes paths with both PC1 and PC2
  - ✓ Does not guarantee that the exact solution will be found
- **ESX (Edge Subset Exclusion)**
  - ✓ Iteratively removes edges from the road network that lie on some already computed alternative path
  - ✓ Computes the shortest path on the updated graph
  - ✓ Continues until a sufficiently dissimilar path is found



1. **Remove edge (n_5,t)**
   $p_1 = \langle (s,n_3)(n_3,n_5)(n_5,n_4)(n_4,t) \rangle$
   Path p1 is not an alternative
2. **Remove edge (n_3,n_5)**
   $p_2 = \langle (s,n_3)(n_3,n_4)(n_4,t) \rangle$
   Path p2 is an alternative

Sample execution of ESX (θ=50%)

EXPERIMENTAL EVALUATION



SUMMARY

MultiPass: computes the optimal result but is practical only for small road networks

OnePass+: good approximation and practical for larger road networks than MultiPass

ESX: less accurate but practical even for large road networks and large values of *k*

[1] T. Chondrogiannis, P. Bouros, J. Gamper, U. Leser. Alternative routing: k-shortest paths with limited overlap. *In Proc. of the 23rd ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, pages 68:1–68:4, 2015.

[2] I. Abraham, D. Delling, A.V. Goldberg, R.F. Werneck. Alternative routes in road networks. *Journal of Experimental Algorithmics*, 18:1–17, 2013.