Comparing Approaches to Preference Dominance for Conversational Recommenders

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***Abstract*—A conversational recommender system iteratively shows a small set of options for its user to choose between. In order to select these options, the system may analyze the queries tried by the user to derive whether one option is dominated by others with respect to the user's preferences. This paper describes a framework for preference dominance. Two instances of the framework are developed for query suggestion in a conversational recommender system. The first instance of the framework is based on a basic quantitative preferences formalism, where products are compared using sums of weights of features. The second is a qualitative preference formalism, using a language that generalizes CP-nets, where models are a kind of generalized lexicographic order. A key feature of both methods is that deductions of preference dominance can be made efficiently, since this procedure needs to be applied for many pairs of products. We show that, by allowing the recommender to focus on undominated options, which are ones that the user is likely to be contemplating, both approaches can dramatically reduce the amount of advice the recommender needs to give to a user compared to what would be given by systems without this kind of reasoning.**

I. Introduction

In an era of overwhelming choice, *recommender systems* are a new source of assistance, helping their users to decide which goods, services or information to purchase or consume. [1], [2]. These systems infer user preferences from data gathered either explicitly, e.g., in the form of product ratings, or implicitly by observing user behaviour.

No matter how good recommender systems become, they are unlikely ever to be sufficiently prescient that their first set of recommendations always satisfies the user. *Conversational recommender systems* allow for this, and recognise that their users may be willing and able to reveal more of their constraints and preferences, over a short dialogue. This is also an opportunity for the recommender system to guide the user by asking questions, giving advice, displaying candidate products, and giving explanations [3], [2], [4], [5].

Conversational recommender systems typically involve iteratively showing the user a small set of options (e.g., products) for them to choose between. To select an appropriate set to display at each stage, from a much larger collection of options, the recommender needs information regarding which options are likely to be preferred to others by the user, based on previous responses the user has given in the dialogue; if one assumes that the user has some kind of preference relation over products, this amounts to determining if certain products are dominated according to this preference relation.

In 2007, Bridge & Ricci introduced a new kind of conversational recommendation, which they call *Information Recommendation* [6]. In Information Recommendation, summarized in Section II, the recommender infers constraints on the user's preference relation from her previous contributions to the dialogue. The recommender must reason with the constraints to determine whether certain products dominate others and to suggest actions the user might take that are compatible with these preferences.

Information Recommendation hence requires a framework for this kind of reasoning, where a set of models of the user is assumed, each with an associated preference ordering, along with a satisfaction relation between models and statements of constraints on preferences expressed in an appropriate language. Within this framework, given a set of constraints, we infer that one product is preferred to another if this preference holds for all models satisfying the constraints. We describe this framework in more detail in Section III.

Two instances of this framework are developed and presented in this paper. The first, described in Section IV, is based on a simple quantitative preferences formalism, involving a sum of weights, with an associated language of linear inequalities. This is a very commonly used model for preference representation, specifically, in Multi-Attribute Utility Theory (MAUT) [7]. It is widely used in recommender systems, e.g. [8]. And it is the approach to preference dominance taken in Bridge & Ricci's original Information Recommendation paper.

The second instance of the framework (Section V) is a qualitative preference formalism, where models are a kind of generalised lexicographic order, and constraints are expressed as comparative preference statements in a language generalising CP-nets [9]. This has not been used in Information

Recommendation or any other recommender system before.

Section VI explains how the two instances of the framework can be used within Information Recommendation. Finally, Section VII describes how the implementations of the approaches have been experimentally tested.

The contributions of this paper include the following. It shows how a comparative preferences approach can be used in a conversational recommender. Approaches of this kind have not been used in recommender systems before. Using them is significant because of their expressiveness. This manifests in two ways. The first, as we show in Section VI, is that there are nuances about user preferences that the recommender can capture in the constraints when using comparative preferences that cannot be captured when using the sum of weights approach. The second, as we discuss in Section VIII, is that the comparative preferences approach allows a recommender to express a wider range of constraints than those envisaged by the original Information Recommendation work, including, e.g., statements about conditional preferences. Furthermore, through our experiments, we reveal the viability of the comparative preferences approach. It is sufficiently efficient in a practical (rather than theoretical) sense and it allows inferences to be drawn that are strong enough on the one hand to give useful advice but not so strong on the other hand to incorrectly eliminate options that the user prefers. Indeed, its inferences are stronger than those drawn in the sum of weights approach, resulting in giving the user shorter advice but still without compromising success in directing the user to the best product, and we find that these experimental results are robust in the sense that they pertain irrespective of how the users true preference are represented.

II. Information Recommendation

Information Recommendation is concerned with helping a user to find a product to purchase or consume. Throughout this paper, we use hotels as the example product. The user repeatedly edits and resubmits a query until she finds a product that she wants. For example, she might submit a query that asks for a hotel that has a swimming pool and a restaurant. If she is advised that hotels that satisfy her query do exist, she might be encouraged to edit her query and see whether there are hotels that additionally allow pets. Perhaps if there are not, she might edit her original query to one that sacrifices the swimming pool in favour of allowing pets. This is a hit-and-miss process that can be improved by the intervention of a recommender system. The recommender system: observes the user's actions (her queries); infers constraints on the user's preferred products; uses these inferences to deduce which queries a user is likely to try next; and advises the user to avoid those that cannot be satisfied. Below we describe this recommender system in more detail.

A. The products

In this paper, we assume a collection of Boolean-valued features $V = \{F_1, \dots, F_n\}$. The features are intended to relate to a set of products that the user is interested in choosing between; for example, in choosing a hotel room, one feature might be whether the hotel has a swimming pool.

Define a configuration α to be a mapping from $\{1, \dots, n\}$ to $\{1, 0\}$. Configuration α can also be thought of as a set of features, i.e., all features F_i such that $\alpha(i) = 1$. For convenience we will write a configuration such as $(1, 0, 1)$ as $f_1 \bar{f}_2 f_3$.

In Information Recommendation, configurations can be thought of as queries over the set of features. Hence, we will often denote them by q. If a user issues query q and if $f_i \in q$, this means that the user is interested in products that have the ith feature. In accordance with most Web-based product search systems, $f_i \notin q$ means only that the user has not (yet) declared any interest in feature F_i; it does not mean that the user wants products that lack the ith feature. So, for example, if q is $f_1 \bar{f}_2 f_3$, the user wants a product that has features f_1 and f_3 and has said nothing yet about f_2.

A subset of the configurations correspond to products that are available to the user. A query is *satisfiable* if and only if there exists a product which has all the features in the query; otherwise, it is *unsatisfiable*. Users cannot be expected to know in advance which queries are satisfiable and which are not, although they may have incomplete knowledge of this.

B. The dialogue

In the kind of system envisaged in [6], the user submits an initial query, typically one that is quite under-specified: 'to test the water'. In our experiments (Section VII) we use an empty initial query. Let this query be known as the *current query*, q.

The recommender system does not know the user's preferences and does not ask about them. It may only infer them from the sequence of queries that the user submits. As the dialogue proceeds, the recommender system will infer constraints on the user's preferences and express them as statements in a language $\mathcal{L}$. We will denote the current set of statements by Φ. Initially Φ may contain a set of 'background' assumptions. In particular, for example, we will want to express the idea that including a feature in a query is at least as good as not including it. Statements will be added to Φ as the dialogue proceeds. For example, if the user's query requests a certain feature, we may plausibly infer that this feature is more important than the ones not included in the query.

The interaction between the user and the recommender system proceeds as follows:

1) The recommender system analyzes q, with particular regard to differences between current query q and the queries the user might alternatively have submitted. The system induces some additional constraint on the user's preferences and adds corresponding statements to Φ.
2) The recommender system generates a set of candidate next possible queries and prunes this set to those that are satisfiable and undominated (see below). It advises the user to confine her next query to this set.
3) The user chooses and submits her next query. This becomes the new current query q. In the experiments reported in Section VII, we arrange that the user always

chooses one of the queries that the system advises (although this might not be so in practice.)

Steps 1–3 are repeated until the user is satisfied with q or the set of undominated, satisfiable candidates is empty, in which case as far as the recommender system is concerned q cannot be bettered. At this point, the user can request to see the products that satisfy q.

The goal of the recommender system is to give the advice that has the greatest value. We consider this to be that which minimizes the total quantity of advice given and the dialogue length, while guiding the user to the best product.

During step 2 above, the recommender system computes the following three sets of queries:

- *Candidates:* Candidate queries are ones which are close, in a particular sense, to the current query. Each is a low-cost edit to the current query. For example, if $f_i \notin q$, the set of candidates will include the query that results from adding just feature f_i to q.
- *Satisfiables:* The recommender system never includes unsatisfiable queries in its advice: they make interaction length longer without leading the user to the best product. Hence, the system eliminates from *Candidates* those queries which are unsatisfiable; the remaining queries are called the *Satisfiables*.
- *Undominated*: Then, the system eliminates from *Satisfiables* each query which is dominated by (i.e., worse than) some other member of *Satisfiables*; the remaining set of queries is called *Undominated*. The dominance relation is based on what is induced in step 1 above.

In effect, the system's advice to the user is to confine her next query to a set which the system knows are satisfiable and believes the user is likely to try next (since, according to what the system has inferred about the user's preferences, they are ones that are not dominated by other satisfiable queries).

We will now explain how to obtain the three sets of queries.

Generating the candidates: Real user behaviour in query editing tends to proceed with modifications of limited 'reach'. Hence, following [6], we define *Candidates* as the set of queries which we obtain by applying three editing operations, *Add*, *Switch* and *Trade*, to the current query. These operations are defined as follows. Given current query q and $f_i \notin q$, then $Add(q, f_i)$ adds just feature f_i to q, giving the new query $q \cup \{f_i\}$, which we sometimes write as q^i. $Switch(q, f_i, f_j)$ where $f_i \in q, f_j \notin q, i \neq j$ discards feature f_i in favour of feature f_j, giving the new query $q \setminus \{f_i\} \cup \{f_j\}$. Finally, $Trade(q, f_i, f_j, f_k)$ where $f_i \in q, f_j \notin q, f_k \notin q, i \neq j, i \neq k, j \neq k$ discards feature f_i and introduces features f_j and f_k.

Checking satisfiability: As defined earlier, a query is satisfiable if and only if there exists a product which has all the features present in the query. If products are stored explicitly in a database, satisfiability of a candidate query can be checked by a scan of the database.

Checking for dominance: The final pruning of the satisfiable candidate queries is performed using one of the two instances of the framework for dominance of preferences that we will explain in Sections III, IV and V. In either case, $q \in$ *Satisfiables* is pruned if it is strictly dominated, i.e., dominated according to relation $\succ_\Phi$, by $q' \in$ *Satisfiables*.

III. A Framework for Dominance of Preferences

We assume a set Ω of possible configurations (as defined in section II). We would like to generate some kind of (partially ordered) preference relation $\succcurlyeq$ on Ω, based on previous information we have received regarding the user's preferences. This section describes a framework for generating such a relation. We define two instances of the framework in Sections IV and V.

In order to make non-trivial inferences regarding the user's relative preferences over configurations, we will have to make some assumptions. We assume:

- A set of models $\mathcal{M}$, each of which is intended to represent a possible user (or way the user could be). Associated with each $M \in \mathcal{M}$ is a total pre-order $\succcurlyeq_M$ on configurations, i.e., a reflexive, transitive and complete relation (so for all configurations α and β, we have either $\alpha \succcurlyeq_M \beta$ or $\beta \succcurlyeq_M \alpha$ or both).
- A formal language $\mathcal{L}$ whose statements express constraints on the user's preferences.
- A relation $\models$ between $\mathcal{M}$ and $\mathcal{L}$. For $M \in \mathcal{M}$ and $\varphi \in \mathcal{L}$, we interpret $M \models \varphi$ to mean that φ holds for the preferences of M.

Given a particular set $\Phi \subseteq \mathcal{L}$ of statements, we consider the orderings on configurations which hold for every model satisfying statements Φ. Formally, we define relation $\succcurlyeq_\Phi$ on configurations as follows: $\alpha \succcurlyeq_\Phi \beta$ if and only if $\alpha \succcurlyeq_M \beta$ for all M satisfying (every member of) Φ. It follows that $\succcurlyeq_\Phi$ is a pre-order (a reflexive and transitive relation) on configurations. Now, $\alpha \succcurlyeq_\Phi \beta$ means that every user who agrees with Φ considers that configuration α is at least as desirable as configuration β (assuming this particular model of users). It is possible that we also have $\beta \succcurlyeq_\Phi \alpha$, in which case every user considers that α and β are equally desirable. We define the relation $\succ_\Phi$ to be the strict part of $\succcurlyeq$, so that $\alpha \succ_\Phi \beta$ if and only if $\alpha \succcurlyeq_\Phi \beta$ and $\beta \not\succcurlyeq_\Phi \alpha$. Relation $\succ_\Phi$ is irreflexive and transitive. We say that *Given* Φ*,* α *strictly dominates* β, if $\alpha \succ_\Phi \beta$, i.e., if all users (represented by models in $\mathcal{M}$) agreeing with Φ regard α as at least as preferable as β, and at least one such user regards α as strictly preferable to β.

IV. Sum of Weights Model of the User

In this section we consider our first kind of model of the user's preferences, where it is assumed that a user assigns a weight to each feature, and configurations are compared on the sum of weights of the associated set of features.

A. Models

The set of models is the set of all vectors of weights $w = (w_1, \dots, w_n)$, where w_i is a non-negative real number. w_i is the weight assigned to feature F_i. Given a weights vector w, the overall value $w(\alpha)$ of a configuration α is the sum of weights of the features included in α, i.e., $w(\alpha) = \sum_{i:\alpha(i)=1} w_i$. This is used to define the ordering on configurations. We define the preference relation $\succcurlyeq_w$ for model w by $\alpha \succcurlyeq_w \beta$ if and only if $w(\alpha) \geq w(\beta)$, i.e., iff $\sum_i w_i(\alpha(i) - \beta(i)) \geq 0$. Thus $\succcurlyeq_w$ is a total pre-order on configurations.

B. Constraint language

The language consists of statements of the form $\alpha \geq \beta$, where α and β are configurations.

C. Dominance relation

Weight vector w is defined to satisfy $\alpha \geq \beta$ if $\alpha \succcurlyeq_w \beta$. Let Φ be a set of statements. The definitions in Section III lead to the following definition of $\succcurlyeq_\Phi$, the induced preference relation given constraint statements Φ:

For configurations α and β, $\alpha \succcurlyeq_\Phi \beta$ if and only if $\alpha \succcurlyeq_w \beta$ for all weight vectors w satisfying Φ. Dominance relation $\succ_\Phi$ is then the strict part of $\succcurlyeq_\Phi$. When we want to make explicit comparisons with the second instance of the framework (Section V), we will use the notation $\succcurlyeq_\Phi^{sw}$ for $\succcurlyeq_\Phi$.

Example 1: Let Φ be the pair of statements: $f_1\bar{f}_2f_3 \geq \bar{f}_1f_2f_3$, and $f_1f_2\bar{f}_3 \geq \bar{f}_1f_2f_3$. Let α be the configuration $f_1\bar{f}_2\bar{f}_3$, and let β be the configuration $\bar{f}_1f_2f_3$. Weights vector w satisfies the constraint $f_1\bar{f}_2f_3 \geq \bar{f}_1f_2f_3$ if and only if $w(f_1\bar{f}_2f_3) \geq w(\bar{f}_1f_2f_3)$, i.e., $w_1 + w_3 \geq w_2 + w_3$, which holds if and only $w_1 \geq w_2$. By similar reasoning, w satisfies Φ if and only if $w_1 \geq w_2$ and $w_1 \geq w_3$. Also, w satisfies $\alpha \geq \beta$ if and only if $w_1 \geq w_2 + w_3$. Thus Φ does not entail $\alpha \geq \beta$, so we do not have $\alpha \succcurlyeq_\Phi^{sw} \beta$, since, for example, weights vector w with $w_1 = 4$, $w_2 = 2$ and $w_3 = 3$ satisfies Φ but does not satisfy $\alpha \geq \beta$. ■

Example 2: Suppose now that there are four features, and let Ψ be the pair of statements $f_1\bar{f}_2f_3f_4 \geq \bar{f}_1f_2f_3f_4$ and $f_1f_2\bar{f}_3\bar{f}_4 \geq f_1\bar{f}_2f_3\bar{f}_4$. With the sum of weights semantics this implies $f_1f_2\bar{f}_3f_4 \geq \bar{f}_1f_2f_3f_4$, since the first statement implies $w_1 \geq w_2$, and the second statement implies $w_2 \geq w_3$, implying $w_1 \geq w_3$, which implies that the third statement is satisfied. We therefore have $f_1f_2\bar{f}_3f_4 \succcurlyeq_\Psi^{sw} \bar{f}_1f_2f_3f_4$. ■

D. Computation of preference

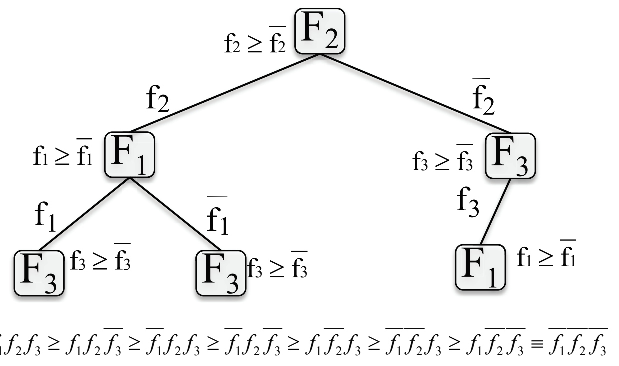
We wish to determine if $\alpha \succcurlyeq_\Phi \beta$, for given configurations α and β. Let *Pos* be the set of constraints $w_i \geq 0$, for $i = 1, \ldots, n$, representing the non-negativity of the weights (and corresponding to the assumption that including a feature is always at least as good as not including it). The definition implies that $\alpha \succcurlyeq_\Phi \beta$ if and only if the linear constraints $\Phi \cup Pos$ (over real-valued variables w_i) entail the constraint $\sum_i(\alpha(i) - \beta(i))w_i \geq 0$.

For implementation of this using a Linear Programming solver, it can be convenient to express it as a linear optimisation problem. Define A_{min} to be the minimum value of $\sum(\alpha(i) - \beta(i))w_i$ subject to constraints $\Phi \cup Pos$. It can be easily shown that $\alpha \succcurlyeq_\Phi \beta$ if and only $\Phi \cup Pos$ entails $\sum_i(\alpha(i) - \beta(i))w_i \geq 0$ if and only if $A_{min} \geq 0$.

V. Comparative Preferences Model of the User

A. Models

In our second approach, models are a kind of generalised lexicographic order, called *cp-trees* [10], which are similar to search trees used for solving constraint satisfaction problems. Figure 1 gives an example of a cp-tree. Each node is labeled with a variable. The root is labeled by the most important variable, F_2 in this example. Each node is associated also with a preference ordering of the values of the variable. This local ordering in the case of the nodes in the example is $f_i \geq \bar{f}_i$, where f_i means F_i is included ($F_i = 1$) and $\bar{f}_i$ means that F_i is not included ($F_i = 0$). This ordering captures the requirement that including a feature is never worse than not including it.



$f_2 \geq \bar{f}_2$
F_2
f_2
$\bar{f}_2$
$f_1 \geq \bar{f}_1$
F_1
$f_3 \geq \bar{f}_3$
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$f_1f_2f_3 \geq f_1f_2\bar{f}_3 \geq \bar{f}_1f_2f_3 \geq \bar{f}_1f_2\bar{f}_3 \geq f_1\bar{f}_2f_3 \geq \bar{f}_1\bar{f}_2f_3 \geq f_1\bar{f}_2\bar{f}_3 \equiv \bar{f}_1\bar{f}_2\bar{f}_3$

Fig. 1. A cp-tree σ, along with its associated ordering on outcomes $\succcurlyeq_\sigma$.

Two configurations α and β are compared first on this most important variable. If they do not agree on this variable then the comparison is settled: in the example, if α contains feature F_2 and β does not, then α is better than β. This happens, for example, if α is $\bar{f}_1f_2\bar{f}_3$ and β is $f_1\bar{f}_2f_3$. Otherwise, α and β agree on the most important variable. The user may then have a next most important variable (labeling a child node); this can depend on the value assigned to the most important variable (signified by the value on the edge from parent to child). For this reason, this model allows *conditional preferences*. If there is no such next important variable, then α and β are considered equally preferable according to this cp-tree. Thus the cp-tree σ generates a total pre-order $\succcurlyeq_\sigma$ on outcomes.

Note that each node in the cp-tree in Figure 1 is associated with a single variable. In fact, we can allow a more general representation, where at most γ variables (where, in this paper, $\gamma = 1$, 2 or 3) are associated with a node, along with a total pre-order over the assignments to that set of at most γ variables. For example, if a node is associated with the pair of variables $Y = \{F_2, F_3\}$ then the local ordering is over assignments to Y, and might be e.g., $f_2f_3 \geq f_2\bar{f}_3 \equiv \bar{f}_2f_3 \geq \bar{f}_2\bar{f}_3$. Let $\mathcal{M}(\gamma)$ be the set of cp-trees over Ω, where the set of variables associated to a node involves at most γ variables. A 1-cp-tree over Ω is defined to be an element of $\mathcal{M}(1)$, i.e., a cp-tree with a single variable being associated with each node. The local ordering associated with a node associated with feature F_i, must then be $f_i \geq \bar{f}_i$, since including a feature is always at least as good as not including it. For full definitions of cp-trees, see [10].

B. Constraint language

The language will include statements that compactly express comparative (and sometimes conditional) preferences among configurations. There is a substantial and fast growing literature on this topic in the AI and Philosophy of Science communities, see e.g., [11], [9], [12], [10].

The language includes comparative preference statements φ of the form $p \geq q \parallel T$, where P, Q and T are subsets of the features V, and p is an assignment to P (i.e., a function from P to $\{0,1\}$), and q is an assignment to Q. Informally, the statement $p \geq q \parallel T$ represents the following: p is preferred to q if T is held constant. More formally, the preference relation $\succcurlyeq$ satisfies the statement $p \geq q \parallel T$ if and only if $\alpha \succcurlyeq \beta$ for all configurations α and β such that (i) α extends p (i.e., α restricted to the subset of the features in P equals p), (ii) β extends q, and (iii) α and β agree on T: $\alpha(T) = \beta(T)$.

A very important kind of statement is one expressing the constraint that one configuration, α, is preferred over another, β. This can be written as $\alpha \geq \beta \parallel \emptyset$; we also write such a preference statement as $\alpha \geq \beta$.

Since including a feature is at least as good as not including it, we always include, in the set Φ, the statement $f_i \geq \overline{f_i} \parallel V \setminus \{F_i\}$ for each feature F_i. Hence our language is strongly related to *Conditional Importance Networks* [13].

C. Dominance relation

We define the set of models of users to be the $\mathcal{M}(\gamma)$ for some $\gamma = 1, 2$, or 3. Each cp-tree σ generates a total pre-order $\succcurlyeq_\sigma$ on configurations. Let σ be a cp-tree and let φ be a statement in the constraint language. σ satisfies φ if and only if $\succcurlyeq_\sigma$ satisfies φ. In the same way as in Section IV, we define, for a given set of statements Φ, $\succcurlyeq_\Phi$ (or more precisely, $\succcurlyeq_\Phi^{cp_\gamma}$), in the following way: $\alpha \succcurlyeq_\Phi \beta$ holds if and only if $\alpha \succcurlyeq_\sigma \beta$ holds for all cp-trees σ in $\mathcal{M}(\gamma)$ satisfying Φ.

Example 1 continued: With the cp-tree semantics, when $\gamma = 1$, the pair of statements Φ ($f_1\bar{f}_2f_3 \geq \bar{f}_1f_2f_3$, and $f_1f_2\bar{f}_3 \geq \bar{f}_1f_2f_3$) implies the preference statement $f_1\bar{f}_2\bar{f}_3 \geq \bar{f}_1f_2f_3$, so we have $f_1\bar{f}_2\bar{f}_3 \succcurlyeq_\Phi^{cp_1} \bar{f}_1f_2f_3$. The reason is that, for any 1-cp-tree σ satisfying $f_1\bar{f}_2f_3 \geq \bar{f}_1f_2f_3$, the most important feature must be either F_1 or F_3. (If F_2 were the most important feature, then we would not have $f_1\bar{f}_2f_3 \succcurlyeq_\sigma \bar{f}_1f_2f_3$, because the local ordering is $f_2 \geq \bar{f}_2$, since the presence of a feature is never worse than its absence.) Similarly, if 1-cp-tree σ satisfies $f_1f_2\bar{f}_3 \geq \bar{f}_1f_2f_3$, then the most important feature must be either F_1 or F_2. Hence for any 1-cp-tree satisfying Φ, F_1 is the most important variable. The root node then decides pair of configurations $f_1\bar{f}_2\bar{f}_3$ and $\bar{f}_1f_2f_3$. Since the local ordering of this node must be $f_1 \geq \bar{f}_1$, we have $f_1\bar{f}_2\bar{f}_3 \succeq_\sigma \bar{f}_1f_2f_3$. Hence we have $f_1\bar{f}_2\bar{f}_3 \succcurlyeq_\Phi^{cp_1} \bar{f}_1f_2f_3$. The qualitative and lexicographic nature of the cp-trees semantics ensures this inference, in contrast with the numerical sum of weights method, which did not. ■

Example 2 continued: In contrast with the sum of weights semantics, Ψ does not imply $f_1f_2\bar{f}_3f_4 \geq \bar{f}_1f_2f_3f_4$. To show this we can construct a 1-cp-tree σ with F_4 as the most important (root node) variable, and where, given f_4, F_3 is more important than F_1 which is more important than F_2, and given $\bar{f}_4$, F_2 is more important than F_3 which is more important than F_1. σ then satisfies Ψ, but not $f_1f_2\bar{f}_3f_4 \geq \bar{f}_1f_2f_3f_4$.

A key issue here is that cp-trees can represent *conditional* preferences: the preferences can be different given f_4 from those given $\bar{f}_4$. In contrast, the sum of weights semantics assumes preferential independence, so preferences are not conditional at all, which is why the inference holds for the sum of weights semantics. ■

The pair of examples shows that the two preference dominance techniques are incomparable: $\succcurlyeq_\Phi^{cp_1}$ can sometimes include preferences not included in $\succcurlyeq_\Phi^{sw}$, and vice versa.

D. Computation of preference

Given set of statements Φ and configurations α and β, we can determine in polynomial time whether or not $\alpha \succcurlyeq_\Phi \beta$ holds, using the algorithm given in [10], as shown by Theorem 1 in [10].

VI. Inducing Constraints on Preferences in Information Recommendation

Now that we have explained the two instances of the framework for dominance of preference, it remains for us to return to Information Recommendation to explain what the system induces in step 1 above (Section II), when it observes the user's queries. We explain this below for each of the two preference models. Due to space limits, we confine our treatment only to what can be inferred when the system observes the user adding a feature f_i to a query q, $Add(q, f_i)$. However, our system induces preference statements for *Switch* and *Trade* as well as for *Add*.

A. Inducing constraints in the sum of weights model

If the user has added feature f_i to query q, giving rise to new query q^i, then statements $q^i \geq q^j$ are induced for all $f_j \notin q$, $i \neq j$ unless $Add(q, f_j) = q^j$ is unsatisfiable. This assumes that the new query is preferred to other satisfiable queries that could have been generated by adding other features. This implies that the weight vector satisfies the linear inequality $w_i \geq w_j$. However, we do not infer $q^i \geq q^j$ in all cases. In particular, we do not infer it if $Add(q, f_j)$ is unsatisfiable. Users may have (incomplete) knowledge of which queries are unsatisfiable: if she knows a query is unsatisfiable, then she will not submit it. We 'play it safe': when q^j is unsatisfiable, in case the user knows this, we do not assume that the query that she does submit has higher weight than this unsatisfiable query.

B. Inducing constraints in the comparative preferences model

Again consider the situation where the user has chosen to add feature f_i rather than feature f_j. For this model, there are alternative statements one might induce from this decision by the user. We consider two, each being a kind of counterpart for the constraint $w_i \geq w_j$ induced for the sum of weights approach. It is an advantage of the comparative preferences model that it can express nuances that the sum of weights model cannot.

- **Basic:** Let q be the current query, let q^i be the current query q with the feature f_i added, and let q^j be q with the feature f_j added. A basic, somewhat conservative, approach is to just model the preference of feature i over feature j by the preference statement: $q^i \geq q^j || \emptyset$, i.e., $q^i \geq q^j$, which just expresses a preference for q^i over q^j.

- **Importance:** Alternatively, and less conservatively, we can induce $f_i \geq \overline{f_i} || V \setminus \{F_i, F_j\}$, which says that the presence or not of the feature F_i is more important than the choice of F_j. Thus, whatever the state of the feature F_j in the query the user will prefer F_i to be present in the query so that this feature is included in the best product.

Note too that in either case we ensure that the recommender system 'plays it safe' when inducing preference statements, in the same way that we explained for the Sum of Weights Model, by not inducing preferences over unsatisfiable queries.

VII. Experiments

In this section, we report experiments with simulated users that demonstrate the feasibility of using both the Sum of Weights Model and the Comparative Preferences Model within the Information Recommendation system. It is a common practice to use simulated interactions to initially test alternative algorithms for conversational systems [14], [15]. Simulations can pinpoint the main deficiencies of the algorithms and can be used to compare a large number of alternative approaches, as in our case. Experiments with real users cannot be used to extensively test alternative dialogue control algorithms, even if it is clear that the ultimate evaluation of the effectiveness of a conversational system has to be made online.

We use two separate product databases, that we scraped from the Web, each describing hotels by their amenities expressed as Boolean features such as *airport shuttle*, *pets permitted*, *restaurant on-site*, etc. The Marriott-NY database records 9 features about 81 hotels; many offer the same amenities, and so there are 36 distinct products in the database. The Trentino-10 database records 10 features for 4056 hotels, of which 133 are distinct.

The simulated users in our experiments behave in the following somewhat idealized way: within a dialogue, they do not try queries that they have tried earlier in the dialogue; they are aware of their own preferences and never choose a next query that would be inferior to the current one; and they take heed of all advice given, i.e., if the recommender system tells them to confine their next query to a certain set, then they do so; indeed they choose the best possible query from this set. (In the terminology of [6], these are *optimizing users*.)

For a simulated user to make choices about which among the queries in the recommender's advice is the best one for it to submit next, the simulated user must be assigned a set of *true preferences*. These are generated randomly. We arrange that they are known to the simulated user and used by that user for query selection, but they are not known to the recommender system, which knows only what it induces about user preferences and adds to Φ when observing user query behaviour.

In the same way that the recommender system can represent induced constraints on users' preferences in either the Sum of Weights Model or the Comparative Preferences Model, equally the simulated users' true preferences can be represented in either model. If the true preferences are represented in the Sum of Weights Model, then recommenders that induce constraints on preferences in the Sum of Weights Model may have an advantage over recommenders that are using the Comparative Preferences Model, and vice versa. We control for this problem by showing results below that pair both ways of representing true preferences with recommenders that use both ways of representing induced preferences.

In the experiments, one recommender system uses the Sum of Weights Model; six use the Comparative Preferences Model, differing first on which of the two alternative preference statements they infer (Basic or Importance), and on their value for γ (1, 2 or 3). For each pairing of a user with a recommender system, we ran 500 simulated dialogues. In total then, we are reporting results for 2 databases $\times$ 2 ways of representing true preferences $\times$ 7 recommenders $\times$ 500 dialogues, which is 14000 runs of the system.

We stress that a recommender that does not remove from its advice those queries that are dominated would suggest to the user a large number of next possible queries, i.e., all those that are satisfiable. This has been a common approach in earlier conversational systems and it is current practice in many conventional Web applications which allow the user to specify a preferred value for a feature (e.g., using a checkbox) and then show the number of products that satisfy that condition. The whole advantage of Information Recommendation rests on being able to prune the dominated queries, showing only the undominated ones, on the basis that these best match the user's preferences. Hence, in the experiments we compare the pruning rates achieved by using the Sum of Weights Model with those achieved by the six recommender systems that use the Comparative Preferences Model. The pruning rate is defined as follows:

$$\textit{pruning rate} = \frac{|\textit{Satisfiables} \setminus \textit{Undominated}|}{|\textit{Satisfiables}|} \times 100$$

It shows the extent to which an approach eliminates what it takes to be inferior satisfiable candidate queries from its advice. Other things being equal, the shorter the advice the better, as this reduces the choice the user has to make.

A. *Representing true preferences in the sum of weights model*

In our first set of experiments, we set the user's true preferences by randomly generating weight vectors over product features. The pruning rates in this case are shown in Table I.

The table shows that, in nearly all settings, the Comparative Preferences approach is pruning non-optimal queries a little more than the Sum of Weights approach. For example, the Comparative Preferences Model using *Basic* preference statements and with $\gamma = 1$ eliminates 87.5% of satisfiable candidates in dialogues about the Marriott-NY database, where the Sum of Weights approach prunes 87.38%. The table also shows that, on the whole, with the Comparative Preference Model, the amount of pruning increases as the preference statements induced become less conservative (from Basic to

TABLE I
THE PRUNING RATES (TRUE PREFERENCES REPRESENTED IN SUM OF WEIGHTS MODEL)

	$\gamma=1$	$\gamma=2$	$\gamma=3$
Marriott-NY			
Comp. Prefs. Basic	87.50	14.48	12.65
Comp. Prefs. Importance	87.50	87.49	87.42
Sum of Weights	87.38		
Trentino-10			
Comp. Prefs. Basic	87.49	16.51	13.98
Comp. Prefs. Importance	87.42	87.57	86.72
Sum of Weights	85.72		

TABLE II
THE PRUNING RATES (TRUE PREFERENCES REPRESENTED IN COMPARATIVE PREFERENCES MODEL)

	$\gamma=1$	$\gamma=2$	$\gamma=3$
Marriott-NY			
Comp. Prefs. Basic	85.77	14.28	14.28
Comp. Prefs. Importance	85.78	85.77	85.75
Sum of Weights	85.73		
Trentino-10			
Comp. Prefs. Basic	86.81	15.03	14.94
Comp. Prefs. Importance	86.81	86.79	85.93
Sum of Weights	85.15		

Importance). For example, in the *Trentino-10* part of Table I, with $\gamma = 2$, pruning goes from 16.51% Basic to 87.57% Importance. (The very slight exception to this for the Trentino-$\gamma = 1$ case is probably due to random variation in tie breaking.)

Furthermore, we see that the parameter γ, (the maximum number of variables that are associated with a node in a cp-tree), affects the degree of pruning. Specifically, as γ increases, the number of queries pruned tends to decrease. For example, in the *Marriott-NY* part of Table I, with preference statements Importance, pruning goes from 87.50% ($\gamma = 1$) to 87.49% ($\gamma = 2$) to 87.42% ($\gamma = 3$). This is a reflection of the monotonicity with respect to γ observed above (however, pruning is to do with strict dominance, which is not necessarily monotonic with respect to γ, but very often will be, because of the monotonicity of dominance.) The effect is especially marked in the Basic model where the pruning rate falls from nearly 90% to around 16.5% or less. When γ is increased from 1 to 2, many queries of the *Trade* form become undominated in the Basic model, because of the more expressive preference relations which can be represented by cp-trees with $\gamma = 2$ (allowing more than one feature to be assigned at a node). With the stronger Importance preference form, these *Trade* queries are still dominated.

What is also of concern from a practical point of view is the average length of the advice that the system gives, i.e., the number of options the user has to choose from; this is inversely related to the pruning rate. Except in the cases where pruning is very low (Basic with $\gamma = 2$ or 3), advice from the Sum of Weights recommenders is very slightly longer than it is in the case of the Comparative Preferences recommenders, being around 10 for both datasets. Dialogue lengths are very similar in the case of all recommenders: around 6 steps on average for Marriott-NY, and around 6.8 steps for Trentino-10.

Of course, it is not enough to know that one approach prunes more than another, or gives shorter advice. If it were doing so to the detriment of other factors, in particular the ability of the user to reach the best product, then the extra pruning would be of little value. We have measured the extent to which the final queries that the user reaches in a dialogue (and hence the final product that she might choose) agree across the different recommenders. Space limits preclude the inclusion of detailed results but we find that the Comparative Preferences approaches agree with the Sum of Weights approach between 91 and 99% of the time, and the more an approach prunes, the less this agreement is. For example, for Trentino-10, Basic $\gamma = 1$ agrees with Sum of Weights 92.6% of the time; this rises to 96.2% for $\gamma = 2$; and it falls to 92% for Importance $\gamma = 1$.

Since true preferences are represented in the Sum of Weights Model, we can also measure the amount by which the utility of the product that the user ultimately chooses falls short of the utility of the best product that she could have reached, normalized by the difference between the products of highest and lowest utility. Unsurprisingly, these follow a similar pattern to the percentage agreements reported in the previous paragraph. The values are very close to zero, ranging from 0 to 0.008.

When $\gamma = 1$, the time taken by the different implementations of the pruning is roughly similar—for example, around 0.2 seconds for a dialogue with the basic comparative preferences pruning for the Trentino dataset—with the sum of weights linear programming algorithm taking a little longer than the two others. The computation time increases exponentially with γ; for example, the Basic-Trentino-$\gamma = 3$ dialogue takes on average around 7.5 seconds.

Overall, for this experimental setup it seems that it is better to use the more restrictive set of models corresponding to $\gamma = 1$, at least for the Basic form, because it generates much greater pruning, leading to manageable sets of options for the user, and is computationally cheaper. However, there may be situations (e.g. other datasets) where the more cautious reasoning corresponding to $\gamma = 2$ or 3 might pay off in terms of the final quality of solutions.

B. Representing true preferences in the comparative preferences model

In our second set of experiments, the user's true preferences are set by randomly generating cp-trees over product features. The pruning rate results in this case are shown in Table II.

The results in Table II pattern in a very similar way to the ones in Table I. For example, again, in nearly all settings, the Comparative Preferences approach is pruning non-optimal queries a little more than the Sum of Weights approach; also, with the Comparative Preference Model, the amount of pruning increases as the preference statements induced become less conservative (from Basic to Importance); and, as γ increases, the number of queries pruned tends to decrease.

With users' true preferences represented as cp-trees, the pruning rate (Table II) is roughly the same as when they are represented by weight vectors (Table I), a little less for the $\gamma = 1$ case. The consequences of this for these experiments is slightly longer advice (around 13 queries on average for both datasets and in all settings except Basic with $\gamma = 2$ or 3, where pruning is very low) and shorter dialogues (around 3.7 steps for Marriott-NY and around 3.9 steps for Trentino-10). Furthermore, in every dialogue, the product that the user ultimately chose was the optimal product for her true preferences (whereas we saw very small shortfalls in utility in the experiment described in the previous section). The dialogue involves features being added incrementally to the empty initial query—with the most important (according to the true cp-tree) first—until the optimal product is reached.

We were interested to see whether mis-matches between the ways in which true preferences and induced preferences are represented would have any effect. One might expect if induced preferences are represented in the same model as the true preferences, then they can more accurately capture the true preferences, resulting in greater pruning. But we are not seeing this in our experiments. We are seeing that, for all but a few settings, irrespective of the way in which true preferences are represented, using the Comparative Preferences approach for induced preferences results in slightly greater pruning. This is a useful result: it gives us confidence that the advantage that the Comparative Preferences approach enjoys is not an artefact of the way we are running the experiments.

VIII. Conclusions

There has been a lot of excellent theoretical work produced on comparative preference formalisms in recent years, for example, the award winning papers [9], [16]; however, development towards applications has been lagging somewhat. A major contribution of this paper is to show how a comparative preferences approach can be adapted for a conversational recommender system. For this kind of application it is important that the preference language allows the expression of direct comparisons between configurations (that one configuration is preferred to another); and also that the inference technique is both efficient and strong enough in its inferences (or else the pruning of possibilities is too weak). The language, inference method and algorithm described in [10] fit these requirements. We have shown that it is possible to implement this inference method so that it is efficient in a practical (rather than theoretical) sense, as part of a form of conversational recommender system. We have tested our method on datasets involving real hotel data, and shown that it leads to strong pruning of possibilities, but without eliminating the best options, even when ‘best’ is defined based on a different semantics (based on a sum of weights of included features).

An attractive feature of the comparative preferences approach is its expressiveness. In the current work, it has allowed us to choose between differently nuanced induced preferences. In future work, it would also allow general conditional preference statements to be expressed by the user, such as *If the hotel is not in the city centre, then I’d like there to be an on-site restaurant*. Such statements can further strengthen the pruning capability, and could potentially be re-used for different searches.

In future work too, we will extend these approaches for other kinds of recommender systems, including for non-boolean features, and for configurable products, where the set of possibilities is expressed implicitly as the solutions of a Constraint Satisfaction Problem. We will also consider other inference procedures. For instance, the weighted sum model can be easily extended to a sum of functions of more than one variable (a GAI representation [17]).

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