Part 5: Scoring, Term Weighting and the Vector Space Model

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Most of these slides comes from the course:

Information Retrieval and Web Search,

Christopher Manning and Prabhakar Raghavan

Content

- Ranked retrieval
- Scoring documents
- Term frequency (in each document)
- Collection statistics
- tf-idf
- Weighting schemes
- Vector space scoring

Boolean retrieval

- Thus far, our queries have all been Boolean:
  - documents either **match** or **don't**
- Good for expert users with precise understanding of their needs and the collection
- Also good for applications: applications can easily consume 1000s of results
- **Not good for the majority of users**
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work)
  - Most users **don't want to wade through 1000s of results**
    - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in **either too few** (=0) **or too many** (1000s) **results**
- Query 1: "*standard user dlink 650*" → 200,000 hits
- Query 2: "*standard user dlink 650 no card found*": 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits
- AND gives too few; OR gives too many.

Ranked retrieval models

- Rather than a **set** of documents satisfying a query expression, in **ranked retrieval models**, the system returns an **ordering** over the (top) documents in the collection with respect to a query
- **Free text queries:** Rather than a query language of **operators** and **expressions**, the user's query **is just one or more words** in a human language
- In principle, there are two separate choices here – the **query language** and the **retrieval model** - but in practice, ranked retrieval models have normally been associated with free text queries.

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, **large result sets are not an issue**
  - Indeed, the size of the result set is not an issue
  - We just show the top k (≈ 10) results
  - We don’t overwhelm the user
  - ***Premise: the ranking algorithm works***

Do you really agree with that?

Scoring as the basis of ranked retrieval

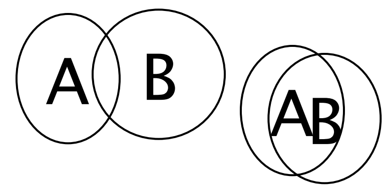
- We wish to return in **order** the documents most **likely to be useful** to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/ document pair
- Let's start with a one-term query
- If the query term does not occur in the document:
  - The score should be 0
  - *Why? Can we do better?*
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this.

Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets *A* and *B*
- jaccard$(A,B) = |A \cap B| / |A \cup B|$
- jaccard$(A,A) = 1$
- jaccard$(A,B) = 0$ if $A \cap B = 0$
- *A* and *B* don't have to be the same size
- Always assigns a number between 0 and 1
- We saw that in the context of k-gram overlap between two words.



Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

- jaccard$(Q,D) = |Q \cap D| \,/\, |Q \cup D|$

- *jaccard(Query, Document1) = 1/6*
- *jaccard(Query, Document2) = 1/5*

Issues with Jaccard for scoring

- Match score decreases as document length grows
- We need a more sophisticated way of normalizing for length
- Later in this lecture, we'll use $|A \bigcap B| / \sqrt{|A \bigcup B|}$
  - . . . instead of |A ∩ B|/|A ∪ B| (Jaccard) for length normalization.
- 1) It doesn't consider *term frequency* (how many times a term occurs in a document)
  - For J.C. documents are **set** of words not **bag** of words
- 2) Rare terms in a collection are more informative than frequent terms - Jaccard doesn't consider this information.

Recall (Part 2): Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$.

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a count vector in $\mathbb{N}^V$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	1
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words **model**

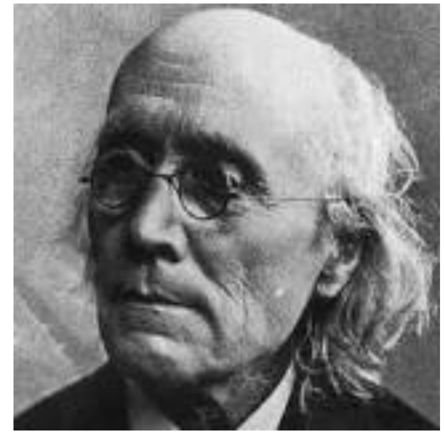
- Vector representation doesn't consider the **ordering** of words in a document
- *"John is quicker than Mary"* and *"Mary is quicker than John"* have the same vectors
- This is called the bag of words model
- In a sense, this is a step back: the positional index was able to distinguish these two documents
- We will look at "recovering" positional information later in this course
- For now: bag of words model.

Term frequency *tf*

- The term frequency $tf_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*
- We want to use tf when computing query-document match scores - but how?
- *Raw term frequency is not what we want:*
  - A document with 10 occurrences of the term **is more relevant** than a document with 1 occurrence of the term
  - **But not 10 times** more relevant
- Relevance does not increase proportionally with term frequency.

frequency in IR = count

Fechner's Project



- Gustav Fechner (1801 - 1887) was obsessed with the relation of mind and matter
- **Variations of a physical quantity** (e.g. energy of light) cause variations in the intensity or quality of the **subjective experience**
- Fechner proposed that for many dimensions the function is **logarithmic**
  - An increase of stimulus intensity by a given **factor** (say 10 times) always yields the same **increment** on the psychological scale
- *If raising the frequency of a term from 10 to 100 increases relevance by 1 then raising the frequency from 100 to 1000 also increases relevance by 1.*

Log-frequency weighting

- The log frequency weight of term *t* in *d* is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.
- Score for a document-query pair: sum over terms *t* in both *q* and *d*:

$$score(d,q) = \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$$

- The score is 0 if none of the query terms is present in the document
- If q' ⊆ q then score(d,q') <= score(d,q) – is this a problem?

Normal vs. Sublinear tf scaling

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- The above formula defined the **sublinear tf-scaling**
- The simplest approach (**normal**) is to use the number of occurrences of the term in the document (frequency)
- But as discussed earlier sublinear tf should be better.

Properties of the Logarithms

- $y = \log_a x$ iff $x = a^y$
- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a (xy) = \log_a x + \log_a y$
- $\log_a (x/y) = \log_a x - \log_a y$
- $\log_a (x^b) = b \log_a x$
- $\log_b x = \log_a x / \log_a b$
- $\log x$ is typically $\log_{10} x$
- $\ln x$ is typically $\log_e x$ (e = 2.7182... Napier or Euler number) – Natural logarithm.

Document frequency

- **Rare terms** – in the whole collection - are **more informative** than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the (information need originating the) query *arachnocentric*
- → We want a high weight for rare terms like *arachnocentric*.

Document frequency, *cont'd*

- Generally frequent terms are **less informative** than rare terms
- Consider a **query** term that is frequent in the collection (e.g., *high*, *increase*, *line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- **But** consider a query containing two terms – e.g.: high *arachnocentric*
- For a **frequent** term in a document, s.a., high, we want a positive weight but **lower** than for terms that are **rare** in the collection, s.a., *arachnocentric*
- We will use **document frequency** (df) to capture this.

http://www.wordfrequency.info

idf weight

- df_t is the document frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t *(the smaller the better)*
  - $df_t \leq N$
- We define the idf (**inverse document frequency**) of t by

$$\text{idf}_t = \log(N/\text{df}_t)$$

Is a function of t only – does not depend on the document

  - We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

idf example, suppose *N* = 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\mathrm{idf}_t = \log\left(N/\mathrm{df}_t\right)$$

There is one idf value for each term *t* in a collection.

Collection vs. Document frequency

- The *collection frequency* of *t* is the number of *occurrences* of *t* in the collection, counting multiple occurrences in the same document
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is (in general) a better search term (and should get a higher weight in a query like "try insurance")?

tf-idf weighting

- The **tf-idf** weight of a term is the **product** of its **tf weight** and its **idf weight**:

$$\mathrm{w}_{t,d} = (1 + \log \mathrm{tf}_{t,d}) \times \log(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf

1. **Increases** with the **number of occurrences within a document**
2. **Increases** with the **rarity of the term in the collection.**

Final ranking of documents for a query

$$\text{Score}(d,q) = \sum_{t \in q \cap d} \text{tf}_{\text{t,d}} \times \text{idf}_{t}$$

We will see some other options for computing the score ...

Effect of idf on ranking

- Can idf have an effect on ranking for one-term queries? E.g. like:
  - iPhone
- idf has no effect on ranking one term queries - since there is one idf value for each term in a collection
  - idf affects the ranking of documents for queries with at least two terms
  - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

Binary → count → weight matrix

dimensions

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

- So we have a |V|-dimensional vector space
  - **Terms** are **axes** of the space
  - **Documents** are **points** or vectors in this space
- Very high-dimensional:
  - 400,000 in RCV1
  - tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.

Queries as vectors

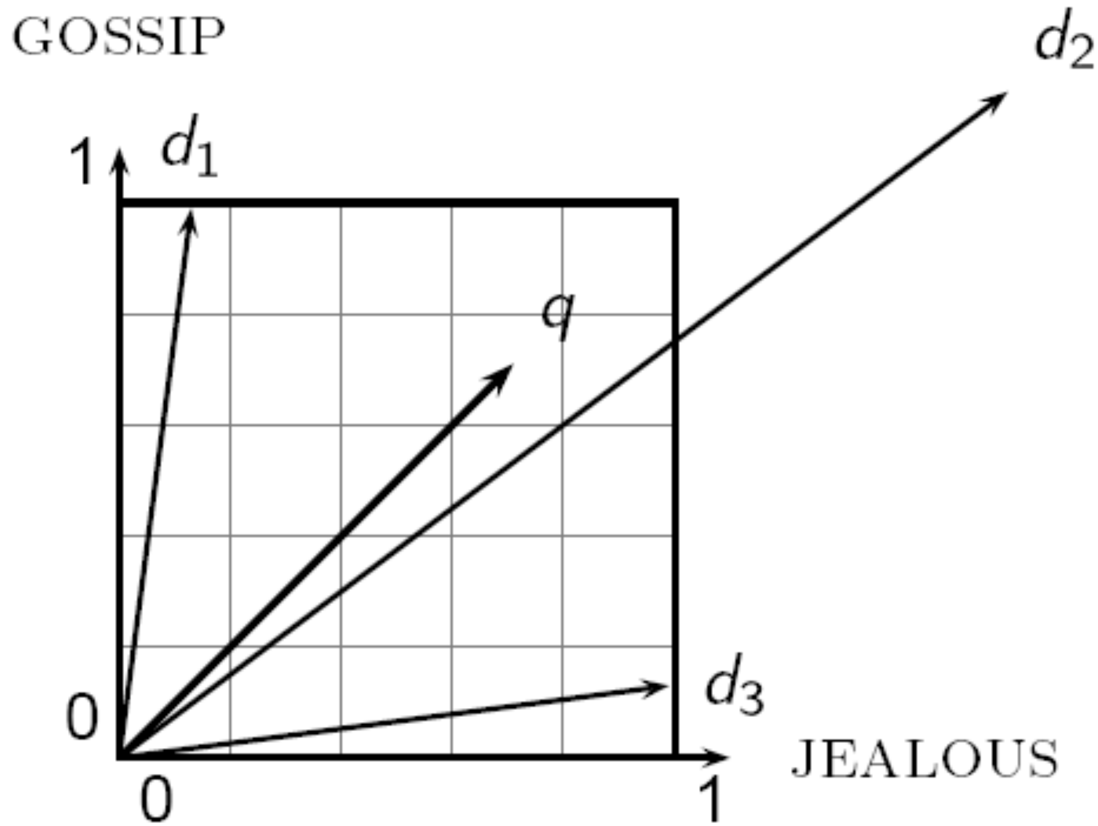
- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model
- Instead: rank more relevant documents higher than less relevant documents.

Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

The Euclidean distance between *q* and d_2 is large even though the distribution of terms in the query *q* and the distribution of terms in the document d_2 are very similar.



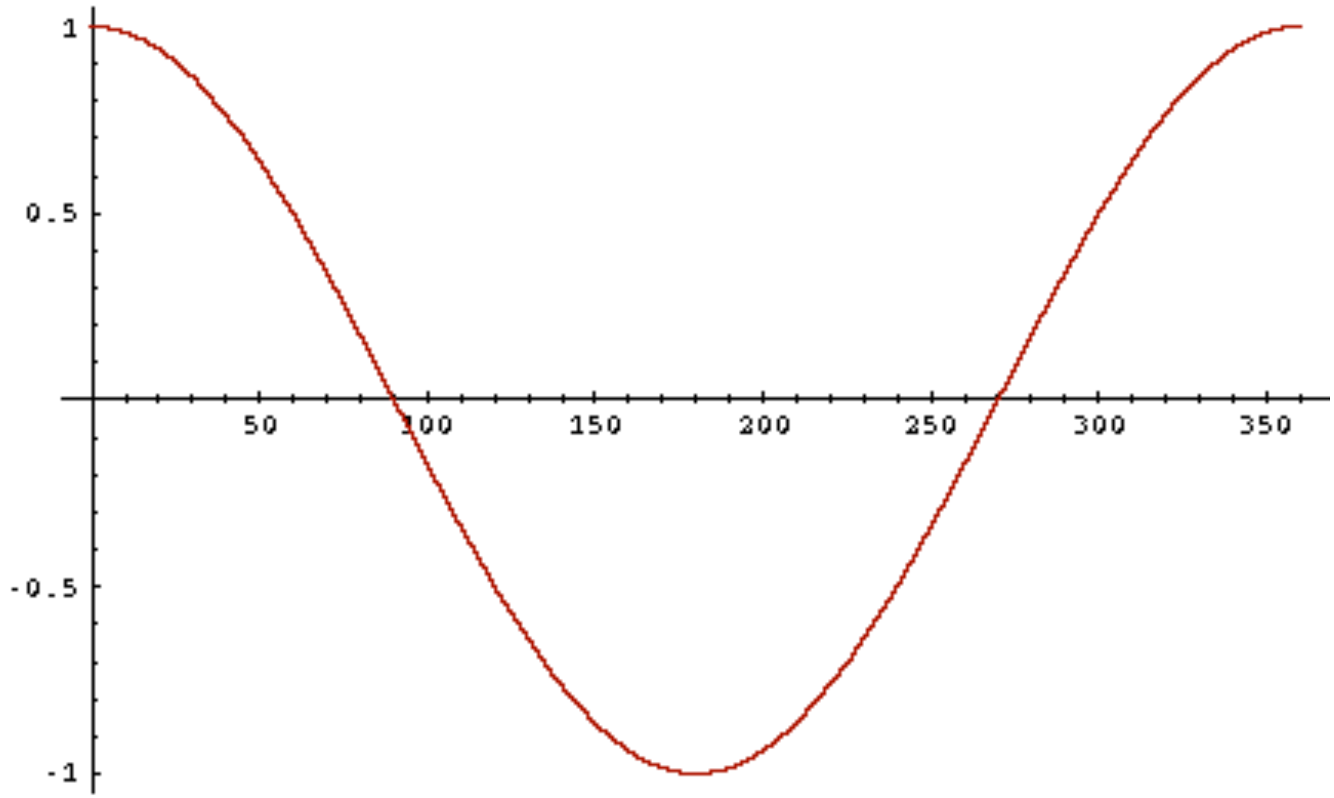
Use angle instead of distance

- Thought experiment: take a document *d* and append it to itself
- Call this document *d′*
- "Semantically" d and d′ have the same content
- The Euclidean distance between the two documents can be quite large
- If the $tf_{t,d}$ representation is used then the angle between the two documents is 0, corresponding to maximal similarity
- Key idea: Rank documents according to angle with query.

From angles to cosines

- The following two notions are equivalent:
  - Rank documents in increasing order of the angle between query and document
  - Rank documents in decreasing order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0^{o}, 180^{o}]

From angles to cosines



- But how – *and why* – should we be computing cosines?

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\|\vec{x}\|_2 = |\vec{x}| = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization
  - Long and short documents now have comparable weights.

cosine(query,document)

Dot product

Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term *i* in the query
d_i is the tf-idf weight of term *i* in the document

$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

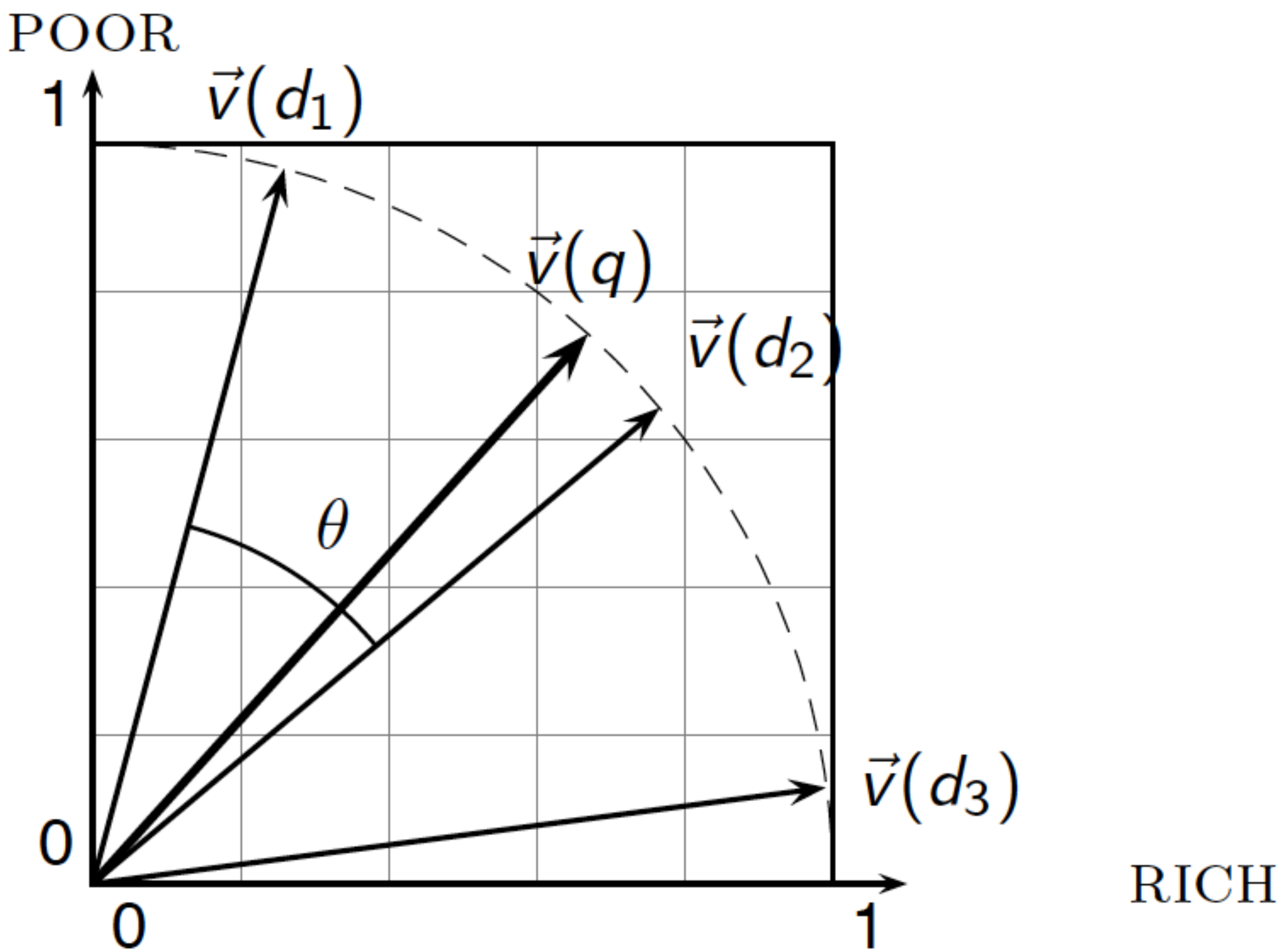
Cosine for length-normalized vectors

- For **length-normalized** vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for q, d length-normalized.

Cosine similarity illustrated



Cosine similarity amongst 3 documents

How similar are the novels
SaS: *Sense and Sensibility*
PaP: *Pride and Prejudice*, and
WH: *Wuthering Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈

$0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0$

≈ 0.94

cos(SaS,WH) ≈ 0.79

cos(PaP,WH) ≈ 0.69

Why do we have cos(SaS,PaP) > cos(SaS,WH)?

Computing cosine scores

```
CosineScore(q)
float Scores[N] = 0
float Length[N]
for each query term t
do calculate w_{t,q} and fetch postings list for t
    for each pair(d, tf_{t,d}) in postings list
    do Scores[d] += w_{t,d} × w_{t,q}
Read the array Length
for each d
do Scores[d] = Scores[d]/Length[d]
return Top K components of Scores[]
```

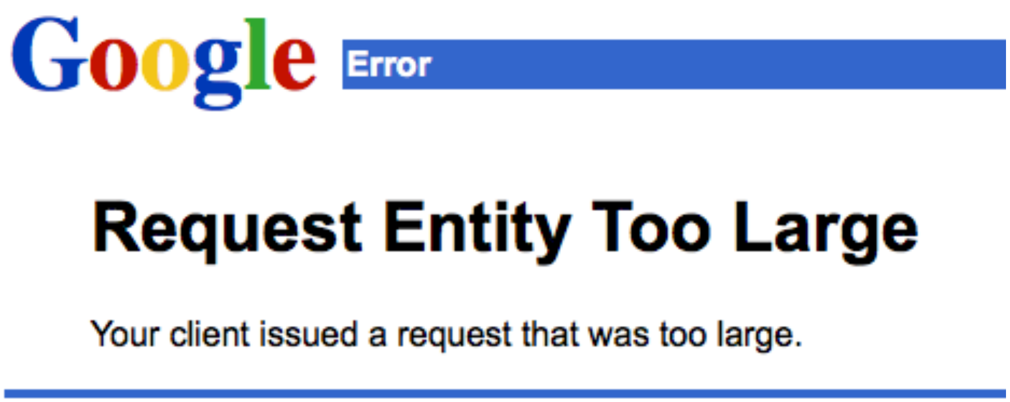
# of documents

This array contains the vector lengths of all the documents

It is not computing the cosine because the score is not divided by the query vector length

Observations

- The inverted index – used for Boolean queries – is still important for retrieving top scoring documents
- It is not feasible to scan all the documents to compute the top K documents (more similar) for a given input query
- Queries are “normally” short documents



Try with a long query!

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1 + \log(\mathrm{tf}_{t,d})}{1 + \log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

# of chars in the document

Columns headed 'n' are acronyms for weight schemes.

Why is the base of the log in idf immaterial?

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation *ddd.qqq*, using the acronyms from the previous table
- A very standard weighting scheme is: lnc.ltc
- Document: logarithmic tf (l as first character), no idf, and cosine normalization

A bad idea?

- Query: logarithmic tf (l in leftmost column), idf (t in second column), cosine normalization …

tf-idf example: lnc.ltc

ddd.qqq

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query						Document				Prod
	tf-raw	tf-wt	df	idf	wt	n'lize	tf-raw	tf-wt	wt	n'lize	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Exercise: what is the value of *N*, i.e., the number of docs?

Doc length $=\sqrt{1^2+0^2+1^2+1.3^2}\approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., $K = 10$) to the user.

Reading Material

- Sections: 6.2, 6.3, 6.4 (excluded 6.4.4)