$\mathcal{E}$-CONNECTIONS AND LOGICS OF DISTANCE
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Abstract

This thesis intends to contribute to two different strands of logic-based Artificial Intelligence (AI) research.

In Part 1 of the thesis, we investigate logics of distance spaces: a family of knowledge representation formalisms aimed to bring a numerical, quantitative concept of distance into the conventional qualitative representation and reasoning.

Part 2 of the thesis is concerned with the study of $\mathcal{E}$-connections: a combination methodology for logical formalism that is widely applicable, and which is very well-behaved computationally.

The two sections that follow contain brief descriptions of the contents of (the two parts of) this thesis.

Logics of Distance. In Part 1, we systematically investigate first-order, modal, and Boolean modal languages intended for reasoning about distances, where 'distance' is understood in a wide, not necessarily spatial sense. The structures in which these languages are interpreted are metric spaces, or more general classes of distance spaces satisfying only a subset of the conditions of metric spaces.

Chapter 1 introduces the first-order languages $\mathcal{LF}[M]$, their two-variable fragments $\mathcal{LF}_2[M]$, as well as a family $\mathcal{LO}_O[M]$ of modal languages parametrised by sets O of distance operators being primitive in the language, and by parameter sets M of subsets of the reals, i.e., the distances that formulae can explicitly refer to. Over different classes of distance spaces, we compare the expressive power of the first-order languages with the modal distance languages, as well as with a variant $\mathcal{LOB}[M]$ of Boolean modal languages. It is shown that the modal language $\mathcal{LO}[M]$ is expressively complete over metric spaces for the two-variable fragment $\mathcal{LF}_2[M]$.

Chapter 2 investigates the computational behaviour of these languages. We show the two-variable fragment of first-order logic to be undecidable when interpreted in metric spaces, and single out an expressive and decidable fragment $\mathcal{LO}_D[M]$ of $\mathcal{LO}[M]$ that has the finite model property.

In Chapter 3, we study logical properties of the modal distance logics introduced. We give complete axiomatisations of modal distance logics, amongst them the counterpart of two-variable first-order distance logic interpreted in metric spaces, and discuss compactness, and the (mostly failing) interpolation property.

$\mathcal{E}$**-Connections.** Part 2 of the thesis is concerned with a new combination technique for logics, called $\mathcal{E}$-connections.

In Chapter 4, we introduce abstract description systems as a framework for studying combinations of logics, introduce the methodology of $\mathcal{E}$-connections, and provide a number of examples.

Chapter 5 studies the computational behaviour of $\mathcal{E}$-connections. It is shown that, unlike for instance products of logics, $\mathcal{E}$-connections exhibit an extremely stable computational behaviour: the basic $\mathcal{E}$-connection of any number of decidable logical systems is again decidable. This result can be refined to show that $\mathcal{E}$-connections of certain subclasses of decidable logics remain decidable, even if the interaction between the component logics is enriched in various ways.

In Chapter 6, we begin by comparing $\mathcal{E}$-connections with the related combination methodology of distributed description logics (DDLs). We show that DDLs can be understood as a special case of $\mathcal{E}$-connections. We then briefly start an investigation into the expressiveness of $\mathcal{E}$-connections by lifting the concept of bisimulations to $\mathcal{E}$-connections, and apply this theory to show that certain properties are not definable by basic $\mathcal{E}$-connections. It is shown, however, that such undefinable properties can be added as 'first-order constraints' to $\mathcal{E}$-connections in a way that, again, preserves decidability.

Finally, we discuss the relationship between $\mathcal{E}$-connections and other combination methodologies such as multi-dimensional products of logics, independent fusions, fibrings, and description logics with concrete domains.