4. Containment and Minimization of Conjunctive Queries

Instructions: Work in groups of 2 students. You can write up your answers by hand (provided your handwriting is legible) or use a word processing system like Latex or Word. However, experience shows that Word is in general difficult to use for this kind of task. Please, include name and email address in your submission.

1. Conjunctive Queries with Disequalities

A disequality is an atom of the form "$s \neq t$". The class of conjunctive queries with disequalities consists of the conjunctive queries with the property that all their built-in atoms are disequalities.

Question: How difficult is to decide containment of conjunctive queries with disequalities?

Hint 1: First, find a characterizing property of containment for this class of queries. Then use this property to establish an upper complexity bound for the problem.

Hint 2: Give a reduction that shows the hardness. Make the reduction plausible without providing a formal proof. (It is highly likely that you can reuse ideas from the analysis of the containment problem for conjunctive queries with comparisons in the lecture slides.)

(15 Points)

2. Injective and Surjective Mappings

For the proofs in Exercise 3, it will be necessary to make use of some facts about injective and surjective mappings on finite sets. The purpose of this exercise is to review those facts.

Recall that a mapping $f\colon X \to Y$ from a set X to a set Y is *injective* if for all x, $x' \in X$ we have that $f(x) = f(x')$ implies that $x = x'$. In other words, an injective mapping maps any two distinct elements of X to distinct elements of Y.

Recall as well that a mapping $f\colon X \to Y$ is called *surjective* if for every $y \in Y$ there exists some $x \in X$ such that $f(x) = y$. In other words, every element y of Y has a preimage in X with respect to f.

Recall as well that for mappings $f\colon X \to Y$ and $g\colon Y \to Z$ the *composition* $g \circ f$ is a mapping from X to Z defined as $(g \circ f)(x) = g(f(x))$.

1. Prove that if $f\colon X \to Y$ and $g\colon Y \to Z$ are injective, then $g \circ f$ is injective.
2. Prove that if $f\colon X \to Y$ and $g\colon Y \to Z$ are surjective, then $g \circ f$ is surjective.
3. If the composition $g \circ f$ is injective, what can you conclude about g and f? Prove your answer.
4. Similarly, if the composition $g \circ f$ is surjective, what can you conclude about g and f? Prove your answer.
5. Suppose X is a finite set and $f\colon X \to X$ is injective. What can you conclude about f? Prove your answer.
6. Similarly, suppose X is a finite set and $f\colon X \to X$ is surjective. What can you conclude about f? Prove your answer.

(3 Points)

3. Minimisation of Conjunctive Queries

Recall that *relational conjunctive queries* (RCQs) are conjunctive queries without equalities and inequalities. Recall as well that a conjunctive query Q_0 is a *subquery* of another conjunctive query Q if Q_0 can be obtained from Q by dropping some of the atoms in the body of Q.

Prove the following two propositions that provide the underpinnings for the algorithm of conjunctive query minimization.

Proposition 1: Let Q be a RCQ with n atoms and Q' be an equivalent RCQ with m atoms where $m < n$. Then there exists a subquery Q_0 of Q such that Q_0 has at most m atoms in the body and Q_0 is equivalent to Q.

(6 Points)

Proposition 2: Let Q and Q' be two equivalent minimal RCQs. Then Q and Q' are identical up to renaming of variables.

(6 Points)

Submission: 30 May 2013, 10:30 am, by email